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Gray-Scott

Programming in Python¹

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Gray-Scott systems

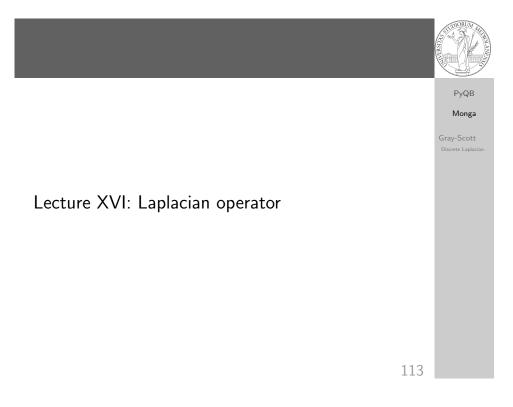
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Systems driven by the Gray-Scott's equation exhibit Turing **patterns** $(D_u, D_v, f, k \text{ are constants}).$

Gray-Scott

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u - uv^2 + f \cdot (1 - u)$$
$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + uv^2 - (f + k) \cdot v$$

- These give the **change** of u and v chemicals over time
- The diffusion term can be approximated on a grid by computing the discrete Laplacian



Discrete Laplacian

$$\nabla^{2} = \nabla \cdot \nabla = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}}$$
• Change on a grid (1-D):

$$\nabla f[n] = f[n+1] - f[n]$$

$$\nabla f[n] = f[n] - f[n-1]$$
• Second order change (1-D):

$$\nabla (\nabla f[n]) = \nabla (f[n+1]) - \nabla (f[n])$$

$$= (f[n+1] - f[n]) - (f[n] - f[n-1])$$

$$= f[n-1] - 2f[n] + f[n+1]$$
• In 2-D we do this independently on the 2 dimensions n, m :

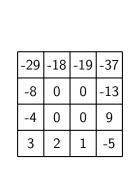
$$\nabla (\nabla f[n,m]) = f[n-1,m] - 2f[n,m] + f[n+1,m] + f[n,m-1] - 2f[n,m] + f[n,m+1]$$

$$= f[n-1,m] + f[n+1,m] + f[n,m-1] + f[n,m+1] - 4f[n,m]$$

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0	0	0	0	0	0
0	13	14	15	16	0
0	9	10	11	12	0
0	5	6	7	8	0
0	1	2	3	4	0
0	0	0	0	0	0



X[1:-1, 2:]

Same trick we used for "life", but we need to compute the 5-point stencil with these weights (see previous derivation):



This way one can compute the Laplacian matrix using only vectorized plus.

Experimental evidence



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Discrete Laplacian

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Discrete Laplacian

Turing proposed his model on a pure theoretical basis, but we have now also some experimental evidence:

Economou, A. D., Ohazama, A., Porntaveetus, T., Sharpe, P. T., Kondo, S., Basson, M. A., Gritli-Linde, A., Cobourne, M. T., Green, J. B. (2012). Periodic stripe formation by a Turing mechanism operating at growth zones in the mammalian palate. Nature genetics, 44(3), 348-351. https://doi.org/10.1038/ ng. 1090

Consider also the diagonals

Another approximation which takes into account also the "diagonals" is the 9-point stencil.

1	1	1
1	-8	1
1	1	1



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