Prerequisites for the course on "Decision methods and models"

The following concepts will be given for granted during the course. Students unfamiliar with these concepts should seriously ponder whether they feel really able to catch up.

Mathematical analysis

Gradient vector of a function

1. Compute the partial derivatives and the gradient vector of a function

$$f(x) = (x_1 - 1)^2 + x_2^2 \Rightarrow \nabla f(x) = ?$$

$$g(x) = x_1 - 3/2 \Rightarrow \nabla g(x) = ?$$

Scalar product of vectors

2. Compute the scalar product of two vectors

$$d_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \; d_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \Rightarrow d_1 \cdot d_2 = ?$$

Graphical representation of constraints

3. Draw the graphical representation of inequality and equality constraints, at least for linear and quadratic functions (lines, circles, ellipses, parabolas and hyperboles)

$$\begin{aligned} g_1(x) &= -x_1^2 - x_2^2 + 4 \leq 0 \\ g_2(x) &= x_1 - 3/2 \leq 0 \\ g_3(x) &= x_1 x_2 \geq 4 \\ g_4(x) &= 4x_1^2 + 9x_2^2 \leq 1 \\ g_5(x) &= x_1 + 2x_2^2 - 4 \leq 0 \end{aligned}$$

Lines in parametric form

4. Compute the tangent vector of a line in parametric form and draw the graphical representation of the line and of its tangent vector

$$\begin{cases} \xi_1(\alpha) = -2\cos(\alpha) \\ \xi_2(\alpha) = 2\sin(\alpha) \end{cases} \quad \text{for } \alpha \in \left[0, \frac{5}{4}\pi\right]$$

$$x'(\alpha) = ?$$

Linear algebra

Linearly (in)dependent vectors

5. Determine whether a given set of vectors are linearly dependent or independent

$$d_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \text{ and } d_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$d_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \text{ and } d_2 = \begin{bmatrix} 1 \\ 2/3 \end{bmatrix}$$

Determinant of a matrix

6. Compute the determinant of a given square matrix, at least of order two and three.

$$D_1 = \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix} \Rightarrow |D_1| =?$$

$$D_2 = \begin{bmatrix} 3 & 1 \\ 2 & 2/3 \end{bmatrix} \Rightarrow |D_2| =?$$

Eigenvalues and eigenvectors

7. Compute the eigenvalues and eigenvectors of a given square matrix of order two and possibly three.

$$D_1 = \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix} \Rightarrow \begin{cases} \lambda_1 =? \\ \lambda_2 =? \end{cases}$$

Probability calculus

8. Compute the conjoint, total and conditional probability of events.

$$\pi(A) = 0.4 \quad \pi(B) = 0.3 \quad \pi(AB) = 0.2 \Rightarrow \pi(A+B) =?$$

$$\pi(A) = 0.4 \quad \pi(B|A) = 0.2 \Rightarrow \pi(AB) =?$$

9. Compute the expected value of a random variable given its possible values and the probabilities of the outcomes.

$$\begin{cases} |\Omega| = 4 \\ \pi(\omega) = [\ 0.1\ 0.3\ 0.4\ 0.2] \\ f(\omega) = [\ 10\ 4\ 6\ 2] \end{cases} \Rightarrow E[f] =?$$

Solutions

1. Compute the partial derivatives and the gradient vector of a function

$$f(x) = (x_1 - 1)^2 + x_2^2 \Rightarrow \nabla f(x) = \begin{bmatrix} 2(x_1 - 1) \\ 2x_2 \end{bmatrix}$$

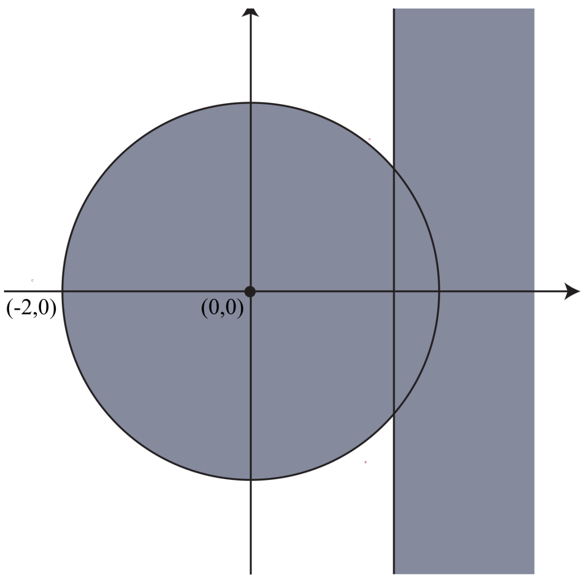
$$g(x) = x_1 - 3/2 \Rightarrow \nabla g(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2. Compute the scalar product of two vectors

$$d_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \; d_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \Rightarrow d_1 \cdot d_2 = 3 \cdot 1 + 2 \cdot (-4) = -5$$

