Planning models

Logistics

Giovanni Righini

Università degli Studi
di Milano

Planning

Production planning is a typical operation at a tactical decision level in production logistics.

Decide the amount of products to be produced on a medium term horizon (one week - one year).

Different levels of planning can be nested for different time periods, from longer to shorter term plans.

Production planning may involve:

- a single product or multiple products
- a single period or multiple periods
- fixed and variable production costs
- constraints on the lot size
- constraints on resource availability
- deadlines or due dates.

Objectives:

- minimize the production cost;
- maximize the expected profit.

The optimal production mix problem

The problem assumes a set P of products and a single period.
Production requires a set R of resources.
For each product $j \in P$ an expected unit profit c_j is known.
For each resource $iinR$ an available amount b_i is known.
A given technological coefficient a_{ij} indicates the amount of each resource needed to produce a unit of each product.
The objective is to maximize the total expected profit.

A (continuous or integer) variable x_j indicates the production level for each product $j \in P$.

$$\begin{aligned} \text{maximize } z = \sum_{j \in N} c_j x_j & \\ \text{subject to } \sum_{j \in N} a_{ij} x_j \leq b_i & \qquad \forall i \in R \\ x_j \geq 0 \text{ (integer)} & \qquad \forall j \in P \end{aligned}$$

Multi-period planning

When several consecutive periods are considered, the common assumption is that a warehouse of limited capacity is available to stock an amount of products that can be distributed or sold later.

Flow conservation constraints are needed to ensure the consistency of produced and stored amounts in consecutive periods.

$$x_t + s_{t-1} = d_t + s_t$$

where

- $x_t \geq 0$ indicates the production in period t;
- $s_t \geq 0$ indicates the amount stored in the warehouse at the end of period t;
- d_t indicates the demand satisfied in period t.

The lot-sizing problem

The problem assumes a single product and a set T of potential production periods.

A demand d_t is known for each period $t \in T$.

A fixed production cost f_t and a variable production cost c_t are given for each period $t \in T$.

A maximum production capacity q_t for each period and a warehouse capacity Q are also given.

A unit inventory cost h_t is known for each period $t \in T$.

The objective is to satisfy the demand at minimum total cost.

The lot-sizing problem

A binary variable y_t indicates whether period t is used or not.
A continuous or integer variable $x_t \geq 0$ indicates the production in period t.
A variable $s_t \geq 0$ indicates the amount stored at the end of period t.

$$
\begin{aligned}
\text{minimize } z = \sum_{t \in T} (f_t y_t + c_t x_t + h_t s_t) & \\
\text{subject to } x_t \leq q_t y_t & \qquad \forall t \in T \\
s_{t-1} + x_t = d_t + s_t & \qquad \forall t \in T : t > 1 \\
s_0 + x_1 = d_1 + s_1 & \\
x_t \geq 0 \text{ (integer)} & \qquad \forall t \in T \\
s_t \geq 0 & \qquad \forall t \in T \\
y_t \in \{0, 1\} & \qquad \forall t \in T
\end{aligned}
$$

Assignment problems

At a more detailed decision level, production lots must be assigned to production units or teams.

This originates assignment problems between a set N of tasks/jobs and a set M of agents/machines.

The decision is typically represented by binary assignment variables:

$$x_{ij} = \begin{cases} 0 & \text{if task } i \in N \text{ is not assigned to machine } j \in M \\ 1 & \text{if task } i \in N \text{ is assigned to machine } j \in M \end{cases}$$

Assignment costs/times are given for each possible assignment (i, j) between a task $i \in N$ and a machine $j \in M$.

The linear assignment problem

A set N of jobs and a set M of agents are given, with $|N| = |M|$.
An assignment cost c_{ij} is given for each $i \in N$ and $j \in M$.
Assign a job to each agent and an agent to each job, minimizing the total assignment cost.

Binary variables x_{ij} are used to represent the selected assignments.

$$
\begin{aligned}
\text{minimize } z = & \sum_{i \in N, j \in M} c_{ij} x_{ij} & \\
\text{subject to } & \sum_{i \in N} x_{ij} = 1 & \forall j \in M \\
& \sum_{j \in M} x_{ij} = 1 & \forall i \in N \\
& x_{ij} \in \{0, 1\} & \forall i \in N, \forall j \in M
\end{aligned}
$$

The generalized assignment problem

A set N of jobs and a set M of agents are given.
An assignment cost c_{ij} and a resource consumption a_{ij} are given for each $i \in N$ and $j \in M$.
A capacity (amount of available resource) b_j is given for each machine $j \in M$.
Assign jobs to machines, minimizing the total assignment cost and complying with capacity constraints.

Binary variables x_{ij} are used to represent the selected assignments.

$$
\begin{aligned}
\text{minimize } z = & \sum_{i \in N, j \in M} c_{ij} x_{ij} & \\
\text{subject to } & \sum_{i \in N} a_{ij} x_{ij} \le b_j & \forall j \in M \\
& \sum_{j \in M} x_{ij} = 1 & \forall i \in N \\
& x_{ij} \in \{0, 1\} & \forall i \in N, \forall j \in M
\end{aligned}
$$