INTERFACES
Vol. 7, No. 1
November 1976

APPLICATION OF THE CUBE-PER-ORDER INDEX RULE FOR STOCK LOCATION IN A DISTRIBUTION WAREHOUSE

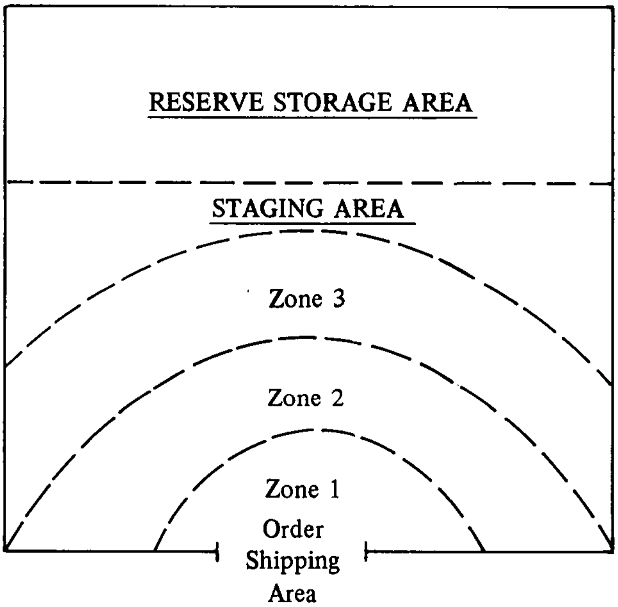
CARL KALLINA
and
JEFFREY LYNN
American Can Company
Greenwich, Conn.

ABSTRACT. Consider a distribution warehouse divided into reserve storage and staging areas. The warehouse stores a variety of items and receives orders for any combination of items. Goods are moved from reserve storage to staging area, where they are selected to fill the given orders. The problem is to locate items in the staging area in order to minimize the expected labor costs of order selection. Several years ago, J. L. Heskett [3] proposed a criterion, called the cube-per-order index (CPO) rule, for solving this problem. The criterion was justified heuristically by means of numerical examples. Recently [5], one of the authors has shown that the class of problems considered by Heskett can be formulated as a linear program, and that the CPO rule is in fact the optimal solution. In this present paper, we will (1) summarize some basic background material, (2) describe the computational steps for implementation of the CPO rule, and (3) discuss some practical conclusions gathered from experience in actually applying the rule to assist in warehouse layout.

I. Introduction and General Problem Definition

In 1963, in a journal relatively unknown to the management science community, J. L. Heskett [3] proposed a criterion, which he called the cube-per-order index (CPO) rule, for the placement of stock in a distribution warehouse so as to minimize the labor cost associated with assembling items from stock in the warehouse staging area to fill customer orders. The criterion was justified heuristically by means of numerical examples, with no claims made in the direction of optimality. Although the CPO rule can be used in a wide variety of materials handling situations, our search of the literature has not revealed any reported application of the rule outside of Heskett's papers [3], [4]. Applications almost surely must exist, but it is safe to say that the CPO rule is not a household word in materials handling circles. One of the present authors [5] has recently shown that the CPO rule is the optimal solution to the class of problems considered by Heskett. In this paper, we forego mathematical analysis in order to discuss some practical conclusions gathered from experience in actually applying the CPO rule. We will first summarize some basic background information needed to place the cube-per-order index rule in perspective. Much of this material can be found in Heskett's lucid articles, but they are somewhat inaccessible and we wish to emphasize the key physical assumptions in such a way as to facilitate our subsequent discussion of empirical results.

Conceptually, the kind of warehouse layout we are discussing is diagrammed in Figure 1. The warehouse is divided into a reserve storage and staging area. The warehouse stores a variety of items, and customers can place orders for any combination of items. We assume that the total warehouse inventory of each item at specified times is given independently, either from some inventory control policy or other considerations such as supply constraints (e.g., canned food arrives in full during the crop pack season). The stock for any given item is split between locations in reserve storage and staging area. When an order for an item is received, an operator of a fork lift truck (or some such conveyance device) travels to the location of that item in the staging area and loads the appropriate amount of the item onto his truck. When he returns to the order shipping area, shown on Figure 1, the item is transferred to some outgoing vehicle for delivery to the customer. This process is called *picking an order.* Similarly, a *pick* refers to one trip of the fork lift truck to a location of one of the items on the order.



