Decision Methods and Models

Master's Degree in Computer Science

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Lesson 16: Decision-making under ignorance

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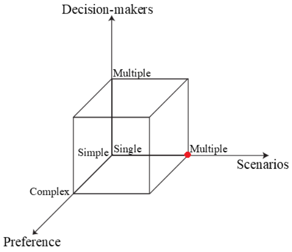
Decision-making under ignorance

We assume

- a preference relation Π that is a weak order with a known consistent value function $u(f)$ (replaced by a cost f)
- a uncertain environment: $|\Omega| > 1$ and we have no other information
- a single decision-maker: $|D| = 1 \Rightarrow \Pi_d$ reduces to Π



The idea is to aggregate all scenarios of Ω and reduce $f(x, \omega)$ to $\phi_\Omega(x)$

- various ways to do that have been proposed
- no approach can satisfy all desirable properties

It is a theoretical impossibility

Worst-case criterium

The Austrian-Hungarian mathematician Abraham Wald proposed the worst-case criterium

$$\min_{x \in X} \phi_{\text{worst}}(x) = \min_{x \in X} \max_{\omega \in \Omega} f(x, \omega) \qquad \text{when } f \text{ is a cost}$$

also called pessimism, Wald, minimax or maximin (for benefits) criterium

In summary:

1. for each alternative $x \in X$, find the worst scenario $\omega^{\dagger}(x)$
2. reduce $f(x, \omega)$ to $\phi_{\text{worst}}(x) = f\left(x, \omega^{\dagger}(x)\right)$
3. rank the alternatives based on $\phi_{\text{worst}}(x)$

It is a conservative approach: avoid losses, even giving up opportunities

Example

1. for each alternative $x \in X$, find the worst scenario $\omega^{\dagger}(x)$
2. replace $f(x,\omega)$ with $\phi_{\text{worst}}(x) = f\left(x, \omega^{\dagger}(x)\right)$
3. rank the alternatives based on $\phi_{\text{worst}}(x)$

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{worst}}(x)$	$\omega^{\dagger}(x)$
$x^{(1)}$	2	2	4	3	4	$\omega^{(3)}$
$x^{(2)}$	3	3	3	3	3	$\omega^{(1)}, \omega^{(2)}, \omega^{(3)}, \omega^{(4)}$
$x^{(3)}$	4	0	4	6	6	$\omega^{(4)}$
$x^{(4)}$	3	1	4	4	4	$\omega^{(3)}, \omega^{(4)}$

The final ranking is

$$x^{(2)} \prec x^{(1)} \sim x^{(4)} \prec x^{(3)}$$

Best-case criterium

The complementary approach is the best-case criterium

$$\min_{x \in X} \phi_{\mathrm{best}}(x) = \min_{x \in X} \min_{\omega \in \Omega} f(x, \omega) \qquad \text{when } f \text{ is a cost}$$

also called optimism, minimin or maximax (for benefits) criterium

In summary:

1. for each alternative $x \in X$, find the best scenario $\omega^* (x)$
2. reduce $f(x, \omega)$ to $\phi_{\mathrm{best}}(x) = f(x, \omega^*(x))$
3. rank the alternatives based on $\phi_{\mathrm{best}}(x)$

It is an opportunistic approach: believe in opportunities ignoring dangers

Example

1. for each alternative $x \in X$, find the best scenario $\omega^*(x)$
2. reduce $f(x,\omega)$ to $\phi_{\text{best}}(x) = f(x, \omega^*(x))$
3. rank the alternatives based on $\phi_{\text{best}}(x)$

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{best}}(x)$	$\omega^*(x)$
$x^{(1)}$	2	2	4	3	2	$\omega^{(1)}, \omega^{(2)}$
$x^{(2)}$	3	3	3	3	3	$\omega^{(1)}, \omega^{(2)}, \omega^{(3)}, \omega^{(4)}$
$x^{(3)}$	4	0	4	6	0	$\omega^{(2)}$
$x^{(4)}$	3	1	4	4	1	$\omega^{(2)}$

