Supplementary material of "A Scatter Search Metaheuristic and Improvements to an Exact Algorithm for the Weighted Safe Set Problem"

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1. Best known results

The following tables report the detailed best known results obtained in all runs of all known algorithms on the benchmark instances available at `https://homes.di.unimi.it/cordone/research/wssp.html`.

Each table refers to a benchmark or benchmark class, identified in the caption. The first three columns identify the graph corresponding to each row, reporting its number of vertices $|V|$ and edges $|E|$ and an identifier. The identifier is usually a progressive number, except for the real-world instances where it is the name of the graph and for the grid instances, where it is a description of its structure. Each graph yields two instances: a weighted one and an unweighted one. Correspondingly, the following two groups of three columns provide the value of the best known solution (UB) and the best lower bound, when known (LB)[1], for the weighted and the unweighted instance.

Tables 1 and 2 consider the 50 small-world graphs of benchmark `SW`, divided by initial degree ($d = 6$ and $d = 10$). Tables 3 and 4 consider the 50 regular graphs of benchmark `Reg`, divided by degree ($d = 5$ and $d = 10$).

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[1] The lower bound quickly degrades as the size of the instance increases, so it has been computed only for the smaller ones. We plan to improve the bounds and add new ones progressively as we find them.

Table 5 considers the 25 planar graphs of benchmark `Pla`. Table 6 considers the 5 two-dimensional and the 5 three-dimensional toroidal grid graphs of benchmark `Grid`.

			Weighted		Unweighted	
$\|V\|$	#	$\|E\|$	UB	LB	UB	LB
100	1	299	111	60	24	11
100	2	298	109	68	23	11
100	3	300	91	65	24	11
100	4	297	113	69	23	11
100	5	300	122	67	23	12
150	1	450	146		38	
150	2	449	141		34	
150	3	449	126		36	
150	4	449	158		33	
150	5	449	153		33	
200	1	600	169		45	
200	2	600	181		43	
200	3	597	193		49	
200	4	600	192		47	
200	5	600	225		47	
250	1	750	253		58	
250	2	749	249		60	
250	3	749	217		59	
250	4	748	255		58	
250	5	749	285		60	
300	1	899	232		64	
300	2	899	267		64	
300	3	899	311		73	
300	4	898	322		71	
300	5	899	287		67	

Table 1: Best known results for the instances with $d = 6$ of benchmark `SW`

			Weighted		Unweighted	
$\|V\|$	#	$\|E\|$	UB	LB	UB	LB
100	1	498	141	85	24	11
100	2	498	116	84	23	11
100	3	496	120	86	24	11
100	4	498	163	95	23	11
100	5	498	138	87	23	12
150	1	748	185		38	
150	2	750	167		34	
150	3	745	194		36	
150	4	749	181		33	
150	5	748	220		33	
200	1	997	269		45	
200	2	998	248		43	
200	3	999	248		49	
200	4	998	258		47	
200	5	999	287		47	
250	1	1248	345		58	
250	2	1248	312		60	
250	3	1249	335		59	
250	4	1246	359		58	
250	5	1249	353		60	
300	1	1497	425		64	
300	2	1497	379		64	
300	3	1500	409		73	
300	4	1500	481		71	
300	5	1496	389		67	

Table 2: Best known results for the instances with $d = 10$ of benchmark `SW`

			Weighted		Unweighted	
$\|V\|$	#	$\|E\|$	UB	LB	UB	LB
100	1	250	158	71	42	10
100	2	250	166	78	41	8
100	3	250	126	64	41	9
100	4	250	178	77	41	9
100	5	250	180	76	41	10
150	1	375	225		65	
150	2	375	248		65	
150	3	375	214		64	
150	4	375	257		65	
150	5	375	285		62	
200	1	500	306		88	
200	2	500	306		88	
200	3	500	298		86	
200	4	500	319		88	
200	5	500	356		87	
250	1	625	372		109	
250	2	625	416		110	
250	3	625	423		110	
250	4	625	452		109	
250	5	625	461		110	
300	1	750	493		133	
300	2	750	502		133	
300	3	750	516		134	
300	4	750	550		134	
300	5	750	523		134	

Table 3: Best known results for the instances with $d = 5$ of benchmark `Reg`

			Weighted		Unweighted	
$\|V\|$	#	$\|E\|$	UB	LB	UB	LB
100	1	500	215	112	47	17
100	2	500	228	117	47	16
100	3	500	178	103	46	16
100	4	500	249	127	46	16
100	5	500	254	129	45	16
150	1	750	338		70	
150	2	750	360		72	
150	3	750	329		70	
150	4	750	384		70	
150	5	750	382		72	
200	1	1000	464		94	
200	2	1000	488		94	
200	3	1000	468		94	
200	4	1000	515		94	
200	5	1000	517		94	
250	1	1250	613		117	
250	2	1250	615		118	
250	3	1250	607		117	
250	4	1250	665		119	
250	5	1250	665		118	
300	1	1500	747		140	
300	2	1500	746		140	
300	3	1500	740		139	
300	4	1500	786		140	
300	5	1500	788		140	