CONCEPTUAL DIAGRAM OF DISTRIBUTION WAREHOUSE

FIGURE 1

Depending on the warehouse design, item bulk and typical item distribution on orders, a number of picking disciplines can be employed at the warehouse. The three basic disciplines would be: (1) *out-and-back* selection of each item, used where order items are picked in large quantities, or where the entire job is carried out with the use of fork-lifts capable of picking only one item during one trip out and back to the order shipping area; (2) *picker routing,* whereby several items on the order are picked on a single trip through the staging area until the truck capacity is reached; (3) *conveyorized system with picking stations,* where a fixed or portable physical conveyor is built in as part of the staging area, and stations are set up along the conveyor route manned by pickers, who select and load items located in their station area. In what follows, we will confine our discussion to warehouses employing either an out-and-back or picker routing discipline. Conveyorized systems are so varied in design and capabilities it would be beyond the scope of this paper to present a general analysis of order picking for such systems. It would seem, however, that much of what follows would apply locally near the picking stations in such systems. Some recent work on storage assignment rules in conveyorized systems may be found in [2]. (In [5], it is shown that a picking station storage assignment rule used in [2] is a special case of the CPO rule.)

The key variable cost element in the order-picking process is the time spent by the fork-lift truck operator in his picks. We assume that with either picking discipline, the average speed of the trucks is the same for all orders. Also, assume that the items to be picked are sufficiently alike in volume, weight and geometric configuration that these factors do not appreciably affect the time taken to lift an item from the floor onto the trucks. It follows, therefore, that under and out-and-back discipline the labor cost of picking a given item is directly proportional to the distance from the order shipping area to the location of that item in the staging area. In the case of a picker-routing discipline, the operator's total time on a single trip can be thought of as distributed across all of the items picked on that trip, with the time allocated to a given item being proportionate to the item's distance from the order shipping area. These times can then be averaged across all the trips made by the operator during some period to arrive at an average picking cost per item. Again, it follows that this average item cost is proportional to the distance between the order shipping area and the item's location in the staging area.

Thus, in order to analyze variable order-picking costs it is sufficient to concentrate on the geometric configuration of the staging area. In Figure 1 we have divided the staging area conceptually into zones. In practice, of course, these zones would conform to the physical characteristics (aisle and bay locations) of the actual warehouse. On average, the distance from the order shipping area to a typical location in Zone 2 is twice as far as the distance to a typical location in Zone 1, etc. The number of zones so configured is arbitrary, except that the larger the number of zones selected the more precise will be the optimal stock locations yielded by application of the cube-per-order index rule, and the more accurate will be the projected cost saving compared to whatever stock location rule is currently employed.

The other important variable cost associated with the operation of such a warehouse is the labor cost of moving items into the reserve storage section of the warehouse initially and then from reserve storage to staging area, either when the amount in the staging area has fallen below some desired level or at some predetermined fixed intervals. Generally, restocking of the staging area would take place at a time when the staging area is not being used for order picking. Also, this restocking would generally be carried out by an out-and-back picking discipline between the item locations in reserve storage and staging areas, since the amount to be restocked would generally be in multiples of full truck load quantites.

