A combinatorial optimization problem arising from text classification

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- bag-of-word approach: simple and language independent, but may lead to a data sparseness problem.
- segmentation based upon a morphological analisys: very refined, but introduces costs for any language used.

A new approach is based upon an optimal reduction of the suffix tree of a training test.

Suffix tree

First a preprocessing on the text is made, reducing the alphabet to lower case letters plus a unique non alphabetic character.

```
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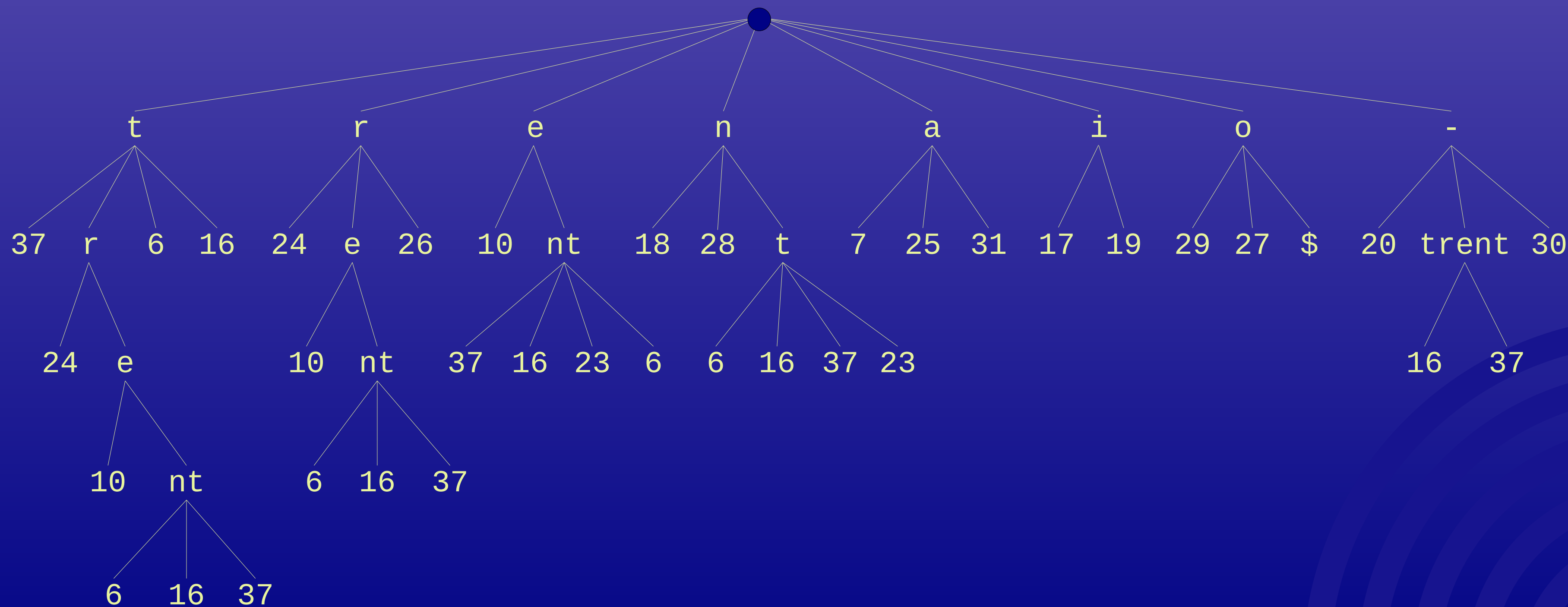
The suffix tree is then built:



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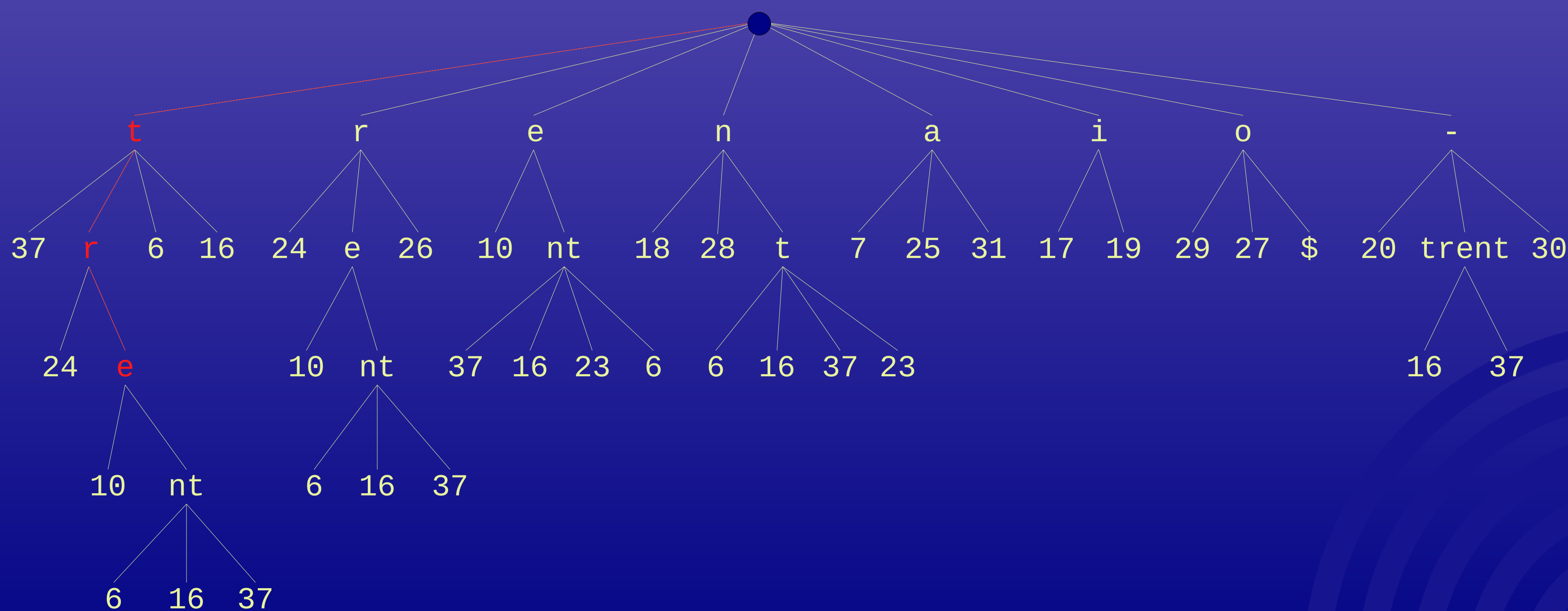

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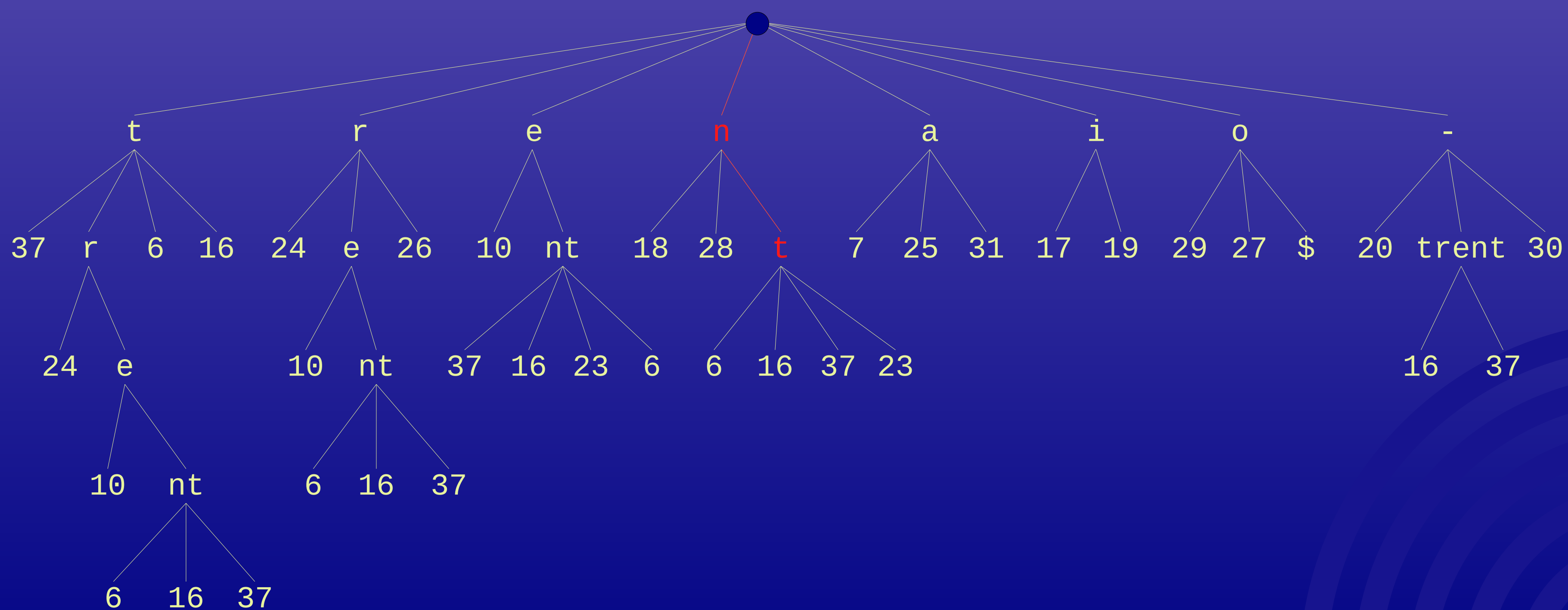

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In this work we have studied a further reduction of the suffix tree that tries to keep only the *important* suffixes, those that describe the training text in the best way.

