# A network flow model of the Northern Italy waterway system

# **Giovanni Righini**

## EURO Journal on Transportation and Logistics

ISSN 2192-4376 Volume 5 Number 2

EURO J Transp Logist (2016) 5:99-122 DOI 10.1007/s13676-014-0068-y





Your article is protected by copyright and all rights are held exclusively by Springer-Verlag Berlin Heidelberg and EURO - The Association of European Operational Research Societies. This e-offprint is for personal use only and shall not be self-archived in electronic repositories. If you wish to self-archive your article, please use the accepted manuscript version for posting on your own website. You may further deposit the accepted manuscript version in any repository, provided it is only made publicly available 12 months after official publication or later and provided acknowledgement is given to the original source of publication and a link is inserted to the published article on Springer's website. The link must be accompanied by the following text: "The final publication is available at link.springer.com".



EURO J Transp Logist (2016) 5:99–122 DOI 10.1007/s13676-014-0068-y



**RESEARCH PAPER** 

# A network flow model of the Northern Italy waterway system

Giovanni Righini

Received: 14 January 2014/Accepted: 21 October 2014/Published online: 4 November 2014 © Springer-Verlag Berlin Heidelberg and EURO - The Association of European Operational Research Societies 2014

Abstract The objective of this study was to develop a mathematical programming model, namely a network flow model, to provide insight into the potential capacity of the Northern Italy waterway system. We estimate the potential flow that can be transferred between the Adriatic sea and inland harbors through the waterway system made of the river Po and its surrounding canals. For this purpose a network flow model was developed, where the capacity of each arc depends on specific characteristics such as the presence of locks or one-way transit bottlenecks. The capacity of the harbors was modeled according to the number of quays and cranes available for freight transfer operations. The mathematical formulation of the problem leads to a variation of the classical maximum flow problem on capacitated networks that is easily solvable to proven optimality in a negligible computing time by any linear programming solver. Several scenarios were studied, with and without navigation in the Adriatic sea, with limited or unlimited navigation along given parts of the river. Future possible scenarios were also considered to evaluate the impact of infrastructure interventions to empower some inland harbors and to make some parts of river Po adapt to higher class barges. This mathematical programming approach based on a network flow model allows for quickly solving realistic problem instances; furthermore it provides quantitative information about bottlenecks, corresponding to binding constraints, owing to post-optimal sensitivity analysis. This provides useful indications for a rational allocation of scarce financial resources to make the waterway system a viable and convenient alternative to other transportation means.

Keywords Waterway transportation · Network flow · Sensitivity analysis

G. Righini (🖂)

Dipartimento di Informatica, Università degli Studi di Milano, Via Bramante 65, 26013 Crema, Italy e-mail: giovanni.righini@unimi.it

### 1 Introduction

The Northern Italy waterway system is made by rivers Po and Mincio and some artificial waterways; they connect some sea harbors on the Adriatic sea coast with some inland harbors in Veneto and Lombardia regions.

The waterway system traverses the Po plain that is the main flat area in the Italian territory. Owing to the fertility of the Po plain, Northern Italy developed a first-class agricultural system in which rivers and artificial canals played and still play a fundamental role. But the regions in Northern Italy are also leading the national economy in industry. Industrial development in Northern Italy did not follow a huband-spoke structure around a single very large city, but rather a reticular structure, based on several cities of some tens or hundreds thousands inhabitants, scattered in the Po plain. The only city with more than one million inhabitants is Milan, which is about 200 km far from the Adriatic sea. Because of this structure, the transportation network in Northern Italy is mainly based on roads and highways, while railways and waterways are less developed and less exploited compared to other European countries. This contributes to make the Po plain one of the most polluted areas on Earth. Therefore, the development of transportation by railways and waterways is on the logistics national agenda as a priority.

In order to foster the use of the Northern Italy waterway system, which would be beneficial from an economic and environmental point of view, public decisionmakers need (a) to know and to communicate what fraction of freight currently transported by trucks could be transported on barges; (b) to know which interventions on the waterway infrastructure could increase its theoretical capacity and to what extent. Therefore, many questions arise like the following: "How much freight can flow from the sea to the inland harbors, if the existing waterway is exploited at its maximum extent?", "Where is the bottleneck of the system?", "Is it worth to excavate the bed of the river Po in some specific points to make it navigable for a longer period along the year?", "Is it worth expanding harbor X and where would be the system bottleneck afterwards?" and many others.

The analysis of these problems was one of the main goals of the EU-funded project "Masterplan" (2014); the project was coordinated by ALOT—"Agenzia della Lombardia Orientale per i Trasporti e la logistica" (East Lombardy Agency for Transportation and logistics), a public agency in charge of developing projects and studies concerning the exploitation and the improvement of current waterways as a viable alternative to the congested, costly and polluting road-based transportation system. The "Masterplan" project was co-funded by the European Union within the European program "Ten-T" on trans-european transportation networks. This paper stems from a collaboration with ALOT within the "Masterplan" project.

In contrast to many studies in logistics optimization, the aim of the study presented in this paper was not to optimize the flow of given goods on given barges from given origins to given destinations, but rather to evaluate the maximum theoretical capacity of the waterway system under examination. The aim of our contribution is threefold: (a) to formally describe in mathematical terms the problem of determining the maximum freight transportation capacity that could be ideally obtained in a waterway system (Sect. 2); (b) to formulate and solve the specific instance of the problem with current data of the Northern Italy waterway system (Sect. 3); (c) to determine the robustness of the optimal solution with respect to some system parameters and to evaluate the possible impact of infrastructure improvements (Sect. 4).