The overall cost minimization problem defined by these considerations involves: (1) the relative assignment of total warehouse space between reserve storage and staging areas; (2) the location and relative amounts of items to store in both areas; (3) the frequency of restocking from reserve storage to staging area, for each item. (Recall that we have assumed the total warehouse supply of an item at specified times to be determined independently.) The solution to this problem would provide a total warehouse layout and restocking rule to minimize the sum of all relevant variable costs. This problem is far from trivial. For example, the decision on the relative amount of space allocated to reserve storage and staging area interacts with the decision on the space to be set aside for each item's stock (expressed as expected demand over some fixed number of days) in the staging area. Also, consider the fact that if the amount of stock held in and the relative size of the staging area were kept very low, in order to keep order-picking costs low by locating these stocks close to the order shipping area, then there would necessarily be higher restocking costs as more frequent trips were required from reserve storage to staging area. A cost tradeoff also arises in the converse scenario where larger stocks are kept in the staging area.

Another aspect to this overall problem not mentioned thus far is the availability of forecasts of customer demand for each item in the warehouse. Further, even assuming this demand is known with certainty, if the demand has clear seasonal patterns then an additional factor to consider is the appropriate time horizon over which the cost minimization solution is to be found.

It is clear, therefore, that the overall problem is quite complex. In [5] a linear programming formulation to one variation of the problem was developed. For our purposes here it is sufficient to note that its solution would generally require extensive computerization for routine application. (Ballou [1] developed an essentially similar linear programming formulation.) Instead of this approach, Heskett proposed that the overall problem be decoupled, and concentration be placed on relatively simple methods for reducing costs in order picking by itself, since it is empirically observed that order-picking costs over time are generally higher than restocking costs. (Intuitively, this is so because as goods move closer to the order shipping area they move in successively smaller quantities, requiring more and more material handling attention per unit.) The *cube-per-order index* is the location rule proposed by Heskett for minimizing order-picking costs in the staging area.

II. The Heskett Problem and Cube-Per-Order Index Rule

Decoupling of the overall problem, in order to focus on cost reduction in order picking, is accomplished by making the following assumptions:

A1: The warehouse is divided into reserve storage and staging areas whose location and relative sizes have been fixed.

A2: A fixed time horizon (e.g., one month) has been determined over which we wish to minimize order-picking costs.

A3: Over this time horizon, customer demand for each item in the warehouse is known with certainty, both in terms of the amount to be shipped and the number of orders to be received.

A4: A determination has been made of the maximum days demand of stock to be kept on hand in the staging area for each item, and adequate total floor space in the staging area is available for the amount specified (the days demand so determined can be different for different items).

With these assumptions, the problem is to locate items in the staging area in order to minimize the variable cost of order picking; equivalently, to minimize the total distance traveled by the truck operators for their picks during the selected time horizon.

There are, in general, four major determinants of this cost: *compatibility, complementarity, popularity* and *space*. Compatible items are those which can be stored next to each other without fear of contamination or other damage, and thus incompatible items (e.g., food and gasoline) must be stored in nonadjacent locations. Complementary items are those which frequently are demanded simultaneously by a customer on the same order (e.g., a bolt and matching nut, or spaghetti and tomato paste), and should be located close to each other. The most popular items, in terms of the average number of picks per day, should be placed closest to the order shipping area, since these items demand the greatest number of trips to their location. This easily understood criterion seems to be the one most commonly recommended for stock location e.g. [6]. Finally, it is desirable that those items requiring the least amount of warehouse space be placed closest to the order shipping area. One heuristic argument for this criterion is as follows: If each item-location is regarded as a point, and distance measured from the shipping dock, then the space criterion would yield a layout with a center of gravity closest to the shipping dock, thus implying the smallest average distance traveled per pick.

Clearly, these four criterion cannot in general all be met simultaneously, and some compromise is needed. First, we assume that all items in the warehouse are compatible. Next, assume that groups of highly complementary items are combined in classes to form new "items" on the master warehouse list. These new item entries may either be in addition to or totally replace the individual items making up the classes, depending on the degree of complementarity. Having done this, we can now specify the steps involved in implementing the cube-per-order index rule, which establishes a quantitative tradeoff between the dual objectives of placing closest to the order shipping area those items taking up the least space, and also those items which are most popular. It will be seen that these steps involve nothing more than a sequence of data tabulations and simple arithmetic calculations.

