# *A branch-and-bound algorithm for the optimization of satellite observations*

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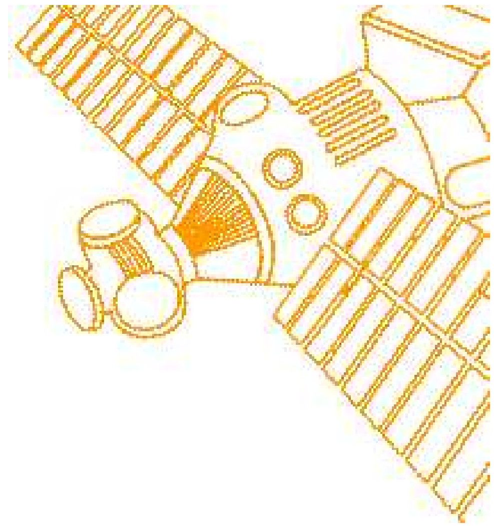
# An overview

- Nadir observations and Polar Operational Environmental Satellites (*POES*)
- The Swath-Segment Selection Problem (*SSSP*)
- An IP formulation
- Lagrangean bounding procedures:
  - Subgradient ascent
  - Lagrangean heuristic
- A branch-and-bound algorithm
- Experimental results (random instances)
  - Small instances (branching performed)
  - Large instances (gap at the root node)



# *Satellite observations*

Satellites observations have several applications

- Leading methereological observations (MetOp and COSMOSkyMed)
- Studying the Earth gravitational field and the geoid morphology (GOCE)
- Building extremely detailed maps (GLOBCOVER)

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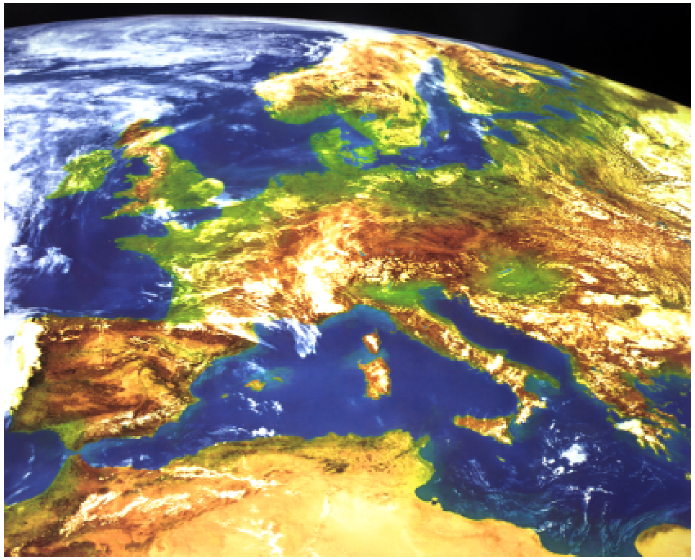

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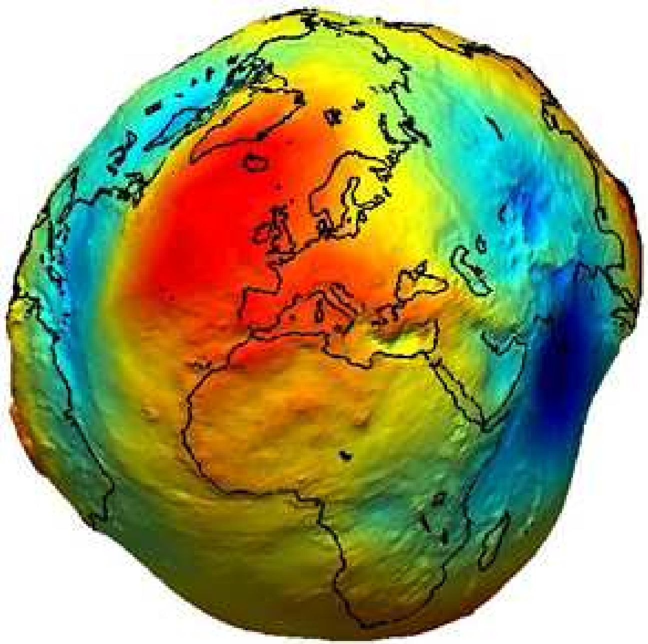