The scientific literature on waterway transportation includes several papers concerning simulation and its applications at a strategic as well as tactical decision level. For instance, Ting and Schonfeld (1998) and Wang and Schonfeld (2005) address the strategic problem of properly selecting and scheduling investments in projects on waterways, using simulation and heuristic optimization algorithms; in Mitchell et al. (2013) the selection of dredging points is also determined via simulation. At a more tactical level, in Carroll and Bronzini (1973) the authors describe a simulation model for studying the operating characteristics of alternative inland waterway transportation systems. Their model processes information concerning commodity flows and waterway fleet characteristics and simulates the movement of tows through ports, locks, pools and channel delay areas in the Illinois waterway system. Most papers on waterway transportation come from the area of economics and transportation engineering; operations research, and mathematical programming in particular, is usually employed only at a tactical or operational level, as in Guenther et al. (2010).

Our study addresses a strategic decision problem and aims at studying and evaluating a target scenario, not at simulating or optimizing operations in the existing one. We follow a typical mathematical programming approach, through the definition of data, variables, constraints and objective functions. The resulting model, illustrated in Sect. 2, is a network flow model, i.e. a particular linear programming model, with which it is possible to optimize different objective functions (flow maximization, cost minimization, etc.).

The data used in this study (specifically in Sect. 3) were kindly provided by ALOT. One of the main outcomes of this study is the assessment about the criticality of exact knowledge of some data and the evaluation of which approximations are acceptable. This is of particular importance because data are not always known with certainty (some of them are intrinsically affected by randomness) and moreover they are subject to changes over time, as discussed in Sect. 4.

In Sect. 5 we outline some directions for further developments of this study.

### 2 The maximum waterway capacity problem

Freight transportation along waterways is operated by barges that are classified into different classes according to their size and capacity. Limits on the load of barges can be imposed by particular characteristics of each waterway segment, typically because the water depth can be different in different periods of the year. The waterway under examination is made of both natural and artificial segments. The natural segments, where water flows according to a natural stream, are found on the river Po and the part of the river Mincio that flows into the river Po near Mantova. The artificial segments are oriented East–West, they have no stream and they include some locks with lifting basins: Milan is 122 m above the sea level, while Cremona, that is currently the west-most inland harbor, is about 45 m above the sea level.

To determine the maximum capacity of the waterway system we represent it as a network, where the capacity of each arc depends on its characteristics. Then we study the flow on the network, imposing flow balance constraints and defining suitable objective functions.

The flow of freight along the arcs of the network under study is limited for different reasons: limited capacity of barges; limited speed of barges; minimum required safety distance between two consecutive barges traveling in the same direction; locks with lifting basins (only along artificial canals); one-way points (only along rivers).

*Barge capacity* We consider different barge classes (in particular classes IV, V and VI) with different given capacities. Furthermore, we consider different types of barges: by barge type we mean a given load level which can be different even for barges of the same class. This is needed because some segments in the waterway (i.e. arcs in the flow network) have characteristics that limit the allowed load on barges, not only their class. To determine the maximum capacity of the system, we assume there is no limit on the number of barges available for each class.

Barges travel at a constant speed of 10 km per hour. However in natural segments of the waterway (rivers Po and Mincio) the natural stream makes barges' speed asymmetric, e.g. 12 km per hour downstream and 8 km per hour upstream.

We assume the safety distance to depend on barge class and speed. In particular we assume a safety distance of 600 m for barges in class IV at normal speed (10 km/h with respect to the stream) and 1,000 m for barges in classes V and VI at the same speed. Furthermore, we assume the safety distance to vary linearly with barge speed. Since our study concerns an ideal case, where no queues arise, we can neglect barge length and consider barges as points. This is not a restrictive hypothesis, because it is possible to insert barge length in the computation of the safety distance if needed, without any major change to the network flow model.

We assume lifting time at basins to be known and constant; in particular we assume 20 min for basin filling and 20 min for basin emptying. In almost all basins it is possible to transfer only one barge per cycle in each direction; there are only two basins which allow to transfer 2 and 4 barges per cycle in each direction.

In some periods of the year, depending on water level in rivers Po and Mincio, there are some points along the rivers in which barges traveling in opposite directions are not allowed to cross. The current policy is to give priority to barges traveling downstream, which implies a waste of time for barges traveling upstream estimated at 5–15 min. In the ideal case, however, there are no queues and no waiting time, because we can assume barges be synchronized. The effect of one-way points is to introduce a link between flows in the two directions so that they cannot attain their maximum allowed value simultaneously and independently. In the remainder we consider one-way points of negligible length, where the time required by barges to traverse them does not depend on the direction of the last barge that traversed them.

Flow in the network is heterogeneous because there are different barge classes and types and not all arcs in the network can carry all barge classes and types. Arcs incident in a same node may allow for barges of different classes or with different load limits. Therefore, it is necessary to impose not only flow conservation for the overall amount of freight but also flow conservation for each barge type. Hence we define multi-commodity flows, i.e. different flows for each barge type.

#### 2.1 Arc capacities

In this subsection we describe how the arc capacities for freight and for barges were determined in different cases: mono-directional or bi-directional flow; homogeneous or heterogeneous flow; flow without obstacles, through lifting basins, through one-way points.