1. Prepare a scale diagram of the entire warehouse, noting aisle locations, storage areas and other significant physical characteristics affecting truck movements, such as obstructions, interior walls, etc. The capacity of each between-aisle storage area is to be indicated, expressed in cubic feet. (Other volume surrogates should be used if they are more natural for the items in the warehouse. For example, in a canned food warehouse, items are packed in cases, and the cases stored in standard pallet loads. For such a warehouse, storage capacity is more conveniently expressed in "number of pallets" rather than cubic feet.)

2. Divide the staging area conceptually into zones which represent different average distances to the order shipping area (as we did, for example, in Figure 1), and note the zones on the scale diagram. Also, the capacity of each zone is to be tabulated.

3. Make a list of all items carried in the warehouse. Next to the item, record the cubic footage required for storage of the smallest shipping unit of that item for which a customer order could be placed.

4. For each item, record the forecast made for the expected number of orders to be received during the time horizon, and the expected number of shipping units to be delivered. We assume that each order for an item represents one pick of that item (if more than one pick-per-order is expected to

occur frequently, then replace the order estimate for the item with an estimate of the total number of picks for the item).

5. Compute, and record separately for each item, the average number of shipping units per order, the average number of cubic feet of storage required per order, and the average number of orders to be received per shipping day during the time horizon. The last two numbers are then multiplied together with the number of days specified as the maximum days demand target to yield the amount of cubic footage of staging area space to be set aside for each item. The *cube-per-order index* is the *ratio* of this latter number to the average number of orders per shipping day. (This definition of the index indicates its function of quantifying the intuitive tradeoff between space and popularity. More directly, the index is defined as the product of the days demand target and average storage space per order.)

6. All items on the list are now ranked based on their cube-per-order index, the item with the lowest index being ranked first. From this ranking, the staging area layout follows immediately. The lowest index item goes in Zone 1, closest to the order shipping area, using up as much space required to accommodate the maximum days demand target of units. If not enough space is available in Zone 1, the amount left over goes in Zone 2. On the other hand, if any space remains in Zone 1, the next lowest index item is also placed in Zone 1 in the appropriate amount up to the capacity of Zone 1. This process continues until all items have been placed in their proper zones, successively further away from the order shipping area.

Numerical examples to illustrate this procedure can be found in [3] and [4]. Formulation of the Heskett problem as a linear program is carried out in [5], and the CPO rule is shown to yield an optimal solution to this program.

III. Experience With Application of the Cube-Per-Order Index Rule

In order to arrive at a problem definition susceptible to solution by the cube-per-order index rule, we have had to make a number of simplifying assumptions with respect to the overall problem in §1. In practice, we have found no fundamental objection to this line of argument on the part of experienced warehouse personnel. They are generally favorable to the key assumption; namely, in the absence of a general solution to the overall problem it is desirable to focus primarily on reducing order-picking costs. This acceptance would probably be achieved regardless of the computer sophistication of the firm in question. It has been enhanced in our applications by the inexperience and even mistrust with computer based analytic models by the firms with which we have dealt.

Several features of the rule seem to favor its acceptance in practice. The rule provides a readily understood structural framework in which to view a complex operational problem. If, in fact, the optimal solution to the problem could only be obtained by application of the simplex algorithm, it is likely that warehouse personnel would prefer living with an easily understood suboptimal solution, such as the popularity or space criterion. In addition, the rule is easy to implement both in terms of time and the level of manpower skill required. It also has a great deal of flexibility to changing conditions.

One empirical observation we have made is that the CPO rule frequently yields staging area layouts radically different from those resulting from application of either the popularity or space criterion. Also, it can be expected that the rule's layout will differ in some degree from the actual layout currently in existence. Specific cost tradeoff calculations with selected items can be carried out to convince skeptical warehouse personnel of the validity of the rule in cases which to them are not intuitively correct.