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The name given to that problem is TCSS, that stands for
Text Covering with Strings Subset.

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For classification purposes the use of long strings is preferable to that of short ones, so the cost coefficient used is the inverse of the string length.

Istances

The number of strings, occurrences and characters obtained for class instances is the following:

CLASS	$\|S\|$	$\|O\|$	$\|W\|$
A	170	2850	8870
B	2200	55700	49000
C	9950	375000	218000

ILP model for TCSS

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s.t. $$\begin{cases} \sum_{j=1}^{|O|} a_{tj} x_j \leqslant 1 & t = 1 \dots |W| \\ x_j - y_{u(j)} \leqslant 0 & j = 1 \dots |O| \\ x_j \in \{0,1\} & j = 1 \dots |O| \\ y_i \in \{0,1\} & i = 1 \dots |S| \end{cases}$$

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Binary variables y for the strings.

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The objective 1 tries to maximize the number of covered characters.

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The objective 2 tries to minimize the cost of the used strings.

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The two objectives are combined with a parameter α.

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Packing constraints.

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For the structure of the matrix $|a_{tj}|$, this problem can be solved in polynomial time.

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Every character is a node. Number the nodes in the same order as the characters.

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Add one node as last node.

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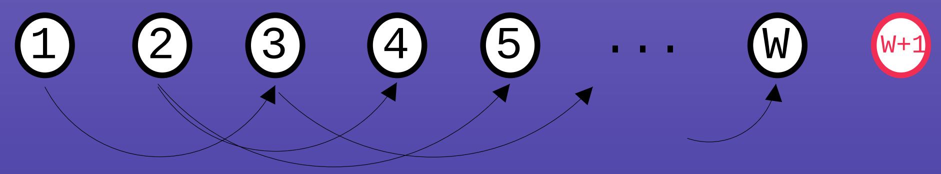

Every occurrence is an arc, outgoing from the node correspondent to its first covered character and entering the node correspondent to the character after the last covered character, and having as weight the number of covered characters.

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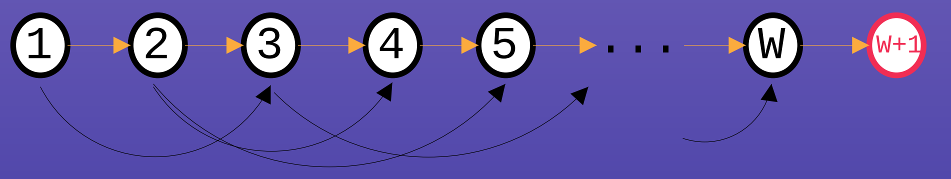

Add one arc from every node the next one (except the last one).

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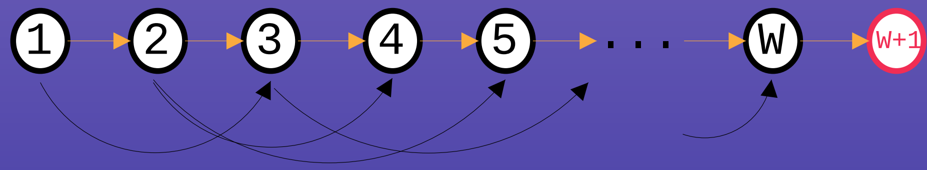

The solution of a Max Path Problem, from the node 1 to the node $|W| + 1$, corresponds to the maximum cover of the text.

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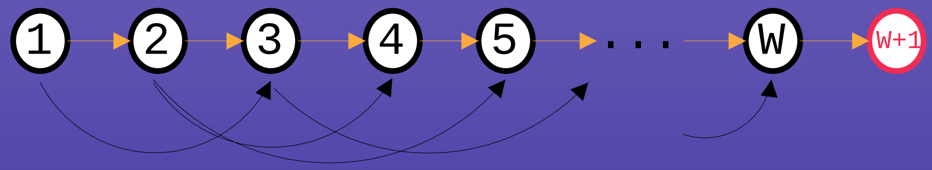

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This graph is acyclic and directed and the optimal solution can be found in polynomial time.

CPLEX 6.5

Three equivalent formulations have been solved with CPLEX:

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Relaxing 1:

$$\text{TCSS}_1) \qquad \max z = \alpha \sum_{j=1}^{|O|} l_{u(j)} x_j - (1-\alpha) \sum_{i=1}^{|S|} \frac{1}{l_i} y_i$$

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Lagrangean relaxation

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Both of them were used to compute dual bounds in a branch and bound framework, solving in an approximate way the Lagrangean dual with Subgradient Optimization.