Mono-directional homogeneous flow without obstacles We consider the simplest case, i.e. a flow of barges of the same type along a waterway without locks and oneway points. We use the following notation: q indicates the load level of each barge [ton]; f indicates the barge flow, i.e. the number of barges traversing a section of the waterway in a unit of time [1/h]; p indicates the flow of freight along the waterway [ton/h]; v indicates the speed with respect to the ground [km/h];  $v^a$  indicates the speed with respect to the stream speed [km/h];  $\delta$  is the constant ratio between the safety distance  $d^{safety}$  [km] and the speed with respect to the water  $v^a$  [km/h]; this ratio is equivalent to the minimum time interval between two consecutive barges [h]. In the remainder a suffix "+" or "-" indicates "upstream" and "downstream", respectively.

The relation between speeds for each barge [km/h] is

$$v^+ = v^a - \overline{v}$$
  $v^- = v^a + \overline{v};$ 

The relation between safety distance and flow [km<sup>-1</sup>] is

$$\frac{v}{f} \ge d^{\text{safety}} \quad d^{\text{safety}} = \delta v^{\text{a}};$$

The relation between freight flow and barge flow [ton/h] is

$$p = qf$$

In each direction the barge flow f is limited by the constraint

$$f \le \frac{v}{\delta v^{\mathrm{a}}}.$$

Hence we obtain the limits on barge flows

$$f^+ \le \frac{v^+}{\delta v^a} \qquad f^- \le \frac{v^-}{\delta v^a}$$

and freight flows

$$p^+ \leq rac{qv^+}{\delta v^{\mathrm{a}}} \qquad p^- \leq rac{qv^-}{\delta v^{\mathrm{a}}}.$$

*Mono-directional homogeneous flow through locks* We now consider the flow through a lock or a sequence of them. We indicate with  $n^b$  the number of barges that can be transferred for each cycle and with  $t^b$  the time needed for a complete cycle (filling and emptying the basin). Since this value is fixed, the frequency with which the barges can traverse the lock is limited by

$$f^{\mathsf{b}} \leq \frac{n^{\mathsf{b}}}{t^{\mathsf{b}}}.$$

The upper limit on the freight flow is given by

$$q f^{\mathsf{b}} \le \frac{q n^{\mathsf{b}}}{t^{\mathsf{b}}}.$$

It is worth noting that the number of locks, the distance between them and the phase difference in cycles do not affect the waterway capacity but only the time needed to traverse it. In steady-state conditions the frequency of the barges is independent of these parameters: the same value of f can be achieved for different values of v and distance between barges. Moreover, the presence of locks does not introduce any asymmetry between upstream and downstream flows.

Mono-directional heterogeneous flow without obstacles Another case of interest concerns different barge types  $k \in K$ . In this case, although traveling at the same speed  $v^a$ , barges have different load levels  $q^k$  and they must respect different safety distances  $\delta^k v^a$ .

For each time unit,  $f^k$  barges of type k traverse each section of the waterway. Their column has an overall length equal to  $\sum_{k \in K} f^k \delta^k v^a$ . This length cannot exceed the distance that each barge travels in the same time unit, i.e. v.

Since  $v^+ = v^a - \overline{v}$  and  $v^- = v^a + \overline{v}$  we get

$$\sum_{k \in K} f^{k+} \delta^k v^a \leq v^a - \overline{v} \qquad \sum_{k \in K} f^{k-} \delta^k v^a \leq v^a + \overline{v}.$$

From these limits on barge flows we can obtain those on freight flows, because  $p = \sum_{k \in K} f^k q^k$  in each direction.

*Mono-directional heterogeneous flow through locks* As in the homogeneous case, the presence of locks limits the frequency of barge transfers. The relation

$$\sum_{k \in K} f^k = \frac{n^{\mathrm{t}}}{t^{\mathrm{b}}}$$

holds on the overall number of barges traversing the lock, since the cycle time is independent of the barge type. The constraint is the same for both upstream and downstream flows. From the limits on barge flows, those on freight flows are derived as in the previous case. *Bi-directional (homogeneous or heterogeneous) flow without obstacles* When no obstacles are there, the flows in the two directions are independent and the relations obtained for mono-directional flows still hold for each of them.

*Bi-directional (homogeneous or heterogeneous) flow through locks* Cycle times at locks are the same for mono-directional and bi-directional flows. Therefore, we can apply to this case the same relations derived for mono-directional flows.

Bi-directional homogeneous flow through one-way points One-way points make flows in opposite directions interdependent. We consider first the case with homogeneous barges. One-way points occur along rivers, where the two flows have different speeds with respect to the ground and hence the time required to traverse a one-way point is also different. We indicate with  $\tau^+$  and  $\tau^-$  the one-way point traversal time of barges traveling upstream and downstream, respectively, i.e. the time between two barges traversing the one-way point consecutively (in either direction). In the remainder we assume that this time does not depend on two consecutive barges traversing the one-way point in the same direction or in opposite directions.

*Remark* We could make the above assumption because one-way points correspond to some bridges along the river Po. Therefore, no time is wasted to stop the barge flow in one direction and start the flow in the opposite direction. The assumption would not be justified in case of one-way segments of significant length, because it would be necessary to forbid the flow in both directions for a certain period of time for each reversal of the transit direction.

