1 Optimal interpolation and identification of sinusoidal functions

Interpolation. An unknown sinusoidal signal has been observed at some given points in time $t_1, t_2, \ldots, t_n$. The corresponding observed values are $y_1, y_2, \ldots, y_n$. Assuming $y(t) = A \sin \omega t + \phi + e_t$, being e_t a small error, one wants to find A, ω and ϕ so that $z = \sum_{i=1}^{n} e_i^2$ is minimum.

The search can be done by invoking a NLP solver, when suitabnle bounds for the parameters are known. Bounding A is not needed, because z is convex in A when ω and ϕ are fixed. Bounding ϕ is trivial, because $-\pi \leq \phi \leq \pi$. Bounding ω is more challenging and it can be done by enumerating a disccrete set of feasible intervals. For easiness of notation, we use $T = \frac{2\pi}{\omega}$. Let call δ_{ij} the duration of time interval between two observations i and $j > i$, i.e. $t_j - t_i$. Let call q_{ij} the number of intersections between $y_t)$ and the line $y = 0$ in the time interval between t_i and t_j.

The following properties hold.

Property 1. $y_i y_j > 0 \;\Leftrightarrow\; q_{ij} \bmod 2 = 0$.

Property 2. $\delta_{ij} \geq \delta_{uv} \;\Rightarrow\; q_{ij} \geq q_{uv} - 1$.

Property 3. $(t_i \leq t_j \leq t_k) \wedge (0 \leq y_i \geq y_j \leq y_k \geq 0) \;\rightarrow\; q_{ik} \geq 2$.

With these properties one can enumerate the feasible values of $q_{i\ i+1}\ \forall i = 1, \ldots, n-1$. From such values, feasible ranges for T can be obtained, using the following property.

Property 4. $\frac{\delta_{ij}}{q_{ij}+1} \leq \frac{T}{2} \leq \frac{\delta_{ij}}{q_{ij}-1}\ \forall i, j$.

A fast enumeration scheme is needed to enumerate the feasible intervals for T, exploiting the above properties. Once the set of feasible intervals for T has been computed, a NLP solver can be called for each interval.

Identification. Assume now that S sinusoidal functions have been observed for each given point in time t_i. One has to decide which observation refers to each function. The goal is to find the assignments that minimize the squared errors using large enough values of the periods or equivalently maximizes the periods obtaining small enough interpolation errors.

A dynamic programming or branch-and-bound algorithm is needed for this purpose.

Il lavoro è adatto ad una tesi triennale. Una tesi triennale è già stata svolta su questo argomento, ma il lavoro va completato.