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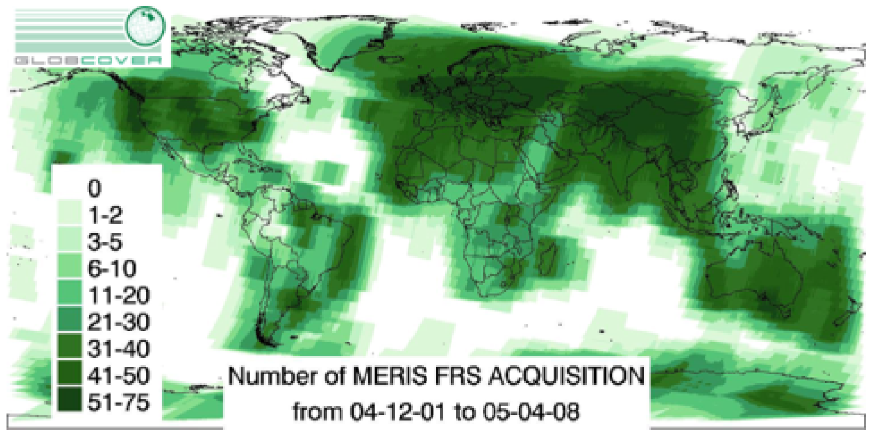
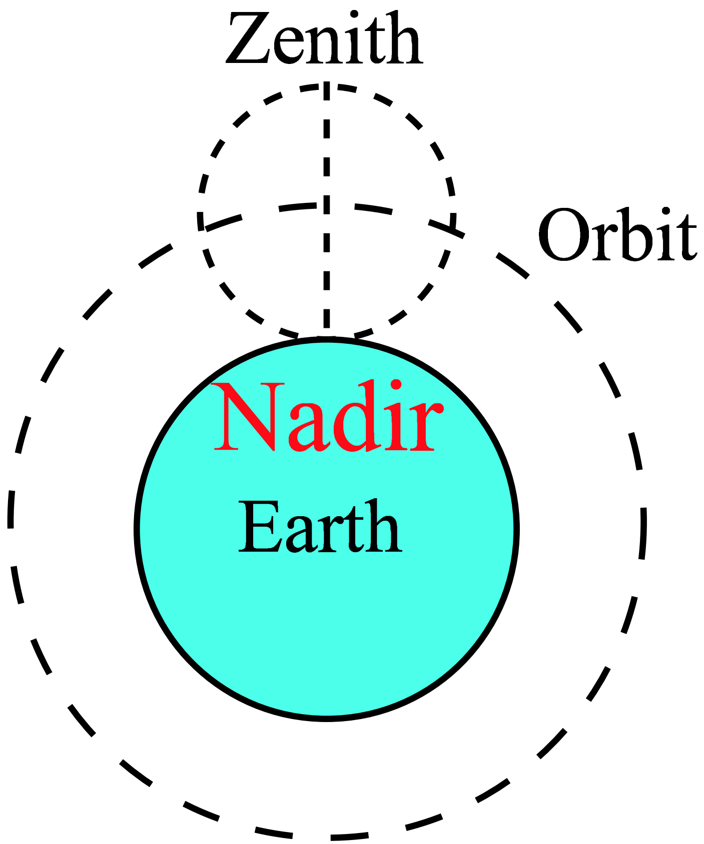


# *Nadir observations*

Nadir is the point on the ground below the satellite



# *Nadir satellites*

They are equipped with a fixed instrumentation nadir-oriented (pointing downwards)