Indicating with  $f^+$  and  $f^-$  the barge flows in the two directions we have the constraint

$$f^+\tau^+ + f^-\tau^- \le 1,$$

because  $f^+\tau^+$  and  $f^-\tau^-$  represent the fractions of time required by barges traversing the one-way point in each direction. In this model we neglect the effect of stop-and-restart under the hypothesis that arrival times can be synchronized to avoid queues.

Therefore, there is a limit on the overall freight flow in the two directions. From the relations

$$p^+ = qf^+ \qquad p^- = qf^-$$

and from the constraint above, we obtain the constraint

$$p^+\tau^+ + p^-\tau^- \le q,$$

which links the maximum freight capacities of the arc in the two directions.

**Observation** In general, if the network representing the waterway is acyclic, then the homogeneous flow balance constraints along each arc imposes that the barge frequencies in the two directions are identical. Imposing  $f^+ = f^-$  one obtains

$$f \le \frac{1}{\tau^+ + \tau^-}$$

which translates into the following constraint on freight capacity in each direction:

$$p \leq \frac{q}{\tau^+ + \tau^-}.$$

On the contrary, if the network is not acyclic (this is the case for the Northern Italy waterway system), then the flow balance must not be imposed on each arc but rather on each cut: in other terms, it is allowed to compensate a higher upstream barge frequency on an arc with a higher downstream barge frequency on another one traversing the same cut that separates the flow sources from the flow sinks.

Bi-directional heterogeneous flow through one-way points In case of a heterogeneous flow of barges the same relations obtained above can be applied to the overall frequency: in other words,  $f^+$  is replaced by the overall value  $\overline{f}^+ = \sum_{k \in K} f^{k+}$  and the same holds for  $\overline{f}^-$  in the other direction. This is possible because traversal times do not depend on barge classes and types, because in turn barge speed does not depend on class and type. Therefore, we obtain

$$\sum_{k\in K} f^{k+}\tau^+ + \sum_{k\in K} f^{k-}\tau^- \leq 1,$$

that is

$$\overline{f}^+\tau^+ + \overline{f}^-\tau^- \le 1$$

which expresses the interdependence between maximum flows of barges in the two directions.

### 2.2 Harbor capacities

Another important element for determining the maximum capacity of the waterway system is harbor capacities. We describe each harbor with the following data: its position in the network; the number of barges  $\lambda$  that can be loaded or unloaded simultaneously; the loading/unloading speed  $\sigma$  [ton/h]. We assume that no barge can be loaded and unloaded simultaneously. With this assumption the harbor capacity is given by  $\lambda \sigma$  [ton/h]. We remark that this capacity constraint at the harbors is a limit on the freight flow, while the constraints due to the characteristics of the arcs of the network are more easily expressed as constraints on barge flows.

### 2.3 A mathematical model

Once the different elements in the system and the characteristics determining their capacity examined, it is now possible to formulate the optimal network flow problem.

Data Data are related to harbors, waterways and barges.

Data about harbors are the following: a set  $N^-$  of nodes corresponding to sea harbors; a set  $N^+$  of nodes corresponding to inland harbors; a loading/unloading speed  $\sigma_i$  for each harbor  $i \in N^+$  [ton/h]; a number of quays  $\lambda_i$  where loading/ unloading operations can be executed in parallel for each harbor  $i \in N^+$ . In our instances sea harbors are considered of infinite capacity. Data about waterways describe a weighted digraph as follows: we are given two dummy nodes *s* and *t*, the former linked to all sea harbors and the latter linked to all inland harbors; a set *B* of nodes corresponding to bifurcations and connections of waterways; a set *A'* of (oriented) arcs corresponding to each waterway and each direction:  $A' \subseteq N \times N$ , with  $N = N^+ \cup N^- \cup B$ ; a set A'' of (oriented) arcs corresponding to links between harbors and dummy nodes *s* and *t*:  $A'' = N^- \times \{s\} \cup \{s\} \times N^- \cup N^+ \times \{t\} \cup \{t\} \times N^+$ ; a maximum capacity  $Q_{ij}^{\max}$ allowed for barges traveling along each arc  $(i,j) \in A'$ ; a value  $\overline{v}_{ij}$  indicating the stream speed [km/h] from *i* to *j* for each  $(i,j) \in A'$  (negative if *i* is downstream with respect to *j*: the relation  $\overline{v}_{ij} + \overline{v}_{ji} = 0$  holds); a value  $t_{ij}^{b}$  indicating the lock basins cycle time [h] along arc  $(i,j) \in A'$ ; a value  $n_{ij}^{b}$  indicating the number of transferred barges for each lock basin cycle along arc  $(i,j) \in A'$ ; a traversal time  $\tau_{ij}$  for one-way points along each arc  $(i,j) \in A'$ . In the remainder we indicate with *A* the set of all arcs in the network, i.e.  $A = A' \cup A''$ .

Finally, data about barges are the following: a set *K* of barge classes; a capacity  $q_k^{\max}$  associated with each class  $k \in K$  [ton]; an ordered set *R* with as many elements as the number of distinct values of class capacity  $q^{\max}$  and waterway segments' capacity  $Q^{\max}$  (each element  $r \in R$  corresponds to a capacity  $q_r$  [ton]); the barge speed  $v^a$  with respect to water [km/h] (independent of barge type); the safety distance  $d_k^{\text{safety}}$  [km] required for barges of class  $k \in K$  at speed  $v_a$ .