Table 4: Best known results for the instances with $d = 10$ of benchmark `Reg`

			Weighted		Unweighted	
$\|V\|$	#	$\|E\|$	UB	LB	UB	LB
100	1	285	85	60	21	11
100	2	281	83	58	21	12
100	3	278	78	67	19	13
100	4	280	89	64	21	12
100	5	284	99	62	21	12
150	1	435	128		31	
150	2	428	128		31	
150	3	438	128		30	
150	4	420	117		29	
150	5	419	120		29	
200	1	577	160		38	
200	2	572	185		37	
200	3	578	135		37	
200	4	570	174		35	
200	5	578	171		36	
250	1	716	198		47	
250	2	715	185		45	
250	3	728	191		43	
250	4	715	212		46	
250	5	716	201		47	
300	1	868	258		57	
300	2	870	245		55	
300	3	865	255		61	
300	4	852	280		54	
300	5	859	241		56	

Table 5: Best known results for the instances of benchmark `Pla`

			Weighted		Unweighted	
$\|V\|$	#	$\|E\|$	UB	LB	UB	LB
100	10x10	200	141		30	
150	10x15	300	198		39	
200	10x20	400	249		50	
250	10x25	500	293		64	
300	15x20	600	324		79	
100	4x5x5	300	178		40	
150	5x5x6	450	275		56	
200	5x5x8	600	373		67	
250	5x5x10	750	426		82	
300	5x6x10	900	534		97	

Table 6: Best known results for the instances of benchmark `Grid`

2. Results obtained by Scatter Search and branch-and-bound

The following tables report the detailed results obtained by the algorithms described in the above mentioned paper. Both algorithms have been run on an Intel Xeon E5-2620 with a 2.1 GHz CPU and 16 GB or RAM. The Scatter Search algorithm has $n_B = n_D = 10$ and $\mu = 0.2$. It runs for a different time, specified in the caption of each table, according to the single benchmark. The branch-and-bound algorithm runs for at most one hour, including the initialisation phase given by Scatter Search.

Tables 7, 8 and 9 consider the 126 random graphs of benchmark `M`, divided by density ($\delta = 0.3$, $\delta = 0.5$, and $\delta = 0.7$). In each table, the first column identifies the graph corresponding to each row, reporting its number of vertices $|V|$. Two groups of four columns follow, referring to the weighted and the unweighted instances, respectively. The first column of each group provides the value returned by Scatter Search (z_{SS}). The other three columns show the results of the branch-and-bound: optimum (z^*), number of branching nodes (BN) and computational time in seconds (CPU).

Tables 10, 11, 12 and 13 consider the 100 random graphs of benchmark `H+`, divided by density ($\delta = 0.1$, $\delta = 0.2$, $\delta = 0.3$ and $\delta = 0.4$). They differ from the previous tables in that, since the density is fixed, the second column provides an integer identifier determining each instance in the group with the same size and density. Moreover, a small number of big instances are not solved to optimality, and therefore replace the optimum with a lower and an upper bound.

Table 14 considers the sparse graphs with 100 vertices of benchmarks `Pla`, `Reg` (with degree 5 and 10) and `SW` (with initial degree 6 and 10. The first three columns provide the name of the benchmark, the number of vertices and the integer identifier. The following two groups of columns, referring to the weighted and unweighted instances, report the value returned by Scatter Search after one minute (z_{SS}) and the final upper and lower bound (UB and LB) when the computation terminates. This always happens for memory exhaustion before the time limit of one hour.

