Exercise 1

Preprocess this MILP instance to tighten its formulation.

$$\min z = 24x_1 + 12x_2 + 16x_3 + 4y_1 + 2y_2 + 3y_3$$

$$\begin{cases} y_1 & +3y_2 & & \geq 15 \\ y_1 & & +2y_3 & \geq 10 \\ 2y_1 & +y_2 & & \geq 20 \\ y_1 & & & \leq 15x_1 \\ y_2 & & & \leq 20x_2 \\ y_3 & & & \leq 5x_3 \\ y \in R_+^3 & & & \\ x \in B^3 & & & \end{cases}$$

Solution

Solving the continuous relaxation of the problem one obtains $z_{LP} = 58.7$.
Consider the binary variables, one at a time, and tentatively fix them at 0 (probing).

$$x_1 = 0 \Rightarrow y_1 = 0 \Rightarrow \begin{cases} (1)\ y_2 \geq 5 \\ (2)\ y_3 \geq 5 \Rightarrow x_3 = 1 \\ (3)\ y_2 \geq 20 \Rightarrow x_2 = 1 \end{cases} \Rightarrow Soluz.\ \ [0,1,1,0,20,5], z = 83$$

We can tighten constraint (1): $x_1 = 0 \Rightarrow y_1 + 3y_2 \geq 60 \Rightarrow (1')\ 45x_1 + y_1 + 3y_2 \geq 60$.

$$x_2 = 0 \Rightarrow y_2 = 0 \Rightarrow \begin{cases} (1)\ y_1 \geq 15 \Rightarrow x_1 = 1 \\ (3)\ y_1 \geq 10 \end{cases}$$

We can tighten constraints (2) and (3):
$x_2 = 0 \Rightarrow y_1 + 2y_3 \geq 15 \Rightarrow (2')\ 5x_2 + y_1 + 2y_3 \geq 15$.
$x_2 = 0 \Rightarrow 2y_1 + y_2 \geq 30 \Rightarrow (3')\ 10x_2 + 2y_1 + y_2 \geq 30$.

$$x_3 = 0 \Rightarrow y_3 = 0 \Rightarrow (2)\ y_1 \geq 10 \Rightarrow x_1 = 1$$

We cannot tighten any constraint.

Replacing constraints (1), (2) and (3) with the stronger constraints (1'), (2') and (3') one obtaines a new formmulation whose linear relaxation has an optimmal solution with value $z' = 79.1$.
The integer optimal solution has value $z^* = 79.33$.

Exercise 2

Search for logical implications and clique inequalities.

$$\begin{cases} (1)\ 4x_1 & +x_2 & -3x_3 & \leq 2 \\ (2)\ 2x_1 & +3x_2 & +3x_3 & \leq 7 \\ (3)\ x_1 & +4x_2 & +2x_3 & \leq 5 \\ (4)\ 3x_1 & +x_2 & +5x_3 & \geq 2 \\ \quad x \in B^3 \end{cases}$$

Solution.

Let introduce the complemented varibales $\bar{x}$ in order to make all coefficients non-negative and all inequalities in the same form ($\leq$), as indicated here below on the left. Thhen we derive the implications reported on the right.

$$\begin{cases} (1)\ 4x_1 & +x_2 & +3\bar{x_3} & \leq 5 & \Rightarrow x_1 + \bar{x_3} \leq 1 \\ (2)\ 2x_1 & +3x_2 & +3x_3 & \leq 7 & \\ (3)\ x_1 & +4x_2 & +2x_3 & \leq 5 & \Rightarrow x_2 + x_3 \leq 1 \\ (4)\ 3\bar{x_1} & +\bar{x_2} & +5\bar{x_3} & \leq 7 & \Rightarrow \bar{x_1} + \bar{x_3} \leq 1 \\ \quad x \in B^3 \end{cases}$$

Now we can build the incompatibility graph and search for maximual cliques in it. Since the graph contains a

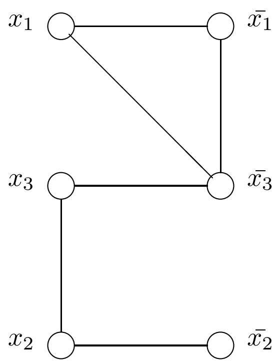

Figure 1: The incompatibility graph.

maximal clique (x_1, $\bar{x_1}$, $\bar{x_3}$), we obtain the clique inequality

$$x_1 + \bar{x_1} + \bar{x_3} \leq 1$$

from which we obtain $\bar{x_3} = 0$, i.e. $x_3 = 1$ and hence $x_2 = 0$. After fixing these two variables, all four constraints become redundant.