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Gray-Scott Discrete Laplacia



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Lecture XIX: Laplacian operator

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### Programming in Python<sup>1</sup>

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Academic year 2024/25, I semester

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## Gray-Scott systems

Systems driven by the Gray-Scott's equation exhibit **Turing** patterns  $(D_u, D_v, f, k \text{ are constants})$ .

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u - u v^2 + f \cdot (1 - u)$$
$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + u v^2 - (f + k) \cdot v$$

- These give the **change** of *u* and *v* chemicals over time
- The diffusion term can be approximated on a grid by computing the discrete Laplacian

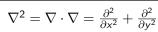


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# Discrete Laplacian



• Change on a grid (1-D):

$$\nabla f[n] = f[n+1] - f[n]$$
$$\nabla f[n] = f[n] - f[n-1]$$

• Second order change (1-D):

$$\nabla(\nabla f[n]) = \nabla(f[n+1]) - \nabla(f[n])$$

$$= (f[n+1] - f[n]) - (f[n] - f[n-1])$$

$$= f[n-1] - 2f[n] + f[n+1]$$

• In 2-D we do this independently on the 2 dimensions n, m:

$$\nabla(\nabla f[n,m]) = f[n-1,m] - 2f[n,m] + f[n+1,m] + f[n,m-1] - 2f[n,m] + f[n,m+1]$$

$$= f[n-1,m] + f[n+1,m] + f[n,m-1] + f[n,m+1] - 4f[n,m]$$

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#### Vectorization



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0	0	0	0	0
13	14	15	16	0
9	10	11	12	0
5	6	7	8	0
1	2	3	4	0
0	0	0	0	0
	9 5 1	9 10 5 6 1 2	9 10 11 5 6 7 1 2 3	9 10 11 12 5 6 7 8 1 2 3 4

-	29	-18	-19	-37
	-8	0	0	-13
	-4	0	0	9
	3	2	1	-5

$$X[1:-1, 2:] X[2:, 1:-1]$$

$$X[1:-1, :-2]$$
  $X[:-2, 1\cdot-1]$   $X[1\cdot-1, 1\cdot-1]$ 

1:-1] X[1:-1, 1:-1]
Same trick we used for "life", but we need to compute the 5-point stencil with these weights (see previous derivation):

0	1	0
1	-4	1
0	1	0

This way one can compute the Laplacian matrix using only vectorized plus.

### Experimental evidence



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Turing proposed his model on a pure theoretical basis, but we

have now also some experimental evidence:

Economou, A. D., Ohazama, A., Porntaveetus, T., Sharpe, P. T., Kondo, S., Basson, M. A., Gritli-Linde, A., Cobourne, M. T., Green, J. B. (2012). Periodic stripe formation by a Turing mechanism operating at growth zones in the mammalian palate. Nature genetics, 44(3), 348–351. https://doi.org/10.1038/ng.1090

### Consider also the diagonals



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Another approximation which takes into account also the "diagonals" is the *9-point stencil*.

1	1	1
1	-8	1
1	1	1

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