Problem P_0 (root problem).

$$\begin{aligned} \text{minimize } z =& 4x_1 + 8x_2 + 9x_3 + 3x_4 + 4x_5 + 10x_6 \\ \text{subject to } & 4x_1 - 5x_2 - 3x_3 - 2x_4 - x_5 + 8x_6 \leq -8 \\ & -5x_1 + 2x_2 + 9x_3 + 8x_4 - 3x_5 + 8x_6 \leq 7 \\ & 8x_1 + 5x_2 - 4x_3 \qquad + x_5 + 6x_6 \leq 6 \end{aligned}$$

$N^0 = N^1 = \emptyset$; $N^{free} = \{1, 2, 3, 4, 5, 6\}$.

$r^T = [-8\ 7\ 6]$. The 0-completion is not feasible.

$V = \{1\}$, $A = \{1, 6\}$, $R = \{2, 3, 4, 5\}$.

$t^T = [-11\ \ -8\ \ -4]$. No constraint is infeasible.

$z^0 = 0$, but no upper bound is known: $\overline{z} = \infty$.

$t_1 + a_{11} > r_1$: we fix $x_1 = 0$.
$t_1 + a_{16} > r_1$: we fix $x_6 = 0$.
$t_1 - a_{12} > r_1$: we can fix $x_2 = 1$, but assume we do not realize it!

It is necessary to branch.

	1	2	3	tot.
x_2	-3	5	1	-3
x_3	-5	-2	10	-7
x_4	-6	-1	6	-7
x_5	-7	10	5	-7

Branching variable: x_2.

Problem P_1.

$$\begin{aligned} \text{minimize } z =& 8 + 9x_3 + 3x_4 + 4x_5 \\ \text{subject to } & -3x_3 - 2x_4 - x_5 \leq -3 \\ & 9x_3 + 8x_4 - 3x_5 \leq 5 \\ & -4x_3 \qquad + x_5 \leq 1 \end{aligned}$$

$N^0 = \{1, 6\}$; $N^1 = \{2\}$; $N^{free} = \{3, 4, 5\}$.

$r^T = [-3\ 5\ 1]$. The 0-completion is not feasible.

$V = \{1\}$, $A = \emptyset$, $R = \{3, 4, 5\}$.

$t^T = [-6\ \ -3\ \ -4]$. No constraint is infeasible.

$z^0 = 8$, but no upper bound is known: $\overline{z} = \infty$.

$t_2 + a_{23} > r_2$: we fix $x_3 = 0$.

$$
\begin{aligned}
\text{minimize } z =& 8 + 3x_4 + 4x_5 \\
\text{subject to } & -2x_4 - x_5 \leq -3 \\
& 8x_4 - 3x_5 \leq 5 \\
& + x_5 \leq 1
\end{aligned}
$$

Constraint 3 is redundant.

$t_1 = r_1 = -3$: we fix $x_4 = 1$ and $x_5 = 1$.

All constraints are satisfied. An upper bound has been found: $x^T = [0\ 1\ 0\ 1\ 1\ 0]$. $\overline{z} = 15$.

It is not necessary to branch.

Problem P_2.

$$
\begin{aligned}
\text{minimize } z =& 9x_3 + 3x_4 + 4x_5 \\
\text{subject to } & -3x_3 - 2x_4 - x_5 \leq -8 \\
& 9x_3 + 8x_4 - 3x_5 \leq 7 \\
& -4x_3 \qquad + x_5 \leq 6
\end{aligned}
$$

$N^0 = \{1, 2, 6\}$; $N^1 = \emptyset$; $N^{free} = \{3, 4, 5\}$.

$r^T = [-8\ 7\ 6]$. The 0-completion is not feasible.

$V = \{1\}$, $A = \emptyset$, $R = \{3, 4, 5\}$.

$t^T = [-6\ \ -3\ \ -4]$. Constraint 1 is infeasible.

It is not necessary to branch: the node is fathomed.

The whole tree has been explored. The optimal solution is $x^* = [0\ 1\ 0\ 1\ 1\ 0]^T$ with cost $z^* = 15$.