3. Draw the graphical representation of inequality and equality constraints, at least for linear and quadratic functions (lines, circles, ellipses, parabolas and hyperboles)

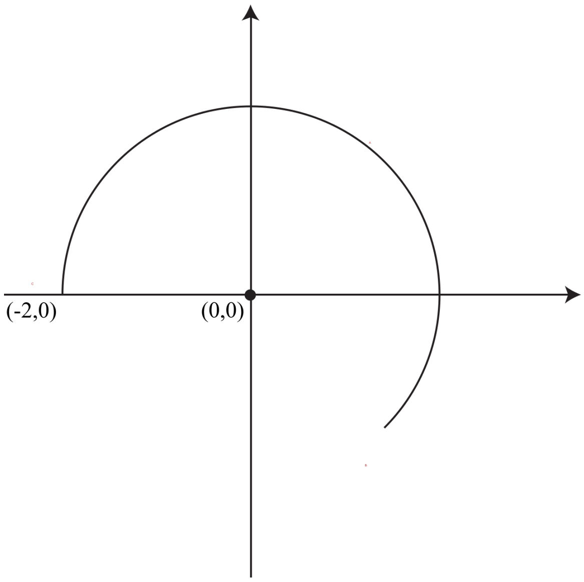
$$\begin{aligned} g_1(x) &= -x_1^2 - x_2^2 + 4 \leq 0 \\ g_2(x) &= x_1 - 3/2 \leq 0 \end{aligned}$$



4. Compute the tangent vector of a line in parametric form and draw the graphical representation of the line and of its tangent vector

$$\begin{cases} \xi_1(\alpha) = -2\cos(\alpha) \\ \xi_2(\alpha) = 2\sin(\alpha) \end{cases} \quad \text{for } \alpha \in \left[0, \frac{5}{4}\pi\right]$$

$$x'(\alpha) = \begin{bmatrix} 2\sin(\alpha) \\ 2\cos(\alpha) \end{bmatrix}$$



5. Determine whether a given set of vectors are linearly dependent or independent

$$d_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \text{ and } d_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$\alpha_1 d_1 + \alpha_2 d_2 = 0 \Rightarrow \begin{cases} 3\alpha_1 + \alpha_2 = 0 \\ 2\alpha_1 - 4\alpha_2 = 0 \end{cases} \Rightarrow \alpha_1 = \alpha_2 = 0 \Rightarrow \text{Independent vectors}$$

$$d_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \text{ and } d_2 = \begin{bmatrix} 1 \\ 2/3 \end{bmatrix}$$

$$\alpha_1 d_1 + \alpha_2 d_2 = 0 \Rightarrow \begin{cases} 3\alpha_1 + \alpha_2 = 0 \\ 2\alpha_1 + \frac{2}{3}\alpha_2 = 0 \end{cases} \Rightarrow \alpha_2 = -3\alpha_1 \Rightarrow \text{Dependent vectors}$$

6. Compute the determinant of a given square matrix, at least of order two and three.

$$D_1 = \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix} \Rightarrow |D_1| = 3 \cdot (-4) - 2 \cdot 1 = -14$$

$$D_2 = \begin{bmatrix} 3 & 1 \\ 2 & 2/3 \end{bmatrix} \Rightarrow |D_2| = 3 \cdot \frac{2}{3} - 2 \cdot 1 = 0$$

7. Compute the eigenvalues and eigenvectors of a given square matrix of order two and possibly three.

$$D_1 = \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix} \Rightarrow |\lambda I - D_1| = (\lambda - 3) \cdot (\lambda + 4) - (-2) \cdot (-1) = \lambda^2 + \lambda - 14 = 0 \text{ etc.}$$

8. Compute the conjoint, total and conditional probability of events.

$$\pi(A) = 0.4 \quad \pi(B) = 0.3 \quad \pi(AB) = 0.2 \Rightarrow \pi(A + B) = 0.5$$

$$\pi\left(A\right)=0.4\quad\pi\left(B|A\right)=0.2\Rightarrow\pi\left(AB\right)=0.4\cdot0.2=0.08$$

9. Compute the expected value of a random variable given its possible values and the probabilities of the outcomes.

$$\begin{cases}|\Omega|=4\\\pi\left(\omega\right)=[\ 0.1\ 0.3\ 0.4\ 0.2]\\f\left(\omega\right)=[\ 10\ 4\ 6\ 2]\end{cases}\quad\Rightarrow E\left[f\right]=10\cdot0.1+4\cdot0.3+6\cdot0.4+2\cdot0.2=5$$