- global coverage of the planet
- low-altitude transits

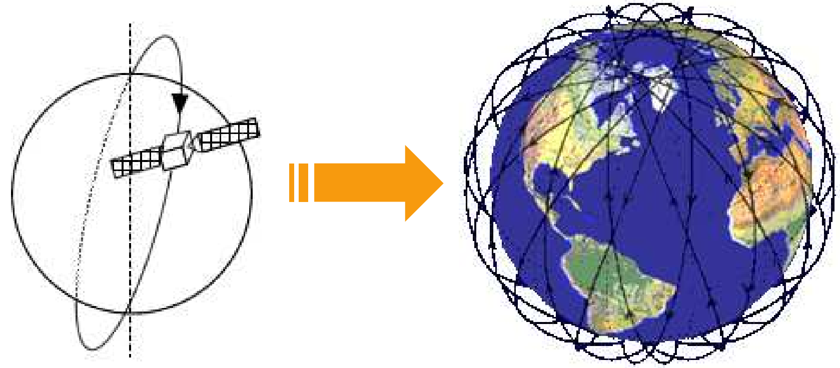
⇒ neat and detailed images

# *POE satellites*

## POES: Polar Operational Environmental Satellites

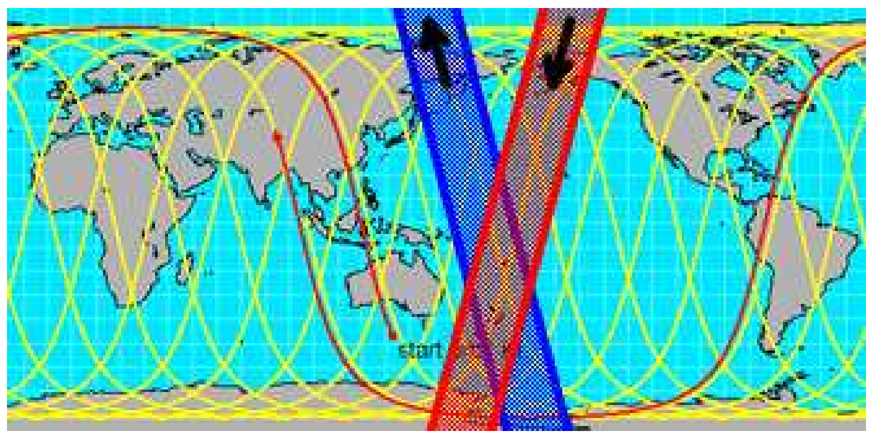
The orbit is *quasi-polar*

Since the Earth rotates,
the tracks of the transits cross
each other forming a grid



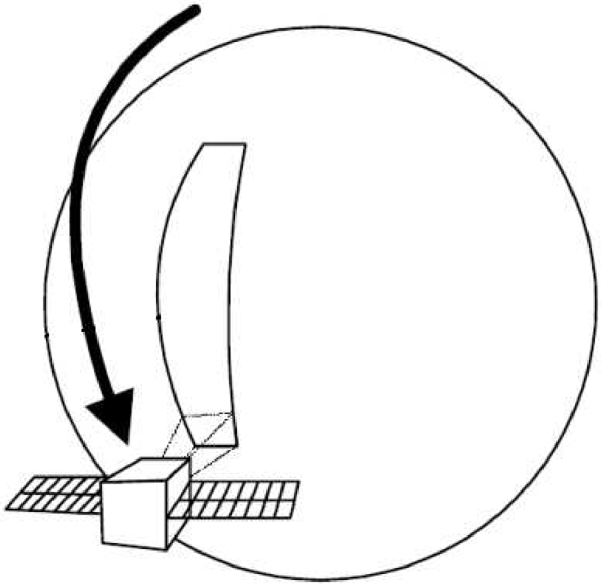
The satellite transits
in two directions