*Variables* To differentiate between flows traveling from sea harbors to inland harbors and vice versa, we use different variables. However, we do not use  $f^+$  and  $f^-$ , because they refer to traveling upstream and downstream and in general it is not guaranteed that the two criteria coincide: in cyclic networks (like the one under study) it may be the case the barges traveling from the sea to the inland find convenient (or necessary) to traverse one or more waterways downstream (or the other way around). This may occur because waterways have different characteristics and in cyclic digraphs multiple paths may exist for each origin–destination pair. Therefore, we introduce continuous non-negative variables  $f_{ij}^{kr}$  and  $\phi_{ij}^{kr}$ , representing barge flows for each class  $k \in K$  and load level  $r \in R$  traveling along each arc (i, j), respectively, from the sea to inland harbors and from inland harbors to the sea. All flows are expressed in number of barges per hour.

*Constraints* The problem has several sets of constraints. Barge flow conservation constraints for each type (k, r) at each node of the digraph impose that the overall incoming flow is equal to the overall outgoing flow, both for ascending flows f and for descending flows  $\phi$ :

$$egin{aligned} &\sum_{i\in N} f_{ij}^{kr} = \sum_{i\in N} f_{ji}^{kr} \quad orall k \in K, \ orall r \in R, \ orall j \in N \ &\sum_{i\in N} \phi_{ij}^{kr} = \sum_{i\in N} \phi_{ji}^{kr} \quad orall k \in K, \ orall r \in R, \ orall j \in N. \end{aligned}$$

Barge flow conservation constraints for each class in dummy nodes *s* and *t* allow to use barges of the same class with different load levels according to the direction:

$$\begin{split} &\sum_{r\in R} f_{it}^{kr} = \sum_{r\in R} \phi_{ti}^{kr} \quad \forall k\in K, \, \forall i\in N^+ \\ &\sum_{r\in R} \phi_{is}^{kr} = \sum_{r\in R} f_{si}^{kr} \quad \forall k\in K, \, \forall i\in N^-. \end{split}$$

Specific constraints forbid ascending flow f to enter s and to leave t and descending flow  $\phi$  to enter t and to leave s:

$$\begin{aligned} f_{ti}^{kr} &= 0 \quad \forall i \in N^+, \, \forall k \in K, \, \forall r \in R \\ f_{is}^{kr} &= 0 \quad \forall i \in N^-, \, \forall k \in K, \, \forall r \in R \\ \phi_{it}^{kr} &= 0 \quad \forall i \in N^+, \, \forall k \in K, \, \forall r \in R \\ \phi_{si}^{kr} &= 0 \quad \forall i \in N^-, \, \forall k \in K, \, \forall r \in R. \end{aligned}$$

The maximum allowed heterogeneous barge flow on each waterway segment without obstacles is imposed by

$$\sum_{k \in K, r \in R} (f_{ij}^{kr} + \phi_{ij}^{kr}) d_k^{\text{safety}} \le v^a + \overline{v}_{ij} \quad \forall (i,j) \in A'.$$

The maximum allowed heterogeneous barge flow on each waterway segment with locks is imposed by

$$\sum_{k \in K, r \in R} (f_{ij}^{kr} + \phi_{ij}^{kr}) \leq \frac{n_{ij}^{\mathrm{b}}}{t_{ij}^{\mathrm{b}}} \quad \forall (i,j) \in A'.$$

The maximum allowed bi-directional heterogeneous barge flow on each waterway segment with one-way points is imposed by

$$\sum_{k \in K, r \in R} (f_{ij}^{kr} + \phi_{ij}^{kr}) \tau_{ij} + \sum_{k \in K, r \in R} (f_{ji}^{kr} + \phi_{ji}^{kr}) \tau_{ji} \le 1 \quad \forall (i,j) \in A'.$$

The maximum allowed load level of barges on each waterway segment is imposed by

$$\begin{aligned} & f_{ij}^{kr} = 0 \quad \forall (i,j) \in A', \, \forall k \in K, \, \forall r \in R : Q_{ij}^{\max} < q_r \\ & \phi_{ij}^{kr} = 0 \quad \forall (i,j) \in A', \, \forall k \in K, \forall r \in R : Q_{ij}^{\max} < q_r. \end{aligned}$$

The maximum allowed load of barges for each class is imposed by

$$\begin{split} f_{ij}^{kr} &= 0 \quad \forall (i,j) \in A', \, \forall k \in K, \, \forall r \in R: \, q_k^{\max} < q_r \\ \phi_{ij}^{kr} &= 0 \quad \forall (i,j) \in A', \, \forall k \in K, \forall r \in R: \, q_k^{\max} < q_r. \end{split}$$

Finally, the constraints on harbor capacities are

$$\sum_{k \in K, r \in R} (f_{it}^{kr} + \phi_{ti}^{kr}) q_r < = \sigma_i \lambda_i \quad \forall i \in N^+.$$

🖄 Springer

Author's personal copy

*Objectives* The model described above can be used to study different optimization problems related to different objective functions: the most natural objectives are network capacity, transportation speed and cost.

The aim of this study was to determine the maximum (current and potential) transportation capacity of the waterway system. Hence our main objective is the maximization of the overall freight flow between sea harbors and inland harbors: the barge flows for each class (not necessarily for each load level) must be the same, but this does not imply that the freight flows are also the same, because each barge could receive different loads when traveling from the sea to inland harbors and vice versa.

