

PyQB Monga

Programming in Python¹

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Gray-Scott systems

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Gray-Scott

Systems driven by the Gray-Scott's equation exhibit **Turing** patterns
$$(D_u, D_v, f, k \text{ are constants})$$
.

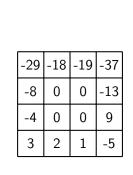
 $\frac{\partial u}{\partial t} = D_u \nabla^2 u - uv^2 + f \cdot (1 - u)$ $\frac{\partial v}{\partial t} = D_v \nabla^2 v + uv^2 - (f + k) \cdot v$

- These give the **change** of u and v chemicals over time
- The diffusion term can be approximated on a grid by computing the discrete Laplacian

Discrete Laplacian $\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ PyQB Monga • Change on a grid (1-D): $\nabla f[n] = f[n+1] - f[n]$ Discrete Laplacian $\nabla f[n] = f[n] - f[n-1]$ • Second order change (1-D): $\nabla(\nabla f[n]) = \nabla(f[n+1]) - \nabla(f[n])$ = (f[n+1] - f[n]) - (f[n] - f[n-1])= f[n-1] - 2f[n] + f[n+1]• In 2-D we do this independently on the 2 dimensions *n*, *m*: $\nabla(\nabla f[n,m]) = f[n-1,m] - 2f[n,m] + f[n+1,m] +$ f[n, m-1] - 2f[n, m] + f[n, m+1]= f[n-1,m] + f[n+1,m] + f[n,m-1] + f[n,m+1] - 4f[n,m]

118

0	0	0	0	0	0
0	13	14	15	16	0
0	9	10	11	12	0
0	5	6	7	8	0
0	1	2	3	4	0
0	0	0	0	0	0



X[1:-1, 2:]

Same trick we used for "life", but we need to compute the *5-point stencil* with these weights (see previous derivation):



This way one can compute the Laplacian matrix using only vectorized plus.

Experimental evidence



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Discrete Laplacian

119

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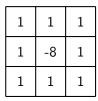
Discrete Laplacian

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Turing proposed his model on a pure theoretical basis, but we have now also some experimental evidence:

Economou, A. D., Ohazama, A., Porntaveetus, T., Sharpe, P. T., Kondo, S., Basson, M. A., Gritli-Linde, A., Cobourne, M. T., Green, J. B. (2012). Periodic stripe formation by a Turing mechanism operating at growth zones in the mammalian palate. Nature genetics, 44(3), 348–351. https://doi.org/10.1038/ ng. 1090 Consider also the diagonals

Another approximation which takes into account also the "diagonals" is the *9-point stencil*.





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120