- descending
- ascending

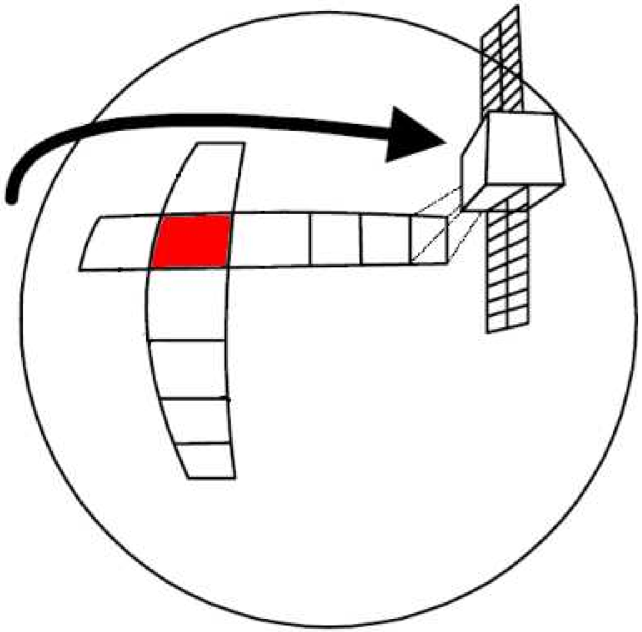


# Definitions (1)

Each satellite transit determines a strip named swath



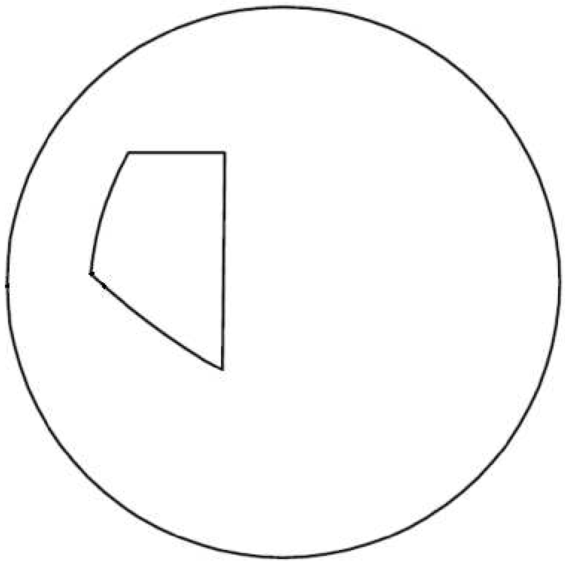
Due to the intersections, each swath is divided into areas named segments

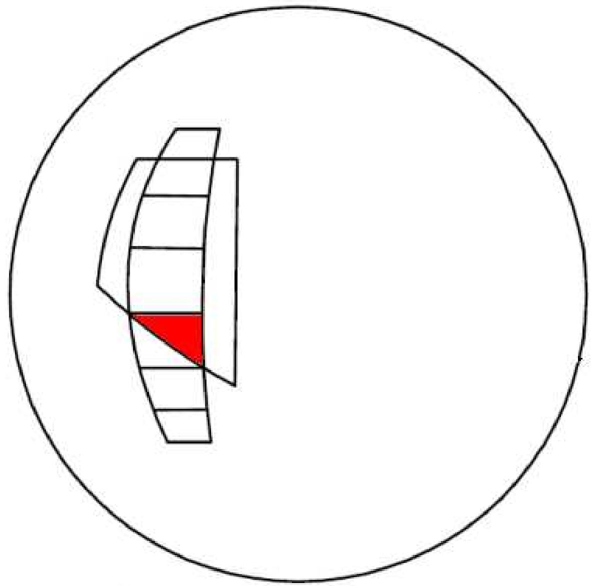


# Definitions (2)

Each portion of land which must be acquired is named target.

The intersection between a segment and a target is named shard.

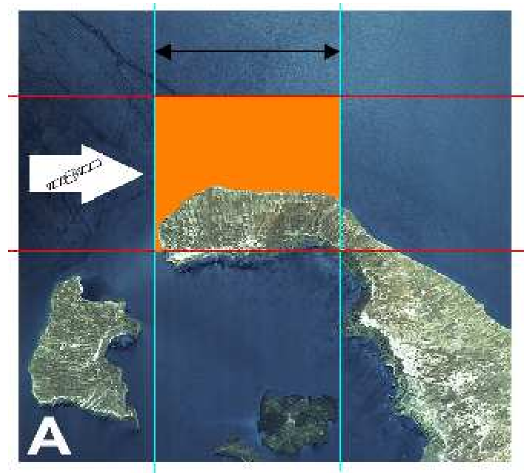




# *The Swath Segment Selection Problem*

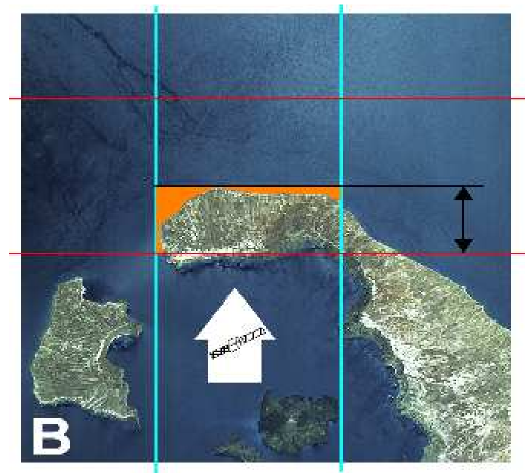
Define for each shard $(i, j)$

- a reward $r_{ij}$
- a horizontal $a_{ij}^{(h)}$ and a vertical $a_{ij}^{(v)}$ area




Objective function

- Acquire the most rewarding subset of images




Constraints

- Downlink capacities $d_i^{(h)}$ and $d_j^{(v)}$
- Acquire each shard at most once

# *References*

There is almost no literature on the subject.