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$$\max z_{LR1}(\lambda) = \sum_{j=1}^{|O|} \left(\alpha l_{u(j)} - \sum_{t=1}^{|W|} \lambda_t a_{tj} \right) x_j - (1-\alpha) \sum_{i=1}^{|S|} \frac{1}{l_i} y_i + \sum_{t=1}^{|W|} \lambda_t$$

$$\text{s.t.} \begin{cases} x_j - y_{u(j)} \leqslant 0 & j = 1 \dots |O| \\ x_j \in \{0,1\} & j = 1 \dots |O| \\ y_i \in \{0,1\} & i = 1 \dots |S| \end{cases}$$

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For any choice of λ the term $\sum_{t=1}^{|W|} \lambda_t$ is constant, and this problem can be decomposed into $|S|$ problems.

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Let c_j be the coefficient of x_j. Every problem is solved fixing:

$$y_i = \begin{cases} 1 & \text{if } \sum_{j \in O_S(i) | c_j > 0} c_j - \frac{1-\alpha}{l_i} > 0 \\ 0 & \text{otherwise} \end{cases}$$

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- *on occurences*. Select $\bar{j} = \arg\max_{j \in V_k} \{c_j\}$. Generate two nodes, fixing respectively $x_{\bar{j}} = 1$ and $x_{\bar{j}} = 0$.
- *on strings*. Select the string $\bar{\imath} = u(\bar{j})$ and generate two nodes fixing respectively $y_{\bar{\imath}} = 1$ and $y_{\bar{\imath}} = 0$.

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The optimal solution (x^*, y^*) of LR1 can violate some of the packing constraints constraints.

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It is possible to obtain a feasible solution by solving an MPP subproblem keeping the values of the y^* fixed:

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$$\textit{s.t.} \begin{cases} \sum_{j=1}^{|O|} a_{tj} x_j \leqslant 1 & t = 1 \dots |W| \\ x_j \leqslant y^*_{u(j)} & j = 1 \dots |O| \\ x_j \in \{0,1\} & j = 1 \dots |O| \end{cases}$$