In certain situations the applicability of the cube-per-order index rule can be enhanced by specific changes to the staging area configuration. For example, in a canned food distribution warehouse cans are packed in cases, and these cases are stacked in standard lot sizes on pallets which in turn are stacked so many deep and so many high in rows. Many of the items in inventory are characterized by low average order size (in cases) but high order frequency, and therefore these items have a very low cube-per-order index. It is therefore desirable to place all such items near the order shipping area. This would be very wasteful of space, however, if the items were stored in a conventional pallet row configuration, since a target level of days demand for each item might take up much less than a normal full pallet row. One solution to this problem is to install multi-level storage racks of some kind close to the order shipping area which would be dedicated to these items. In this way small quantities of the items can be stored together compactly and still be easily accessible for picking.

If the staging area of the warehouse is initially empty, then the optimality of the cube-per-order index rule is a self-evident cost justification for implementation. If, as is more likely, the staging area is initially filled, then a calculation must be made of the expected saving in variable picking costs of the index rule layout compared to the current layout. This saving must be sufficient to offset the one-time cost of moving items from where they are now to where they should be using the rule. If the chosen layout time horizon is relatively long, this cost can be avoided by the expedient of gradually moving items to their proper locations as determined by the rule as these locations are emptied out, in the course of order-picking, of whatever is presently in them.

IV. Considerations on the Overall Problem

As discussed in §1, the overall problem of minimizing total variable costs of stock location and movement is quite complex compared with the reduced Heskett problem. Except for actually producing a computer-generated solution to a linear program, it is difficult to offer any heuristic guidelines for attacking the overall problem. Heskett and also Ballou [1] offer some guidelines based on their experiences, but we have found much of their advice inapplicable to the specific situations we have encountered.

The best advice we can offer is to begin with implementation of the cube-per-order index rule, and plan for periodic re-examination of and experimentation with the basic assumptions behind the rule. For example, simulations can be carried out using varying values for the layout time horizon, the target days demand in the staging area for each item, and the relative amount of warehouse space assigned to reserve storage and staging areas. In these simulations the goal would be to reduce overall costs, not just order-picking costs alone. Also, it is evident that the required estimates of future demand will never be perfect, and some efforts to improve forecast accuracy will be appropriate. Finally, the occurrence of complementary items should be watched for and such items should be grouped together in the staging area.

The key determinants of overall layout costs not considered in developing the cube-per-order index rule are the layout of the reserve storage area, the frequency of restocking from storage to staging area, and the overall inventory control policy in effect at the warehouse. Once again, gradual experimentation and simulation seems to be the best approach to manipulating these factors in order to reduce overall costs. Heskett [4] provides some general guidelines and numerical examples in this regard which may prove appropriate in some situations.

V. A Specific Cost Analysis

Although the CPO rule is the optimal solution to the Heskett problem, it is not advisable to resort solely to this argument when trying to persuade skeptical warehouse personnel to adopt the CPO rule for their operations. However tedious it may be, specific cost comparisons should be made using actual company data. It is reassuring, however, and quite unusual, to know that some cost saving can always be achieved no matter what data is to be analyzed.

We will summarize the results of a specific cost analysis similar to one actually carried out for a canned food distribution warehouse. This warehouse has a high seasonality in its demand pattern, and a one month time horizon was chosen for minimization of order-picking costs. It was further determined that a restock frequency of one or two days for each item was desirable. Those 18 items having the highest annual number of orders were selected for analysis out of a total of 200 items. The selected items accounted for about 35% of annual orders. For each of two months, January and July, the required estimates of demand were developed. For each of the two restock policies, adequate floor space in the picking area was set aside to accommodate the necessary amount of stock. The floor space in each case was divided into 10 zones and relative picking costs from 1 to 10 assigned.

Three different assignment rules — CPO, popularity and space — were applied to yield staging area layouts. Average daily picking costs were calculated in each case. The results, normalized with CPO at a base of 100, are presented in Figure 2. From examples such as this, and a general understanding of the canned food processing industry, we would guess that in most canned food distribution warehouses a savings in order-picking costs of about 5-10% could be achieved by implementation of the CPO rule.

