

# Decision Methods and Models

## Master's Degree in Computer Science

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Schedule: **Tuesday 15.30 - 18.30 in classroom V4**  
**Friday 13.30 - 16.30 in classroom Beta**

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Web page: **<https://homes.di.unimi.it/cordone/courses/2025-mmd/2025-mmd.html>**

Ariel site: **<https://myariel.unimi.it/course/view.php?id=7438>**

# Aims of the course

A decision problem intrinsically implies two fundamental conditions:

- ① **freedom**, that is the **availability of different choices**  
(*otherwise, there is no decision*)
- ② **rationality**, that is the **existence of preference criteria**  
(*otherwise, the choice cannot be motivated*)

The focus is on practical decisions where

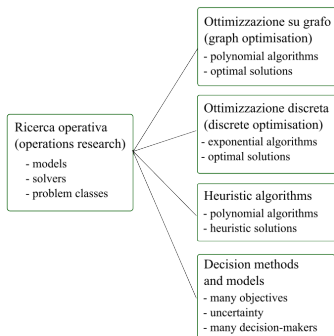
- a large amount of data must be taken into account
- there are many possible choices
- the costs of a wrong choice are high

This course aims to

- ① discuss some of the **factors that make a decision complicated**
- ② present the **mathematical models to describe complicated situations**
- ③ present the **mathematical methods to deal with such situations**
- ④ discuss **limits and errors of such models and methods**

# Relations with other courses

The Computer Science course includes naturally related courses



This gives a specific slant to the presentation of the subject

but the students do not need an Operations Research background  
*(check the list of prerequisites!)*

# What about other students?

Students not specifically interested in optimisation could be interested

Decision models are a guide for action in many application fields

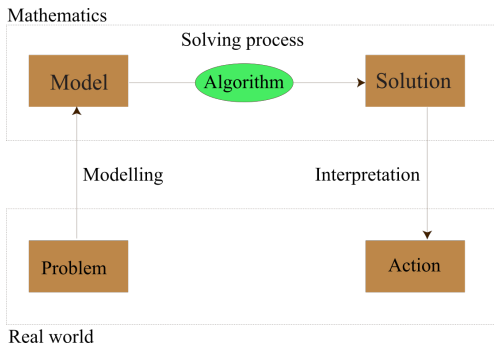
- finance, to define investment strategies
- marketing, to define advertising campaigns or pricing strategies
- natural resource management, to define harvesting strategies
- videogames, to define game strategies
- public or private works, to define the alternative to implement

This is not a technical course on optimisation models and algorithms

- it focuses on the factors that complicate in principle the possibility of building a model
- it presents some impossibility results to build satisfactory models
- it helps to use models as a support in decision-making
  - on the one hand recognising their utility
  - on the other hand using their results with critical sensibility

# The modelling approach

The correct strategy is **first make a model, then compute, finally decide**



This is a classical concept, but nowadays it is strongly enhanced by

- **Big Data:** huge amounts of precise, structured and cheap data, from which to extract information
- **Cloud Computing:** pervasive capacity to access and process data
- **Business Analytics:** a business culture open to the use of models
- **new theory:** online, stochastic, robust programming, etc. . .

# Decision models: terminology

**System** is the portion of the world affected by the decision

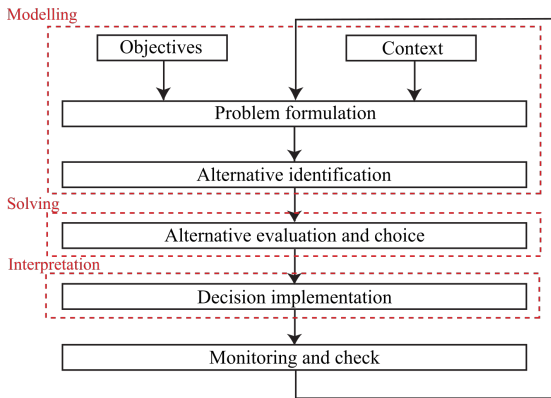
- a system should allow different **configurations** that combine
  - an **alternative** or **solution**, describing all its **controllable aspects**
  - a **scenario** or **outcome**, describing all its **uncontrollable aspects**
- each configuration is associated to an **impact**, that describes all aspects which are relevant for the decision
- **decision-maker** or **stakeholder** is everybody who contributes to the choice of an alternative
- the **preference** describes the relative satisfaction between impacts

A decision problem requires to **choose an alternative**

- so as to **move the system** into a configuration
- such that **the decision-makers prefer** its associated impact to those of the other configurations
- keeping into account that **the actual configuration depends on the alternative, but also on the scenario**

# Feedback loops in the decision process

The decision process often occurs by an iterative correction approach



H. A. Simon, "Models of bounded rationality" (1982)

# Phases of the decision process

- 1 **Problem formulation:** delineate the system, the decision-makers, the scenarios (**Context**), the impacts and preferences (**Objectives**)
- 2 **Alternative identification:** define the set of feasible alternatives
- 3 **Alternative evaluation and choice:**
  - evaluate the impact associated to each alternative and scenario
  - select an alternative based on the preferences of the decision-makers
- 4 **Decision implementation:**  
apply in practice, or simulate, the alternative selected
- 5 **Monitoring and check:**
  - observe (or simulate) the outcomes of the choice
  - if unsatisfactory, correct and repeat the process with new or modified alternatives, scenarios, decision-makers, objectives, models, methods