Relaxation of vub constraints

Relaxing the variable upper bound constraints we obtain:

LR2) $$\max z_{LR2}(\mu) = \sum_{j=1}^{|O|} \left(\alpha l_{u(j)} - \mu_j\right) x_j + \sum_{i=1}^{|S|} \left(-\frac{1-\alpha}{l_i} + \sum_{j \in O_S(i)} \mu_j \right) y_i$$

$$s.t. \begin{cases} \sum_{j=1}^{|O|} a_{tj} x_j \leqslant 1 & t = 1 \dots |W| \\ x_j \in \{0,1\} & j = 1 \dots |O| \\ y_i \in \{0,1\} & i = 1 \dots |S| \end{cases}$$

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that can be decomposed into two independent problems:

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that can be decomposed into two independent problems:

- LR2y is a trivial problem

Relaxation of vub constraints

Relaxing the variable upper bound constraints we obtain:

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- LR2y is a trivial problem
- LR2x is an MPP instance whose coefficients depend on μ values

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The branching variable is the one that maximizes the minimum decrement of $z_{LR2}^*(\mu)$:

$$i = \arg\max_k \left\{ \min \left\{ \sigma_k^0, \sigma_k^1 \right\} \right\}$$

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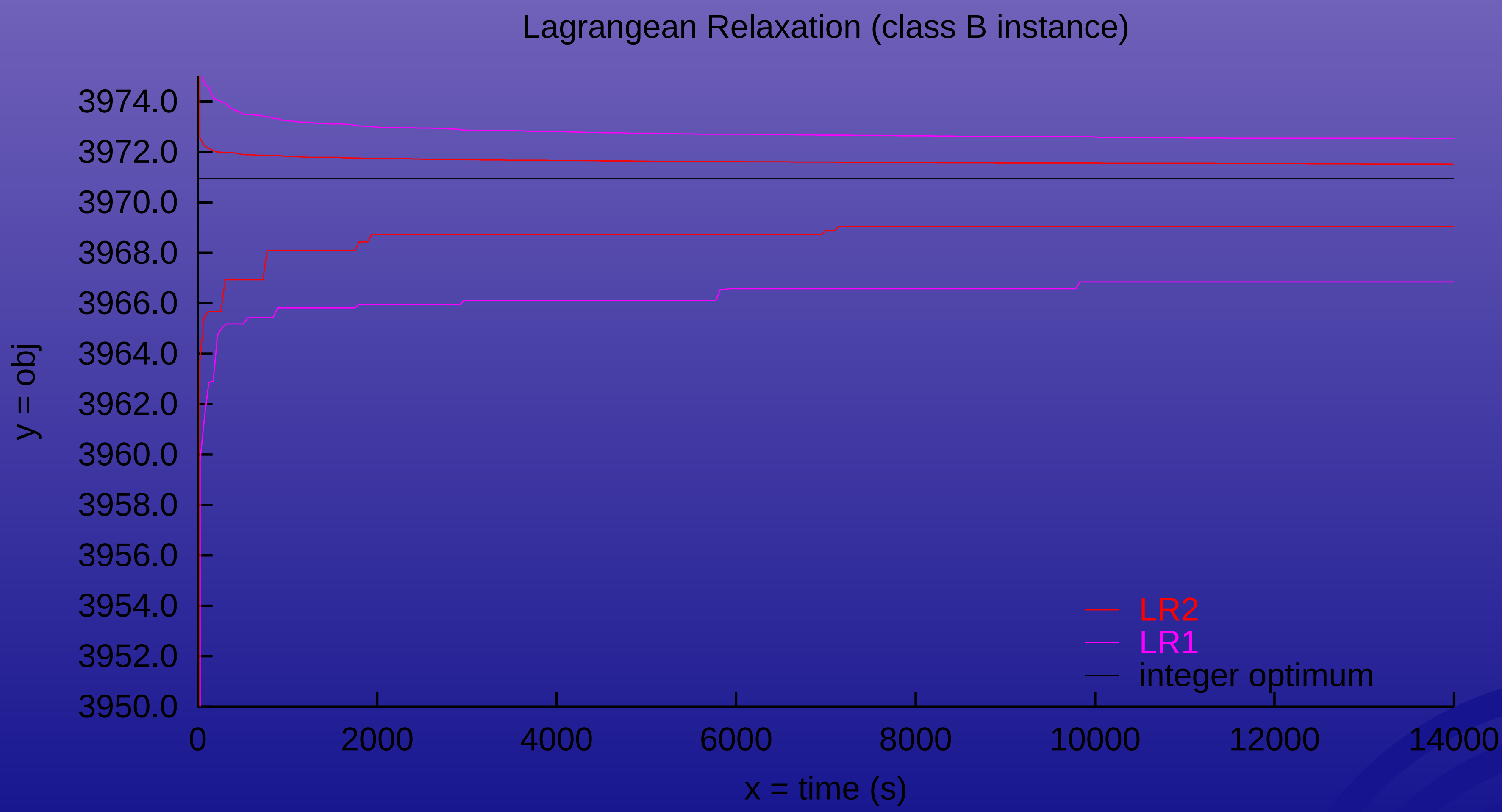
A more sophisticated heuristic consists of solving an MPP problem on a graph containing only the arcs corresponding to the occurrences of the strings such that $y_i^* = 1$.

Comparison of the LR1 and LR2



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- hybrid strategies. Find $n_i^* = \arg \max_{n \in \tilde{N}_i(s_t) \subseteq N(s_t)} z(n)$.

