1 Optimal order picking in automated warehouses

An automated warehouses contains a matrix of locations where identical boxes are stored. A given position in the matrix is the I/O interface. A crane can visit the locations and the I/O position to execute pick-up and delivery operations.

Consider the following taxonomy:

1. Capacity of the crane: 1, 2, $k > 2$.
2. Dimensions of the warehouse: 1 (locations along oone or two lines with an endpoint in the I/O position), 2 (locations organized in a matrix), 3 (every location has double depth and can host two boxes: one in the front position is directly accessible from the rail; the other one, in the rear position, can be accessed only after picking-up the box in the front position);
3. Operations: pick-up only (P): they can be executed in any order; delivery only (D): they must be executed according to the input sequence; both pick-up and delivery (PD);
4. Fixed (F) or variable (V) sites; in the latter case not a single site but a subset of sites is associated with each order; one of themm must be visited to satisfy the order.
5. Deadlines: without deadlines (N) or with deadlines (Y); deadlines are expressed as max number of trips before satisfying the order or as maximum allowed completion time.
6. Objectives: minimize the total time (t), minimize the total energy consumption (e).

Some simple variants have been proven to be polynomially solvable. All complex (NP-hard) variants are still to be solved.

Assuming:

1. Capacity: 2, $k > 2$.
2. Dimensions: 1, 2, 3.
3. Operazions: pickup (P), delivery (D), mixed (PD).
4. Sites: fixed (F) or variable (V).
5. No deadlines.
6. Objective: minimize the total distance traveled.

we can classify models with a four fields notation.

Tha basic variation $2/1/P/F$ is polynomial.

The variants with one complicating feature are polynomial: $k/1/P/F$, $2/2/P/F$ and $2/3/P/F$, $2/1/D/F$ and $2/1/PD/F$, $2/1/P/V$.

Among the variants with two complicating features, these are polynomial: $2/2/D/F$ and $2/3/D/F$, $2/2/P/V$ and $2/3/P/V$, $2/1/D/V$.

Variant $k/1/D/F$ is polynomial for k fixed.

The variants $k/2/P/F$ e $k/3/P/F$ are likely to be NP-hard because they are special cases of the Capacitated VRP with unit demand (that is NP-hard).

The variant $k/1/P/V$ with two lines is polynomial.

The variant $2/1/PD/F$ on a single line is still open.

Among the variants with three complicating features these are polynomial: $2/2/D/V$ and $2/3/D/V$.

All other variants are open. An optimization algorithm (dynamic programming, branch-and-bound...) is needed for each of them.

Reference: Barbato et al. (2019)

Suitable for a project for the O.R. Complements course or a master thesis.