# Why a formal approach?

A formal approach allows to

- 1 predict in a more certain and precise way the impact of a decision using descriptive models instead of intuition and experience
- 2 accelerate the decision process using informatic tools
- 3 consider a much larger number of possible alternatives
- 4 clarifying and certifying the decision process
  - explicitating the assumptions made on alternatives, scenarios, preferences and decision-makers and their relative weight  
*Sometimes it is required to prove that a decision was based on data*
  - guarantee the repeatability of the process
  - allow focused changes to the process without restarting from scratch

The formal approach is based on

- models, to take decisions and to predict their outcomes
- methods, to build models, to solve them (algorithms) and to interpret their results

# Prescriptive and descriptive models

Decision models usually combine

① **prescriptive models** that

- receive impacts and preferences in input
- return a suggested alternative in output

*If this is the case, you should do that*

② **descriptive/predictive models**

- receive the system, an alternative and a scenario in input
- return an impact in output

*If you do this and if this happens, you will obtain that*

The descriptive ones range from trivial (e.g.:  $c_{\text{tot}} = c_1x_1 + c_2x_2$ ) to very refined (e.g.: simulation models, differential equations, etc. . . )

The two families of models can have subtle and complex interactions  
For instance:

- a model prescribes a decision *(close or open streets)*
- based on models that describe a system *(the amount of traffic)*
- including decisions prescribed by models *(satellite navigators)*

# Complicated decision problems: examples (1)

- 1 the search for a parking before a meeting:
  - system is the local street network, with the set of all potential parking places
  - alternative is every possible trajectory of the car (path and schedule)
  - scenario is every possible distribution of the free parking places over space and time
  - impact are the driving time and the walking time after parking
  - decision-maker is the driver (or also the passengers?)
- 2 the thermostat regulation:
  - system is the classroom
  - alternative is the position of the thermostat knob
  - scenario is the external temperature and the exposition of the classroom to the sun
  - impact is the internal temperature of the classroom (but its humidity is also relevant)
  - decision-makers are the people dwelling in the classroom (all of them, or just the teacher?)

# Complicated decision problems: examples (2)

## 3 buying a car:

- system is the local market of cars, petrol, repairs, etc. . .
- alternative is the car bought (possibly, more than one or none!)
- scenario are the stock and the prices of the car dealers, the occurrence of accidents, the prices of petrol and car repairs, etc. . .
- impact are the characteristics of the car throughout its life-cycle
- decision-maker is the buyer (and possibly other family members)

## 4 playing a round of the Risk (Risiko) game:

- system is the map with the distribution of territories, armies, cards
- alternative are the attacking and defending territories and the number of attacking and defending armies in each round
- scenario is the outcome of the dice at each attack
- impact is the number of armies lost by each player
- decision-makers are the players

# What makes a decision problem complicated?

A decision problem can be complicated due to

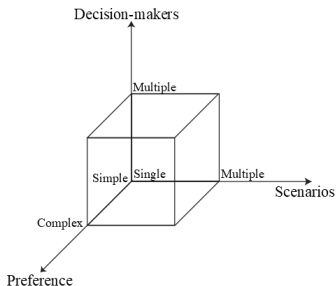
- ① an **insufficient model** of the system:
- ② complicating features of the model
  - **complex preference structure**, insufficient to define an optimum
  - **uncertain environment**: the impact depends also on an unknown scenario
  - **multiple decision-makers**, with potentially conflicting preferences
- ③ a **computationally complex** model
  - everything is clearly defined, but no efficient algorithm is known to solve the problem

This course focuses on the second source of complexity

# Classification of decision problems

The three main complexity sources for decision problems are

- 1 **preference structure**: simple or complex
  - 2 **uncertainty**: a single scenario or many
  - 3 **decision-makers**: a single decision-maker or many
- give rise to  $2^3 = 8$  families of decision problems



After discussing **case studies** and formally defining **basic concepts**, we consider four basic families of **prescriptive models**

- ① simple preference, a single scenario and a single decision-maker
  - **multiple-attribute utility theory**
  - **mathematical programming**
- ② **complex preference**, a single scenario and a single decision-maker:
  - **Paretian preferences**
  - **weak rationality models** (*AHP* and *ELECTRE* methods)
- ③ simple preference, **multiple scenarios** and a single decision-maker:
  - decisions in conditions of **ignorance** (robust programming)
  - decisions in conditions of **risk** (stochastic programming)
- ④ simple preference, a single scenario and **multiple decision-makers**:
  - independent decision-makers (**game theory**)
  - cooperating decision-makers (**group decisions**)

The lecture notes also consider some important families of **descriptive models**  
(*not required for the exam*)