  If not improving consider another subset $\tilde{N}_{i+1} \subseteq N(s_t)$.

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- *Starting solution*. The starting solution is $y_i^{s_0} = 1 \quad \forall i$. Also an initialization to zero and a random initialization have been tested.

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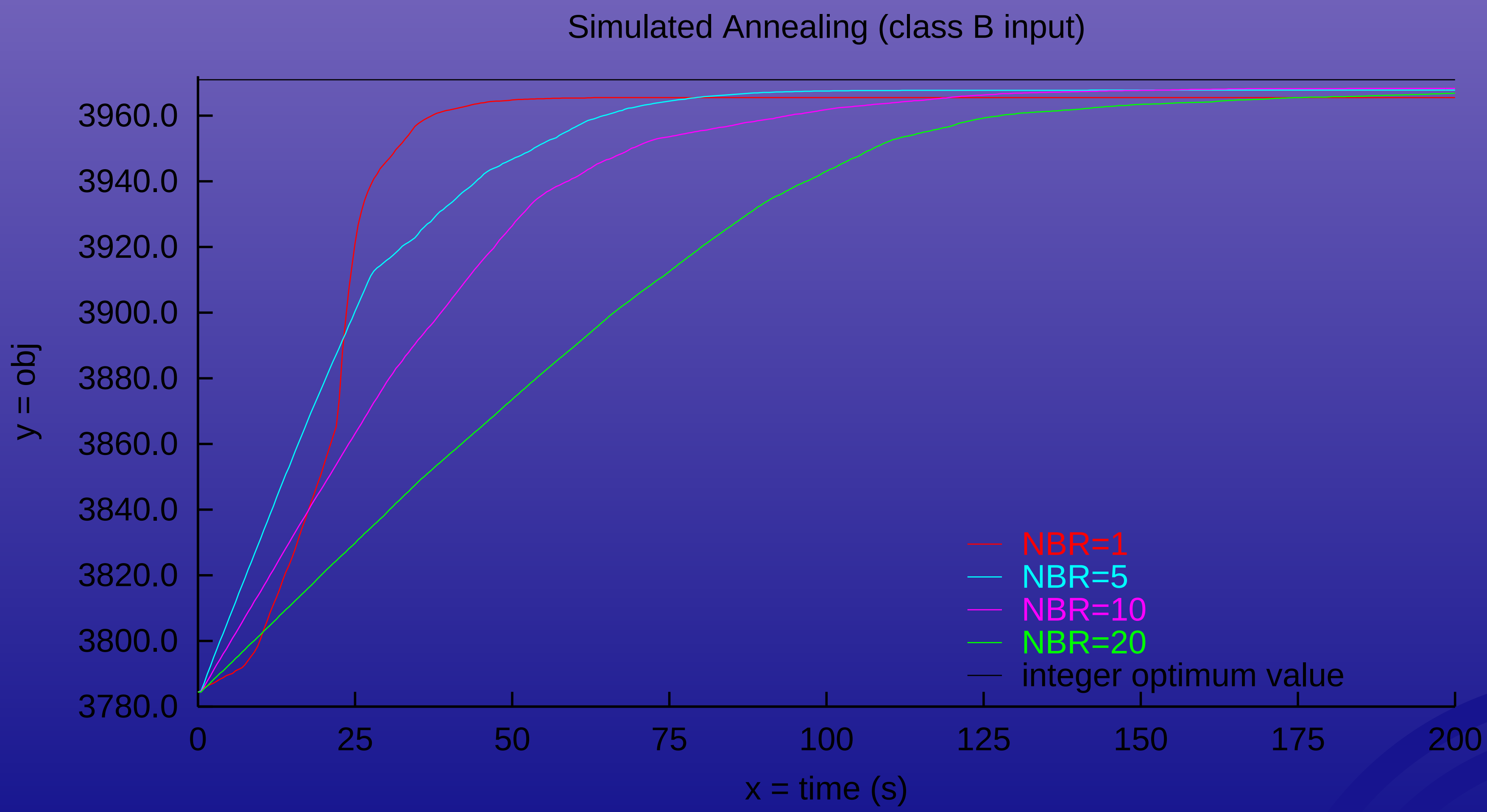
When the selected neighbor is not accepted the subneighborhood enlarges itself. In that way every time the test is made against the most probable neighbor.

Comparisons



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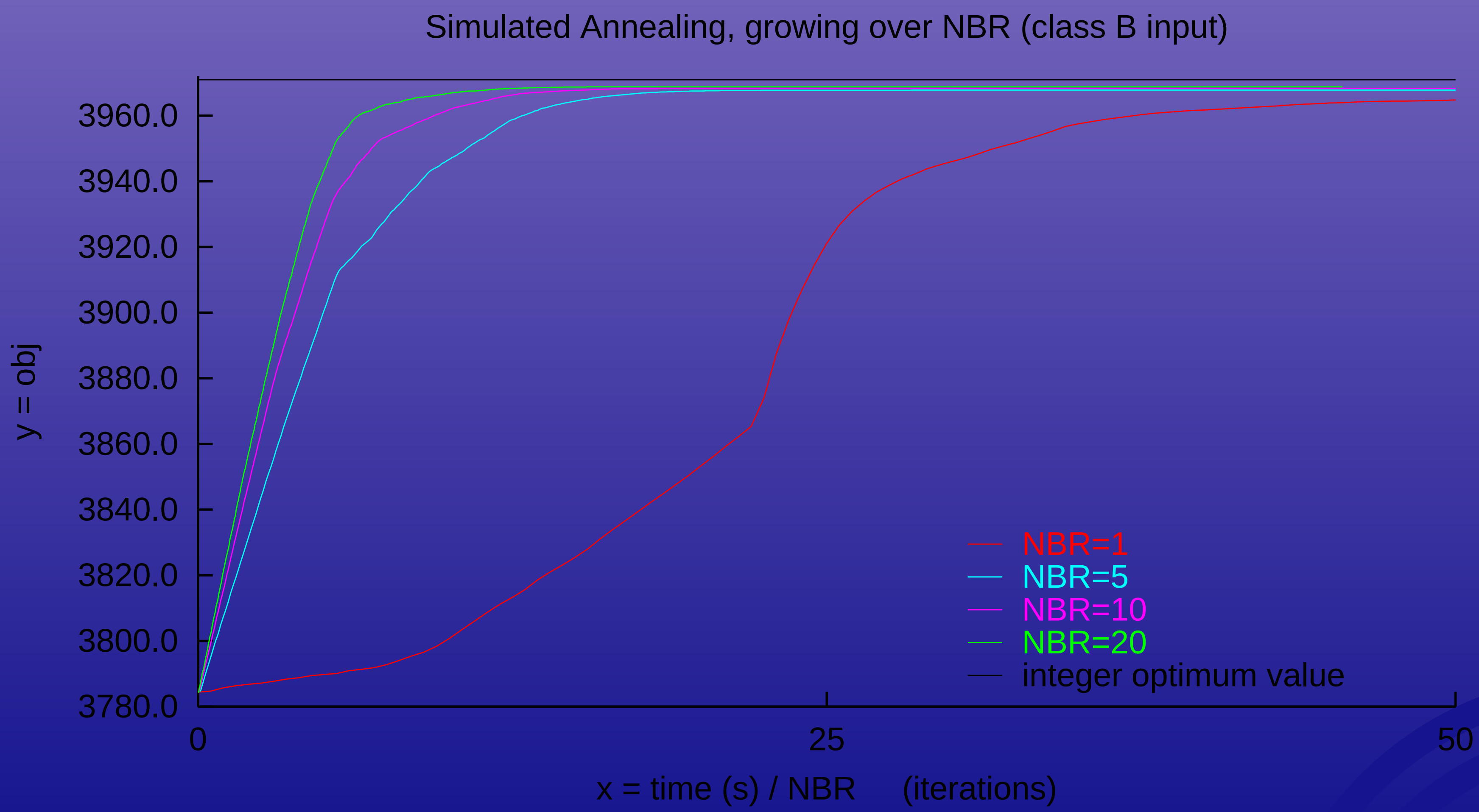
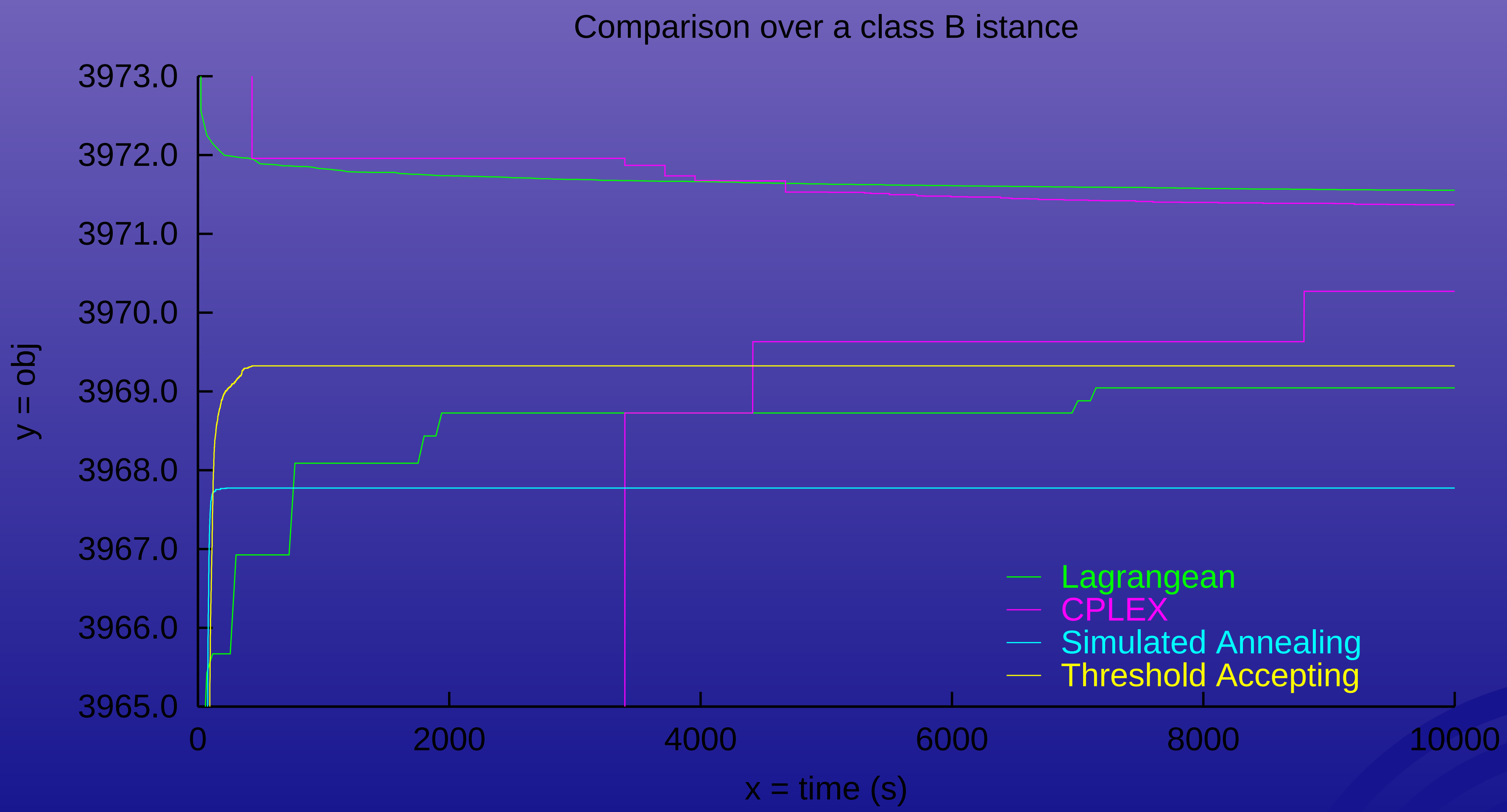


Comparisons of all the methods



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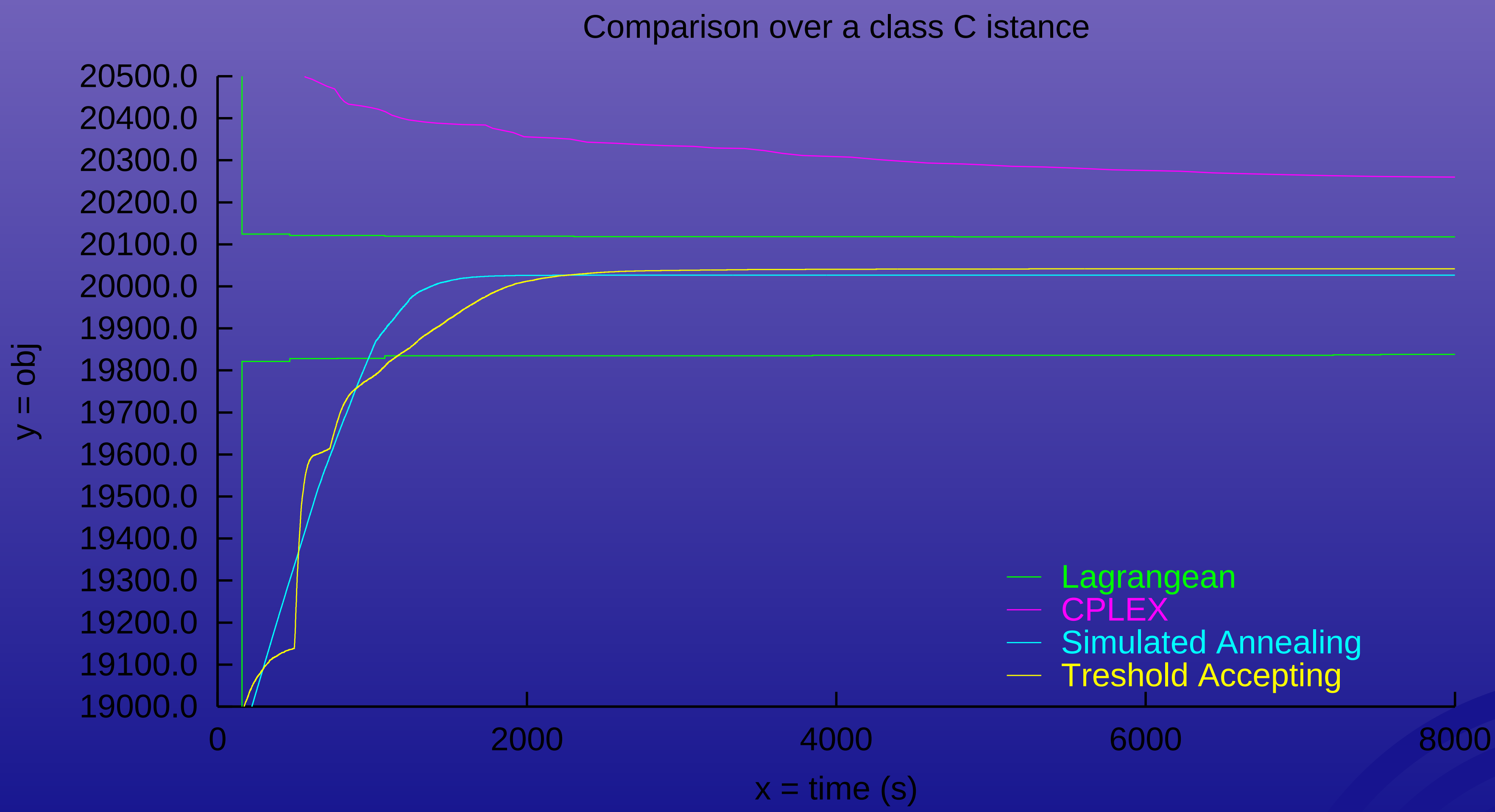

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