Long-distance transportation: examples

Logistics

Giovanni Righini

Università degli Studi
di Milano

Renting vehicles

For a company that owns a private fleet of vehicles as well as for a carrier, a strategic level decision consists in deciding the right size of its fleet, assuming the demand can vary.

Assume all vehicles are identical.
Assume the forecasted demand is variable but periodic, and assume there are n time slots (e.g. weeks of the year) in each period.

- The number of vehicles required in each time slot $t = 1, \dots, n$ is v_t.
- c_f and c_v are the fixed and variable costs for each owned vehicle.
- c_h is the cost for renting a vehicle for a time slot ($c_h > c_f + c_v$).

Renting vehicles

An integer variable x indicates the number of owned vehicles.
An integer variable y_t indicates the number of rented vehicles for each time slot $t = 1, \dots, n$.
An integer variable u_t indicates the number of owned vehicles actually used in each time slot $t = 1, \dots, n$.

Ths cost is given by

$$z = nc_f x + \sum_{t=1}^{n} c_v u_t + \sum_{t=1}^{n} c_h y_t$$

Constraints:

$$\begin{aligned}
& u_t + y_t = v_t \quad \forall t = 1, \dots, n \\
& u_t \leq x \quad \forall t = 1, \dots, n \\
& x \in \mathbb{Z}_+ \\
& u_t \geq 0 \quad \forall t = 1, \dots, n \\
& y_t \geq 0 \quad \forall t = 1, \dots, n
\end{aligned}$$

Flow problems

Long-distance transportation can be often modelled by flow networks.

A flow network is an oriented graph (or *digraph*) $D = (N, A)$, with a set of nodes N and a set of arcs A.

Nodes can be either origins, or destinations or transshipment nodes.

Origins (set O) have an excess e of flow, that leaves the nodes.
Destinations (set D) have a deficit d of flow, that enters the nodes.
Transshipment nodes (set T) are traversed by the flow.

Flow balance constraints:

$$\sum_{i \in N} x_{ij} - \sum_{k \in N} x_{jk} = \begin{cases} -e_j & \forall j \in O \\ 0 & \forall j \in T \\ d_j & \forall j \in D \end{cases}$$

Flow problems

Arcs may have an associated capacity u_{ij} such that

$$0 \leq x_{ij} \leq u_{ij}$$

are the capacity constraints.

Nodes may also have capacities, originating constraints like

$$0 \leq \sum_{i in N} x_{ij} \leq u_j$$

Usually unit costs c_{ij} are associated with the arcs, so that the cost of the flow is

$$z = \sum_{(i,j) \in A} c_{ij} x_{ij}.$$

Fixed costs may be associated with the use of some arcs (Fixed-charge network flow models).

Flow variables x_{ij} can be continuous or discrete.
Flow problems can be single-commodity or multi-commodity.

Complexity

From a computational complexity point of view, max-flow and min-cost-flow problems are easy to solve, because they translate into linear programming models.

When binary variables are introduced, as in fixed-charge network flow models, then the problems are *NP*-hard (integer linear programming) and therefore (very) difficult to solve.