	Weighted				Unweighted			
$\|V\|$	z_{SS}	z^*	CPU	BN	z_{SS}	z^*	CPU	BN
10		249	0.003	23		4	0.001	37
11		214	0.002	41		4	0.001	43
12		197	0.001	33		4	0.001	43
13		116	0.001	5		5	0.001	69
14		235	0.001	65		5	0.001	119
15		251	0.003	155		6	0.002	227
16		216	0.002	87		6	0.003	193
17		257	0.003	145		6	0.002	245
18		210	0.002	47		6	0.002	137
19		281	0.003	295		7	0.004	585
20		433	0.005	617		8	0.010	1335
21		374	0.007	509		8	0.006	671
22		421	0.009	1223		8	0.006	687
23		297	0.004	459		9	0.016	1631
24		427	0.013	1499		10	0.037	5091
25		425	0.019	2191		11	0.107	12595
26		510	0.045	4673		11	0.091	10339
27		587	0.075	7921		11	0.097	10099
28		444	0.029	2955		12	0.231	24905
29		595	0.201	18599		12	0.228	23123
30		562	0.189	16081		13	0.475	43927

Table 7: Results of Scatter Search (0.001 seconds) and branch-and-bound for the instances of density $\delta = 0.3$ of benchmark `M`

	Weighted				Unweighted			
$\|V\|$	z_{SS}	z^*	CPU	BN	z_{SS}	z^*	CPU	BN
10		254	0.003	27		5	0.001	113
11		214	0.001	15		4	0.001	25
12		251	0.002	67		6	0.002	227
13		149	0.001	17		6	0.002	153
14		299	0.002	91		6	0.002	127
15		301	0.005	341		7	0.002	249
16		279	0.003	127		7	0.005	797
17		336	0.003	281		8	0.004	477
18		326	0.003	163		8	0.002	217
19		378	0.005	315		9	0.009	1099
20		489	0.004	419		9	0.007	779
21		545	0.023	2427		10	0.014	1211
22		524	0.018	1807		10	0.011	1145
23		410	0.010	927		11	0.024	2331
24		536	0.019	1461		11	0.105	10825
25		539	0.033	2691		12	0.070	6417
26		638	0.073	5817		13	0.133	11315
27		725	0.136	10267		13	0.069	5635
28		572	0.064	4303		13	0.076	5591
29		735	0.294	19579		14	0.186	13045
30		710	0.279	17171		15	0.397	27233

Table 8: Results of Scatter Search (0.001 seconds) and branch-and-bound for the instances of density $\delta = 0.5$ of benchmark `M`

	Weighted				Unweighted			
$\|V\|$	z_{SS}	z^*	CPU	BN	z_{SS}	z^*	CPU	BN
10		312	0.001	53		5	0.001	39
11		260	0.003	43		5	0.001	23
12		300	0.002	69		6	0.001	77
13		173	0.001	33		6	0.002	287
14		320	0.003	145		7	0.002	113
15		348	0.004	415		7	0.002	85
16		323	0.003	267		8	0.003	335
17		377	0.004	187		8	0.002	81
18		380	0.004	221		9	0.003	237
19		420	0.011	817		9	0.009	905
20		533	0.007	575		10	0.005	343
21		606	0.026	2341		10	0.003	171
22		571	0.017	1313		11	0.007	541
23		410	0.007	519		11	0.018	1365
24		624	0.032	2211		12	0.011	799
25		583	0.027	1759		12	0.072	5317
26		681	0.033	1799		13	0.015	955
27		756	0.056	3247		13	0.180	12235
28		636	0.042	2271		14	0.034	1799
29		775	0.144	7265		15	0.453	26593
30		759	0.278	13021		15	0.050	2381

Table 9: Results of Scatter Search (0.001 seconds) and branch-and-bound for the instances of density $\delta = 0.7$ of benchmark `M`

		Weighted				Unweighted			
$\|V\|$	#	z_{SS}	z^*	CPU	BN	z_{SS}	z^*	CPU	BN
20	1								
20	2								
20	3								
20	4								
20	5								
25	1								
25	2								
25	3								
25	4								
25	5								
30	1								
30	2								
30	3								
30	4								
30	5								
35	1								
35	2								
35	3								
35	4								
35	5								
40	1		64	8.3			13	45.757	
40	2		54	0.9			12	10.128	
40	3		61	0.3			11	3.496	
40	4		52	1.3			13	34.196	
40	5		53	0.9			13	34.755	
50	1		73	37.3			15	264.844	
50	2		79	104.4			16	1357.119	
50	3		59	33.3			15	693.004	
50	4		87	230.3			16	1204.841	
50	5		82	94.2			17	2691.198	
60	1		77	424.3			[18,20]	3600.000	
60	2		79	73.7			[19,22]	OM	
60	3		83	343.6			[17,19]	3600.000	
60	4		98	2459.7			[18,21]	OM	
60	5		86	921.8			[18,21]	OM	

Table 10: Results of Scatter Search (0.001 seconds) and branch-and-bound for the instances of density $\delta = 0.1$ of benchmark $\mathtt{H}^+$