The objective function corresponding to the overall system capacity, indicated by *z*, is the following:

$$\text{Maximize } z = \sum_{k \in K, r \in R} q_r \sum_{i \in N^+} \phi_{ti}^{kr} + \sum_{k \in K, r \in R} q_r \sum_{i \in N^-} f_{si}^{kr}.$$

A mathematical programming model The resulting mathematical programming model is a max-flow problem, that is a linear programming model, and it is easily solvable to proven optimality with existing solvers. Given the small size of the network under examination it is also possible to use free solvers. The results reported in the next section have been obtained with the free GLPK solver (2014), MathProg modeling language and Gusek interface.



Fig. 1 The Northern Italy waterway system network

#### **3** The Northern Italy waterway system instance

We applied the above formal description to the study of the waterway system made of rivers Po and Mincio and the surrounding artificial canals in Northern Italy. The waterway system network is represented in Fig. 1. On each arc some characteristics are indicated: locks, one-way points, maximum allowed load level [tons].

In the remainder we show the optimal solutions (maximum flows) in different scenarios.

The standard scenario refers to ideal conditions both along river Po and in the Adriatic sea. In this scenario it is not necessary to consider one-way points along river Po; the load limit along the rivers is 1,100 tons and it is allowed for barges to navigate along the sea coast.

Freight flows [ton/h] represented in Fig. 2 refer to both directions (from the sea to inland and vice versa). The bottleneck is determined by the inland harbors capacities, which are all saturated. For this reason, since the objective is capacity maximization, not time minimization, there are multiple equivalent (optimal) solutions. Hence the optimal value, 1,075 ton/h, is significant, but the optimal solution is not so much. In order to obtain a significant optimal solution (the one reported in Fig. 2) we solved a max-flow min-cost problem, after associating a cost with each arc. This was done using arc lengths as an approximated indicator of arc costs.

In the scenario with reduced navigability we consider a low water level in river Po, which induces one-way points. The maximum allowed load along rivers is the



#### Fig. 2 Optimal solution in standard conditions

same of the base scenario. Results are reported in Fig. 3. In this scenario the arc between Cremona and Foce Mincio becomes saturated instead of Cremona harbor. For all the other inland harbors the flows remain unchanged. The overall system capacity decreases from 1,075 to 930.6 ton/h compared to the standard scenario.

The scenario with no navigability on the river Po corresponds to the flood period. The corresponding results are reported in Fig. 4. In this scenario the capacity of Rovigo and Mantova harbors is saturated, while the other two inland harbors remain disconnected from the network.

In another scenario we consider rough seas conditions, preventing barges to enter the river Po from the Adriatic sea. The corresponding results are reported in Fig. 5. We obtain the same flow as in standard conditions, with the difference that only barges of class IV are used, since they can traverse the alternative path between Venice and river Po (Brondolo artificial canal). Therefore, costs are higher but system capacity does not decrease.

In the combined scenario with reduced navigability on the river Po and rough sea conditions in the Adriatic sea (see Fig. 6) the overall capacity remains equal to 930.6 ton/h, but only 1,100 tons loads are used. Cremona harbor is not saturated because one-way points along river Po are the bottleneck.

The worst possible scenario considers no navigability on the river Po and rough seas (see Fig. 7). In this case the overall system capacity is the same as with navigability in the sea (700 ton/h). All freight is transported on 1,100 ton loaded barges. Artificial canals incident to Venice harbor are used instead of the sea.



Fig. 3 Optimal solution with reduced navigability

The six scenarios examined above occur during the year because of the seasonality in the water level of river Po (that can be reliably forecasted) and because of meteorological conditions in the Adriatic sea (that cannot be reliably forecasted). On the basis of the observations in the past years we assume that river Po between Voltagrimana and Cremona is navigable without constraints for 215 days/year (59 % of time), navigable with one-way points for 130 days/year (35,5 % of time) and not navigable for 20 days/year (5,5 % of time). We also assume that the Adriatic sea is not navigable by barges for 10 % of time, independently of navigability conditions of river Po. This assumption, however, has no significant effect because maximum capacity does not depend on sea navigability in any scenario.

Therefore we obtain the weights to be associated with each scenario, as reported in Table 1.

By a weighted combination of maximum flows for each scenario, outlined in Table 2, it is possible to obtain the average flow of the waterway system along the year, which turns out to be equal to 1,003.1 ton/h.

#### 4 Post-optimal analysis

One of the main advantages of a mathematical programming approach to this problem is the possibility of performing sensitivity analysis, i.e. to study how the optimal value depends on certain system parameters.



Fig. 4 Optimal solution with no navigability on river Po



Fig. 5 Optimal solution with rough seas conditions

Some useful pieces of information of this type are already available from the inspection of the optimal solutions in the different scenarios: binding constraints reveal what are the bottlenecks, i.e. the elements in the waterway system whose limited capacity is saturated, limiting the overall system capacity.

We now consider a hypothetical scenario, obtained with some variations to the data of the problem, related to harbors and waterways capacities as a result of infrastructure improvements.

### 4.1 Harbor capacities

One possible analysis of interest concerns the capacity of the waterway system without considering harbor capacities, which are currently the bottleneck in the system. We solved the maximum flow problem assuming one inland harbor at a time having infinite capacity. For the purpose of this analysis we considered the scenario with standard navigability conditions. Finally, we also studied the case in which all inland harbors have infinite capacity, to evaluate the current capacity allowed by the waterway.