RELATIVE PICKING COSTS OF THREE STOCK LOCATION RULES—ONE MONTH TIME HORIZON

		CPO	Popularity	Space
Case I:	January 1 day demand target	100	104.4	110.5
Case II:	January 2 day demand target	100	104.7	103.8
Case III:	July 1 day demand target	100	104.2	113.1
Case IV:	July 2 day demand target	100	109.2	102.5

FIGURE 2

This particular study also exhibited the phenomenon referred to above; namely, that the CPO can yield layouts substantially different from those achieved by the popularity or space criterion. Considering Case I of Figure 2, and assigning letters A through R to the 18 items according to their CPO rule ranking (the lowest CPO index item is assigned to A), we have the following three layouts:

CPO:	$A\ B\ C\ D\ E\ F\ G\ H\ I\ J\ K\ L\ M\ N\ O\ P\ Q\ R$
Popularity:	$C\ E\ G\ I\ J\ L\ M\ N\ A\ B\ P\ D\ Q\ F\ H\ K\ O\ R$
Space:	$R\ O\ K\ H\ F\ D\ B\ A\ M\ P\ Q\ L\ J\ I\ N\ E\ C\ G$

Note, for example, that the two items with the lowest CPO indices rank only ninth and tenth, respectively, using the popularity criterion, and no better than seventh by the space criterion. Also, item R is first by the space criterion but last by the CPO index rule.

The specific application which prompted this cost analysis had some related aspects worth mentioning. The study was commissioned by a new company manager of operations with no prior experience in materials handling. The warehouse supervisor he inherited had previously installed storage racks in the picking area dedicated to items with low order size but high order frequency, and also established his own heuristic system for stock location. Unfortunately, he was unable to explain to the manager why he was doing what he was doing, and the manager had become fearful that labor costs were out of control in the warehouse. As a result of our analysis, it became clear that the storage racks had been a brilliant idea, for reasons discussed in §III. Also, it turned out that the actual stock locations were remarkably similar to what was called for by the CPO rule. *Thus the story had a happy ending all around: the manager was assured that warehouse costs were under control and the warehouse supervisor got the manager off his back.*

VI. Conclusions

We have no illusions that the optimality of the CPO rule represents a major breakthrough in materials handling methodology. We have emphasized in this paper that the rule applies to a restricted version of a much broader and difficult problem. Nevertheless, it seems noteworthy to have come across a complex real world situation where heuristic methods achieved an optimal solution before the problem was considered theoretically. Such evolution, of course, has been the rule in many quantitative physical sciences, most noteably astronomy and mechanics. In OR/MS, however, for a variety or reasons the recent direction has been from theory to practice, with the transition not always being successful. The present paper may indicate that this "cutting off of the roots" direction of the profession is not inevitable, but that there are still numerous real world situations waiting to be recognized as candidates for theoretical OR/MS analysis, to the benefit of theoreticians and practitioners alike.

References

1. Ballou, Ronald H., "Improving the Physical Layout of Merchandise in Warehouses," *Journal of Marketing*, Vol. 31, July, 1967, pp. 60-64.
2. Hausman, W. H., Schwarz, L. B., and Graves, S. C., "Optimal Storage Assignment in Automatic Warehousing Systems," Management Science, Vol. 22, No. 6 (February, 1976), pp. 629-638.

3. Heskett, J. L., "Cube-Per-Order Index — A Key To Warehouse Stock Location," *Transportation and Distribution Management,* Vol. 3, April, 1963, pp. 27-31.
4. ———, "Putting The Cube-Per-Order Index To Work in Warehouse Layout," *Transportation and Distribution Management,* Vol. 4, August, 1964, pp. 23-30.
5. Kallina, C., Optimality of the Cube-Per-Order Index Rule for Stock Location in a Distribution Warehouse," Working Paper, American Can Company, March, 1976.
6. Neal, F. L., "Controlling Warehouse Handling Costs by Means of Stock Location Audits," *Transportation and Distribution Management,* Vol. 2, May, 1962, pp. 31-33.