		Weighted				Unweighted			
$\|V\|$	#	z_{SS}	z^*	CPU	BN	z_{SS}	z^*	CPU	BN
20	1								
20	2								
20	3								
20	4								
20	5								
25	1								
25	2								
25	3								
25	4								
25	5								
30	1								
30	2								
30	3								
30	4								
30	5								
35	1								
35	2								
35	3								
35	4								
35	5								
40	1		82	9.839			16	26.095	
40	2		76	2.345			15	6.754	
40	3		68	2.169			16	38.434	
40	4		62	1.266			16	17.456	
40	5		75	3.194			16	29.260	
50	1		104	42.842			22	1280.171	
50	2		126	321.441			21	1167.173	
50	3		102	156.782			21	1263.579	
50	4		114	277.170			22	1134.055	
50	5		130	627.599			22	1756.308	
60	1		120	1983.960			[25,27]	3600.000	
60	2		111	561.308			[25,28]	3600.000	
60	3		[136,138]	3600.000			[25,27]	3600.000	
60	4		131	1341.329			[25,28]	3600.000	
60	5		[129,134]	3600.000			[25,28]	3600.000	

Table 11: Results of Scatter Search (0.001 seconds) and branch-and-bound for the instances of density $\delta = 0.2$ of benchmark H$^+$

		Weighted				Unweighted			
$\|V\|$	#	z_{SS}	z^*	CPU	BN	z_{SS}	z^*	CPU	BN
20	1								
20	2								
20	3								
20	4								
20	5								
25	1								
25	2								
25	3								
25	4								
25	5								
30	1								
30	2								
30	3								
30	4								
30	5								
35	1								
35	2								
35	3								
35	4								
35	5								
40	1		95	1.561			18	15.902	
40	2		108	10.046			18	14.893	
40	3		94	4.461			18	12.733	
40	4		84	1.850			18	11.446	
40	5		105	6.782			18	10.702	
50	1		115	65.740			23	143.080	
50	2		117	54.425			23	153.226	
50	3		119	36.523			24	861.727	
50	4		110	27.546			23	174.425	
50	5		110	38.228			24	846.141	
60	1		129	416.106			[28,29]	3600.000	
60	2		124	197.596			[28,29]	3600.000	
60	3		157	1605.779			[28,30]	3600.000	
60	4		149	392.899			[28,29]	3600.000	
60	5		144	632.324			[28,29]	3600.000	

Table 12: Results of Scatter Search (0.001 seconds) and branch-and-bound for the instances of density $\delta = 0.3$ of benchmark H$^+$

		Weighted				Unweighted			
$\|V\|$	#	z_{SS}	z^*	CPU	BN	z_{SS}	z^*	CPU	BN
20	1								
20	2								
20	3								
20	4								
20	5								
25	1								
25	2								
25	3								
25	4								
25	5								
30	1								
30	2								
30	3								
30	4								
30	5								
35	1								
35	2								
35	3								
35	4								
35	5								
40	1		98	3.164			19	6.906	
40	2		96	4.279			19	5.483	
40	3		105	1.060			19	5.613	
40	4		108	2.107			19	4.289	
40	5		93	0.899			19	5.286	
50	1		127	25.144			24	62.439	
50	2		142	50.309			24	76.613	
50	3		122	15.607			24	59.572	
50	4		123	21.321			24	51.919	
50	5		116	18.442			24	50.881	
60	1		137	138.074			29	675.141	
60	2		130	96.738			29	523.879	
60	3		163	589.745			29	904.702	
60	4		154	172.378			29	322.758	
60	5		155	356.728			29	1522.325	

Table 13: Results of Scatter Search (0.001 seconds) and branch-and-bound for the instances of density $\delta = 0.4$ of benchmark $\mathtt{H}^+$

			Weighted			Unweighted		
Benchmark	$\|V\|$	#	z_{SS}	UB	LB	z_{SS}	UB	LB
	100	1		85	60		22	11
	100	2		83	58		21	12
Pla	100	3		78	67		19	13
	100	4		91	64		22	12
	100	5		100	62		21	12
	100	1		164	71		43	10
	100	2		166	78		43	8
Reg5	100	3		126	64		43	9
	100	4		178	77		43	9
	100	5		181	76		43	10
	100	1		240	112		48	17
	100	2		241	117		47	16
Reg10	100	3		196	103		47	16
	100	4		261	127		48	16
	100	5		272	129		48	16
	100	1		111	60		24	11
	100	2		110	68		24	11
SW6	100	3		91	65		25	11
	100	4		118	69		23	11
	100	5		122	67		25	12
	100	1		157	85		28	14
	100	2		141	84		28	12
SW10	100	3		126	86		28	14
	100	4		168	95		29	13
	100	5		153	87		30	13

Table 14: Results of Scatter Search (60 seconds) and branch-and-bound for the instances of the sparse benchmarks