When we consider single harbors with infinite capacity we obtain the following results:

*Rovigo* The maximum flow that can traverse Rovigo harbor is equal to 8,096 ton/ h (summing up the flows in both directions); the saturated arcs are the Ferrara waterway and the Governolo-Voltagrimana canal on both sides of Rovigo, owing to the lifting basins along it.



Fig. 6 Optimal solution with reduced navigability on river Po and no navigability in the sea

*Mantova* The maximum flow that can traverse Mantova harbor is equal to 17,841 ton/h (summing up the flows in both directions); the saturated arcs are the same as above and also the arcs of river Mincio, owing to the lifting basin near Mantova.

*Cremona* The maximum flow that can traverse Cremona harbor is equal to 19,459 ton/h (summing up the flows in both directions); the saturated arcs are the Ferrara waterway, the Voltagrimana lifting basin and the canal part between Rovigo and Voltagrimana. Also in this case the lifting basins are bottlenecks for the overall system.

*Porto Nogaro* The maximum flow that can traverse Porto Nogaro harbor is equal to 60,000 ton/h (summing up the flows in both directions); the saturated arc is the Aussa-Corno canal.

When we relax capacity constraints on all inland harbors, the optimal flow raises to 81,289.35 [ton/h] and the saturated arcs are the Ferrara waterway, the Voltagrimana lifting basin, the Brondolo canal and the canal segment between Rovigo and Voltagrimana.

### 4.2 Infrastructure interventions

We consider the following possible infrastructure interventions: an artificial canal between Cremona and Milan; new harbors in Ferrara, Boretto and Milan; improvements to harbor capacities; improvements to canal capacities.



Fig. 7 Optimal solution with no navigability on river Po and in the sea

Each modification can be considered independently of the others. In the remainder we refer to an ideal scenario in which all interventions listed above have been made. In particular we consider the following data: a new waterway between Cremona and Milan, with locks, no natural stream and no one-way points, allowing barges loaded with up to 1,800 tons; the upgrade to class V of artificial waterways Mantova-Porto Levante, Brondolo canal and Ferrara waterway, to allow for barges loaded with up to 1,800 tons (instead of 1,600 or 1,100 as in the current scenario); navigability improvement of river Po with a threefold effect: (a) extension of the standard conditions period from 215 to 320 days/year, and reduction of the reduced navigability period from 130 to 25 days/year; (b) elimination of one-way points in the reduced navigability period; (c) increase of the maximum allowed load in standard conditions from 1,100 to 1,800 tons; capacity increase in the Aussa-Corno canal, to allow for barges with 4,000 ton capacity (instead of 3,000 tons); possibility of loading/unloading three barges simultaneously instead of two in Porto Nogaro and Cremona and two barges instead of one in Mantova and Rovigo; new harbors as described in Table 3.

Also this scenario was studied in the six different cases as the current one. The corresponding flow network is represented in Fig. 8.

Results referred to standard conditions are reported in Fig. 9. The bottleneck is determined by inland harbor capacities. The optimal flow is equal to 2,350 ton/h and it is obtained with a suitable combination of load levels equal to 1,800 tons (class V or VI) and 4,000 tons (class VI).

Table 1   Percentage weights for different navigability scenarios	Weights	Po = yes	Po = reduced	Po = no
	Sea = yes (%)	53.1	31.95	4.95
	Sea = no $(\%)$	5.9	3.55	0.55
Table 2 Capacity of the current waterway system in different navigability scenarios	Weights	Po = yes	Po = reduced	Po = no
	Sea = yes	1,075	930.6	700
	Sea = no	1,075	930.6	700
Table 3 Characteristics of new harbors	Harbor	N. of quays	Loading/unloading sp	beed [tons/h]
	Ferrara	2	150	
	Boretto	1	125	
	Milan	3	125	



Fig. 8 The flow network in the ideal future scenario

As opposed to the current scenario, in the future scenario there are no one-way points when the water level of river Po is low (reduced navigability). The corresponding results are reported in Fig. 10. The allowed load level on the river in



Fig. 9 Optimal solution in standard conditions

this case is only 1,100 tons. The optimal solution is achieved with a combination of three load levels: 1,100, 1,800 and 4,000 tons. All inland harbors are saturated.

Results for the scenario with no navigability on river Po are reported in Fig. 11. The maximum flow decreases to 1,350 ton/h. The harbors in Rovigo, Mantova, Ferrara and Porto Nogaro are saturated, while the others remain disconnected.

Results for the scenario with rough sea conditions are reported in Fig. 12. The flow is the same as in the standard scenario, with the only difference that the connection between Venice and Porto Nogaro requires barges with load of 1,100 tons traveling along the artificial canal, instead of barges with load of 4,000 tons traveling in the sea. This illustrates that also in the hypothetical future system the meteorological conditions of the Adriatic sea would affect transportation costs, but not system capacity.

Results for the scenario with reduced navigability and rough sea conditions are reported in Fig. 13. The overall capacity remains equal to 2,350 ton/h with the only difference that Ravenna is now disconnected and all the flow goes through Venice. With respect to the case in which the barges can travel in the sea, the missing flow on Ferrara waterway goes through the Brondolo canal, Voltagrimana and river Po up to Ferrara. Hence, also in this case the conditions of the sea affect costs, but not the capacity.

Results for the scenario with no navigability on the river Po and rough sea conditions are reported in Fig. 14. The disconnection of Ravenna and Ferrara harbors reduces the flow by 75 ton/h in each direction along the Ferrara waterway.