- Greedy algorithm

  Muraoka H., Cohen R.H., Ohno T., Doi N.
  *ASTER observation scheduling algorithm*
  SpaceOps 98, Tokio 1998

- Branch-and-bound based on linear relaxation, solved by max-flow (requires $a_{ij}^{(h)} = a_{ij}^{(v)} \propto r_{ij}$!)

  Knight R., Smith B.
  *Optimal nadir observation scheduling*
  Fourth International Workshop on Planning and Scheduling for Space, Darmstadt 2004

# An IP formulation for the SSSP

$$\max \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij} \left( x_{ij}^{(h)} + x_{ij}^{(v)} \right)$$

$$\sum_{j=1}^{n} a_{ij}^{(h)} x_{ij}^{(h)} \leq d_i^{(h)} \qquad i = 1, \ldots, m$$

$$\sum_{i=1}^{m} a_{ij}^{(v)} x_{ij}^{(v)} \leq d_j^{(v)} \qquad j = 1, \ldots, n$$

$$x_{ij}^{(h)} + x_{ij}^{(v)} \leq 1 \qquad i = 1, \ldots, m \quad j = 1, \ldots, n$$

$$x_{ij}^{(h)}, x_{ij}^{(v)} \in \{0, 1\} \qquad i = 1, \ldots, m \quad j = 1, \ldots, n$$

- $x_{ij}^{(h)} = 1$ if shard $(i, j)$ is acquired along a horizontal transit
- $x_{ij}^{(v)} = 1$ if shard $(i, j)$ is acquired along a vertical transit

# *Lagrangean relaxation*

$$\max \qquad z = \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}\left(x_{ij}^{(h)} + x_{ij}^{(v)}\right)$$

$$\sum_{j=1}^{n} a_{ij}^{(h)} x_{ij}^{(h)} \leq d_i^{(h)} \qquad i = 1, \ldots, m$$

$$\sum_{i=1}^{m} a_{ij}^{(v)} x_{ij}^{(v)} \leq d_j^{(v)} \qquad j = 1, \ldots, n$$

$$x_{ij}^{(h)} + x_{ij}^{(v)} \leq 1 \qquad i = 1, \ldots, m \quad j = 1, \ldots, n$$

$$x_{ij}^{(h)}, x_{ij}^{(v)} \in \{0, 1\} \qquad i = 1, \ldots, m \quad j = 1, \ldots, n$$

# Lagrangean relaxation

$$\max \quad z = \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}\left(x_{ij}^{(h)} + x_{ij}^{(v)}\right) - \sum_{i=1}^{m}\sum_{j=1}^{n} \lambda_{ij}\underbrace{\left(x_{ij}^{(h)} + x_{ij}^{(v)} - 1\right)}$$

$$\sum_{j=1}^{n} a_{ij}^{(h)} x_{ij}^{(h)} \leq d_i^{(h)} \qquad i = 1, \ldots, m$$

$$\sum_{i=1}^{m} a_{ij}^{(v)} x_{ij}^{(v)} \leq d_j^{(v)} \qquad j = 1, \ldots, n$$

$$\sout{x_{ij}^{(h)} + x_{ij}^{(v)} \leq 1 \qquad i = 1, \ldots, m \quad j = 1, \ldots, n}$$

$$x_{ij}^{(h)}, x_{ij}^{(v)} \in \{0, 1\} \qquad i = 1, \ldots, m \quad j = 1, \ldots, n$$