Fig. 10 Optimal solution with reduced navigability

The other flows remain unchanged. System capacity decreases from 1,350 to 1,200 ton/h. The harbor capacities in Rovigo, Mantova and Porto Nogaro are saturated.

*Combination of different scenarios* In the hypothetical waterway system the weights used to combine the six scenarios are more favorable than today, because the periods in which the river Po can be navigated without restrictions are longer, as reported in Table 4.

With the optimal flow values reported in Table 5, we can obtain the average capacity of the system, that turns out to be equal to 2,294.175 ton/h.

### 5 Conclusions and extensions

This case study was the operations research contribution to the European project "Masterplan" whose aim was to study the usability of the Northern Italy waterway system as a viable and sustainable alternative to road transportation. To foster transportation on waterways it is important to have a quantitative indication about the capacity of the existing system which is currently under-utilized. Since it is known that there are natural and artificial obstacles to the barge flow, some interventions are currently under examination for possibly improving the infra-structure. These interventions would imply huge investments from private and public sources. Therefore, it is of paramount importance to have reliable indications



Fig. 11 Optimal solution with no navigability on river Po

on the effects of each possible intervention on the waterway system infrastructure. The network flow model presented here is intended to provide a rational and quantitative basis to the decision process yielding justifiable estimates of the capacity of the resulting system in different operational conditions.

This work can be extended in several possible ways: some of them are listed hereafter.

Cost minimization and time minimization are two relevant problems to be addressed to make waterways competitive with other transportation means. A network flow model like the one illustrated in this paper can be used to evaluate the optimal transportation time and the optimal transportation cost between sea and inland harbors, two key indicators of the system service level. However, for this purpose some additional data are needed. Moreover, it is also necessary to consider the limit imposed by the finite number of barges available for each class.

While this study was focused on a strategic problem, the optimization of transportation time and cost is mainly a tactical problem and it requires ad hoc models and algorithms, as those developed in Guenther et al. (2010). For instance an important problem at a tactical level is the optimization of operations in harbors, where the use of limited resources (cranes, quays, personnel) must be assigned and scheduled to maximize throughput and minimize cost. A related sub-problem concerns workforce management to define optimal work shifts: this problem arises



Fig. 12 Optimal solution with rough sea conditions



Fig. 13 Optimal solution with reduced navigability and rough sea conditions



Fig. 14 Optimal solution with no navigability on river Po and rough sea conditions

Table 4 Percentage weights of the six scenarios in the hypothetical system	Weights	Po = yes	Po = reduced	Po = no
	Sea = yes (%) Sea = no (%)	78.9 8.8	6.12 0.68	4.95 0.55
Table 5   System capacity in the six scenarios (hypothetical	Weights	Po = yes	Po = reduced	Po = no

both at a tactical level (periodic planning) and at an operational level (real-time replanning when needed to tackle unforeseen events).

For each given level of capacity at system level, the speed of barges and the distance between them can be different. A real-time tuning of barge speed in different points of the waterway system can be instrumental to prevent queues at the bottlenecks such as lifting basins, one-way points and harbors. The forthcoming River Information Service (RIS) will provide real-time data on the position of each barge in the system. These data could be used by a real-time optimization algorithm to suggest optimal speed to each barge. A possible strategic problem related to the

RIS is to optimally locate its elements along the waterways (see Agnetis et al. 2009 for an example of a problem of locating facilities along a river).

The advantage of mathematical programming is to formally define the optimization problem and to guarantee optimality of the solutions obtained from the model. However, its results are reliable when all data are deterministically known. This assumption is acceptable to study ideal scenarios or average values on long-term planning horizons, but it does not take into account randomness and fluctuations in a real system: for instance, loading/unloading time could be described by a probability distribution and not by a constant value as well as navigation time between sea harbors. For this reason it can be useful to combine mathematical programming with different optimization techniques, such as queuing theory and simulation, to analyze the existing and the future waterway transportation system in more detail.

**Acknowledgments** This study was developed for ALOT—Agenzia della Lombardia Orientale per i Trasporti e la logistica through the consortium Crema Ricerche. The author acknowledges the kind collaboration of Guido Piccoli (ALOT), Roberto Zaglio (ALOT) and Alessandra Ginelli (Crema Ricerche). The observations and comments of three anonymous referees on the original manuscript allowed for significant improvements.

#### References

- Agnetis A, Grande E, Mirchandani PB, Pacifici A (2009) Covering a line segment with variable radius discs. Comput Oper Res 36:1423–1436
- Carroll JL, Bronzini MS (1973) Waterway transportation simulation models: development and application. Water Resour Res 9(1):51–63
- GNU Linear Programming Kit (2014) http://www.gnu.org/software/glpk/. Accessed 9 Jan 2014
- Guenther E, Luebbecke ME, Moehring RH (2010) Ship traffic optimization for the Kiel canal. In: Proceedings of the seventh triennial symposium on transportation analysis
- Mitchell K, Wang B, Khodakarami M (2013) Selection of dredging projects for maximizing waterway system performance. Transp Res Rec 2330(1):39–46
- Ting C-J, Schonfeld P (1998) Optimization through simulation of waterway transportation investments. Transp Res Rec 1620(1):11–16
- Wang S, Schonfeld P (2005) Scheduling interdependent waterway projects through simulation and genetic optimization. J Waterw Port Coast Ocean Eng 131(3):89–97

Masterplan project (2014) http://www.alot.it/en/masterplan. Accessed 3 Aug 2014