# The Lagrangean subproblem (1)

$$
LR : \max z = \sum_{i=1}^{m} \left\{ \begin{array}{lll} \max & \xi_i^{(\lambda)} = \sum_{j=1}^{n} r_{ij}^{(\lambda)} x_{ij}^{(h)} & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij}^{(h)} x_{ij}^{(h)} \leq d_i^{(h)} & \\ & x_{ij}^{(h)} \in \{0,1\} & j = 1, \ldots, n \end{array} \right. +
$$

$$
+ \sum_{j=1}^{n} \left\{ \begin{array}{lll} \max & \phi_j^{(\lambda)} = \sum_{i=1}^{m} r_{ij}^{(\lambda)} x_{ij}^{(v)} & \\ \text{s.t.} & \sum_{i=1}^{m} a_{ij}^{(v)} x_{ij}^{(v)} \leq d_j^{(v)} & \\ & x_{ij}^{(v)} \in \{0,1\} & i = 1, \ldots, m \end{array} \right. +
$$

$$
+ \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}
$$

where $r_{ij}^{(\lambda)} = r_{ij} - \lambda_{ij}$

# *The Lagrangean subproblem (2)*

$$LR : \max z = \sum_{i=1}^{m} \xi_i^{*(\lambda)} + \sum_{j=1}^{n} \phi_j^{*(\lambda)} + \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}$$

where

- $\xi_i^{*(\lambda)}$: optimum of a knapsack problem on horizontal swath $i$
- $\phi_j^{*(\lambda)}$: optimum of a knapsack problem for vertical swath $j$
- $\sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}$ is constant (for a given multiplier matrix $\lambda$)

# *Lagrangean heuristic (1)*

- Horizontal heuristic:
  1. solve the "horizontal" knapsack problems to get $x_{ij}^{(h)}$
  2. set $a_{ij}^{(v)} = +\infty$ to remove all shards acquired ($x_{ij}^{(h)} = 1$)
  3. solve the "vertical" knapsack problems to get $x_{ij}^{(v)}$
- Vertical heuristic:
  1. solve the "vertical" knapsack problems to get $x_{ij}^{(v)}$
  2. set $a_{ij}^{(h)} = +\infty$ to remove all shards acquired ($x_{ij}^{(v)} = 1$)
  3. solve the "horizontal" knapsack problems to get $x_{ij}^{(h)}$
- Choose the best among the two solutions

# *Lagrangean heuristic (2)*

If $\lambda \neq 0$

- fix the shards acquired once in the Lagrangean solution
  - reduce the downlink capacities
  - solve the knapsack sub-problems considering the shards acquired twice and those not acquired
- use Lagrangean rewards $r_{ij}^{(\lambda)}$ instead of $r_{ij}$

Real-valued rewards $\Rightarrow$ Pisinger-Ceselli code

# Subgradient ascent

Given a heuristic solution $x_{LB}$ of reward $z_{LB}$

1. Solve the Lagrangean subproblem $\Rightarrow x_{UB}$ (reward $z_{UB}$)
2. Compute the current violation of the relaxed constraints

$$s = x_{UB}^{(h)} + x_{UB}^{(v)} - 1$$

3. Compute the update step $\quad T = t \, \dfrac{z_{UB} - LB}{||s||^2}$
4. Update the multipliers

$$\lambda'_{ij} = \begin{cases} 0 & \text{if} \quad \lambda_{ij} + Ts_{ij} < 0 \\ \lambda_{ij} + Ts_{ij} & \text{if} \quad 0 \leq \lambda_{ij} + Ts_{ij} \leq r_{ij} \\ r_{ij} & \text{if} \quad \lambda_{ij} + Ts_{ij} > r_{ij} \end{cases}$$

# The branching strategy

The branching shard $(i, j)$ is the one with maximum $\lambda_{ij}$

**(a)** acquired in both directions in the Lagrangean solution $(x^{(h)}_{LB_{ij}} = x^{(v)}_{LB_{ij}} = 1)$

**(b)** not acquired in the Lagrangean solution $(x^{(h)}_{LB_{ij}} = x^{(v)}_{LB_{ij}} = 0)$

Three subproblems

1. Horizontal acquisition: set $x^{(h)}_{ij} = 1$ and $x^{(v)}_{ij} = 0$
2. Vertical acquisition: set $x^{(h)}_{ij} = 0$ and $x^{(v)}_{ij} = 1$
3. No acquisition: set $x^{(h)}_{ij} = 0$ and $x^{(v)}_{ij} = 0$

The visit strategy is best-bound-first

# The benchmark problems

Two size classes

- 10 small sizes (100 to 10 000 shards)
- 4 large sizes (from 40 000 to 250 000 shards)

Three downlink capacities

- 20% or 30% or 40% of the total reward

Two ranges for rewards and areas:

- small (S): values in $[1; 100]$
- large (L): values in $[51; 100]$

($a_{ij}^{(h)} = a_{ij}^{(v)}$ or not)

14 sizes

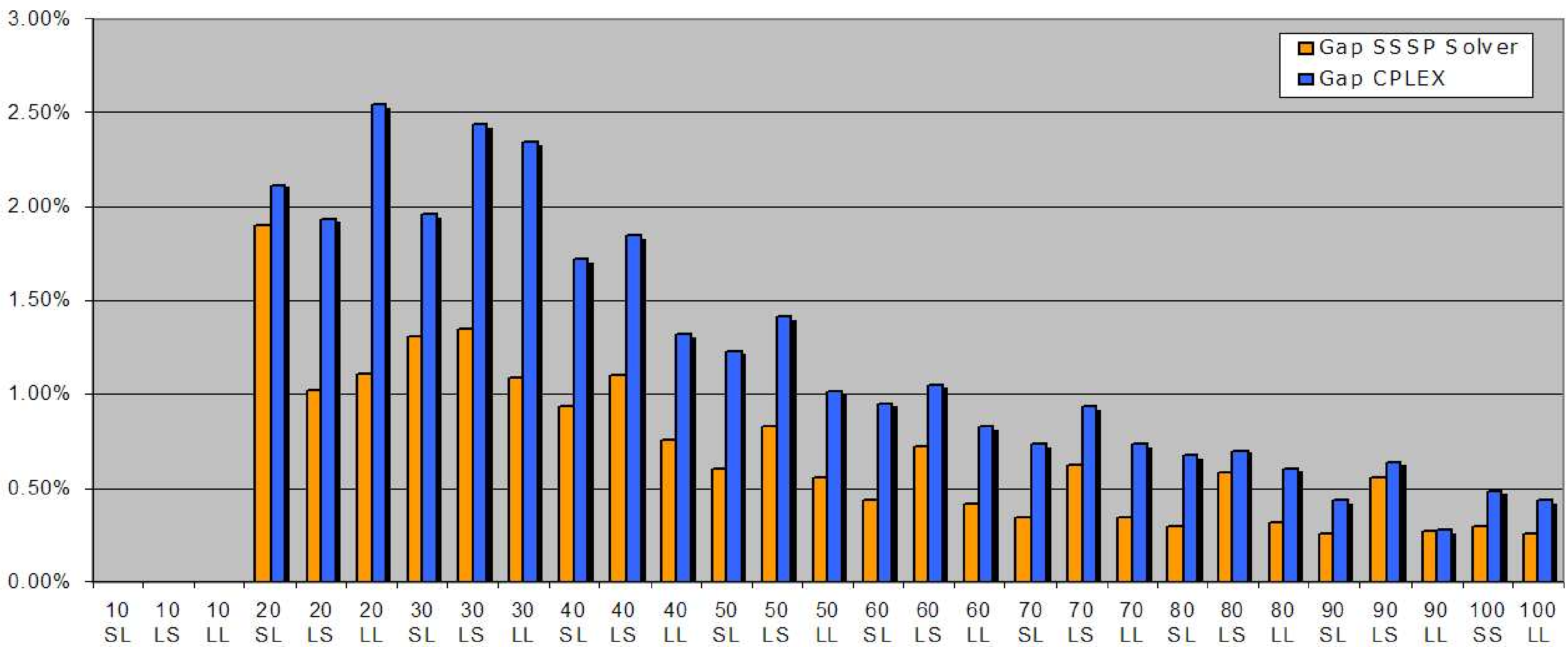
3 capacities

8 areas
and rewards

336
instances

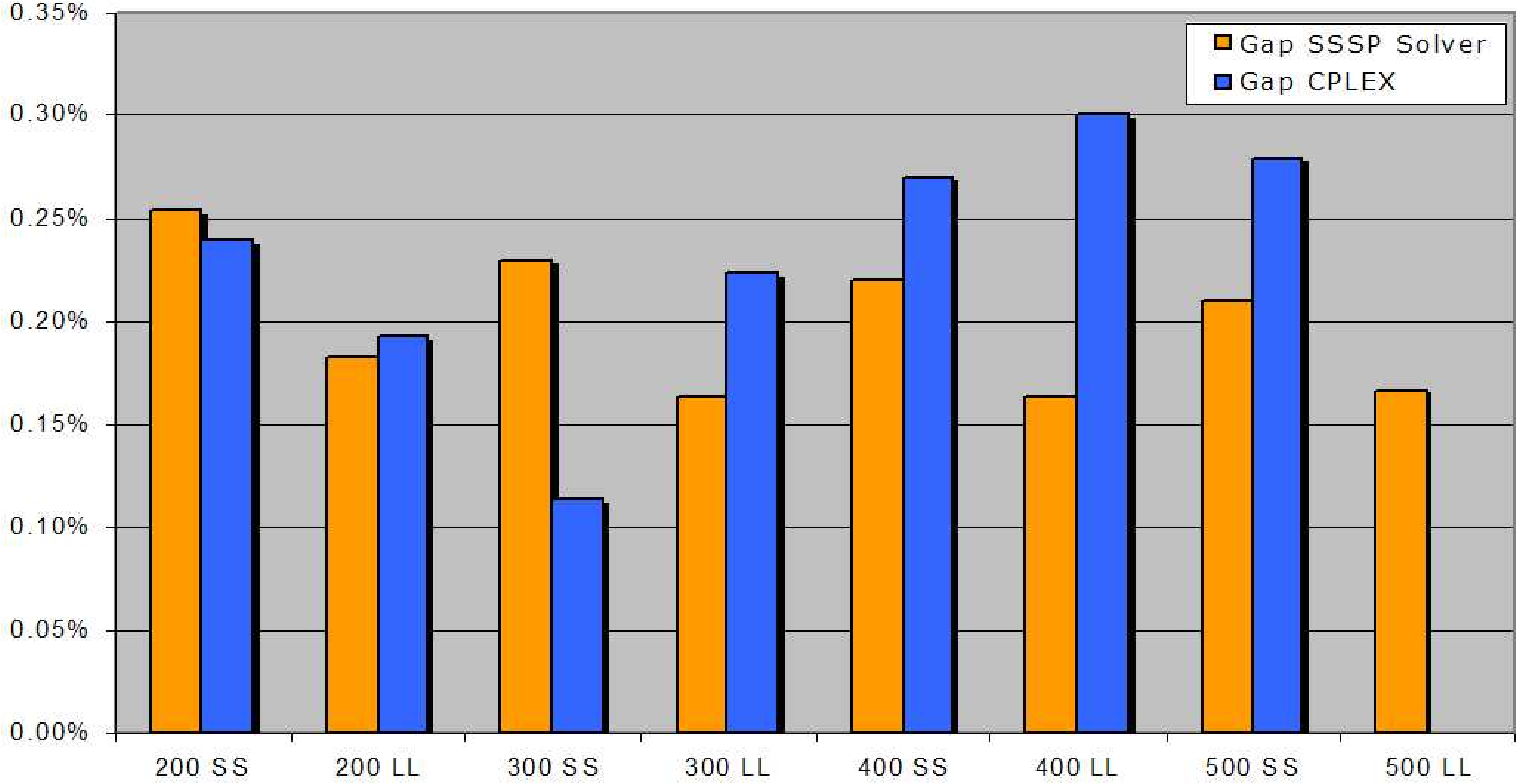
# *Experimental results (1)*

Gap achieved in 30 minutes by CPLEX 8.0 and SSSP solver on small instances



# Experimental results (2)

Gap achieved at the root node by CPLEX 8.0 and SSSP solver on large instances



# *Experimental results (3)*

- SSSP solver
  - required from 1 to 6 minutes on the SS instances
  - was stopped after 30 minutes on the LL instances
- CPLEX 8.0
  - always required more than 30 minutes
  - could not solve the root node in 60 minutes on the 500 LL instances

# *Conclusions*

- The problem is hard, even for rather small sizes
- Hardness increases from SS to SL, to LS and to LL instances (smaller range)
- The algorithm yields tighter bounds than CPLEX (both better heuristic solutions and better upper bounds)
- The Lagrangean relaxation is tighter than the linear one (though enhanced by general-purpose cuts)
- The branching proposed improves the bounds, while CPLEX branching most of the time does not

# *More on this...*

...can be found in:

R. Cordone, F. Gandellini, G. Righini

*Solving the swath segment selection problem through Lagrangean relaxation*

Computers and O.R., to appear.