The final ranking is

$$x^{(3)} \prec x^{(4)} \prec x^{(1)} \prec x^{(2)}$$

Hurwicz criterium

The two previous criteria are too biased towards extreme conditions

The Polish mathematician Leonid Hurwicz proposed
the Hurwicz criterium that merges them into a convex combination

$$\min_{x \in X} \phi_{\mathrm{Hurwicz}}(x) = \min_{x \in X} \rho\, \phi_{\mathrm{worst}}(x) + (1-\rho)\, \phi_{\mathrm{best}}(x) \qquad \text{when } f \text{ is a cost}$$

where $\rho \in [0,1]$ is the pessimism coefficient

- $\rho = 1$ reduces the Hurwicz criterium to the worst-case criterium
- $\rho = 0$ reduces the Hurwicz criterium to the best-case criterium

Example

A simple way is to tune ρ is

1. find a pair of indifferent solutions x and x'
2. impose $\phi_{\text{Hurwicz}}(x) = \phi_{\text{Hurwicz}}(x')$
3. solve the resulting linear equation in ρ

Suppose that $\rho = 0.6$

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{Hurwicz}}(x)$	$\omega^*(x)$
$x^{(1)}$	2	2	4	3	3.2	$\omega^{(1)}, \omega^{(2)}$
$x^{(2)}$	3	3	3	3	3	$\omega^{(1)}, \omega^{(2)}, \omega^{(3)}, \omega^{(4)}$
$x^{(3)}$	4	0	4	6	3.6	$\omega^{(2)}$
$x^{(4)}$	3	1	4	4	2.8	$\omega^{(2)}$

The final ranking is

$$x^{(4)} \prec x^{(2)} \prec x^{(1)} \prec x^{(3)}$$

Sensitivity analysis

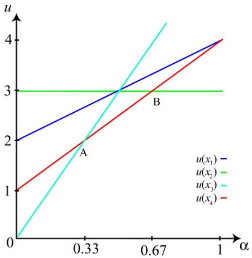
If the ranking is unclear and the value of ρ imprecise, find the support of each solution x, i.e. the range of ρ where x is optimal for $\phi_{\mathrm{Hurwicz}}(x)$

$$\mathrm{Supp}\,(x) = \left\{ \rho \in [0,1] : x \in \arg\min_{x \in X} \phi_{\mathrm{Hurwicz}}(x) \right\}$$

The choice criterium $\phi_{\mathrm{Hurwicz}}(x)$ becomes a linear function in ρ

- $\phi_{\mathrm{Hurwicz}}(x^{(1)}) = 4\alpha + 2(1-\alpha) = 4\alpha + 2 - 2\alpha = 2\alpha + 2$
- $\phi_{\mathrm{Hurwicz}}(x^{(2)}) = 3\alpha + 3(1-\alpha) = 3\alpha + 3 - 3\alpha = 3$
- $\phi_{\mathrm{Hurwicz}}(x^{(3)}) = 6\alpha + 0(1-\alpha) = 6\alpha$
- $\phi_{\mathrm{Hurwicz}}(x^{(4)}) = 4\alpha + 1(1-\alpha) = 4\alpha + 1 - 1\alpha = 3\alpha + 1$

The lower envelope of their profiles identifies the supports



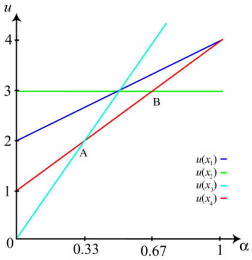
Sensitivity analysis

Notice that

- the strictly dominated solutions are never optimal
- also some nondominated solutions have empty support (unsupported)

This is similar to the weighted sum method for Paretianity, but stronger

- even solutions that are the best in a scenario can have empty support (*a_1 is the best in ω_1, but still unsupported*)



The solution is x_3 for $\rho \in \left[0, \frac{1}{3}\right]$, x_4 for $\rho \in \left[\frac{1}{3}, \frac{2}{3}\right]$, x_2 for $\rho \in \left[\frac{2}{3}, 1\right]$

Notice that the result of all criteria considered depends on Ω

Laplace criterium

The French mathematician Pierre-Simon Laplace suggested that the best estimation of an unknown probability in the worst case is the uniform one

This leads to the formulation of his choice criterium,
also known as equiprobability criterium

$$\min_{x \in X} \phi_{\text{Laplace}}(x) = \min_{x \in X} \frac{\sum_{\omega \in \Omega} f(x, \omega)}{|\Omega|} \qquad \text{when } f \text{ is a cost}$$

which leads to a simple arithmetic mean of the impacts on the scenarios

Of course, this is possible only for finite scenario sets

(*a limitation that might apply also to the previous criteria*)

It is a balanced approach: keep all scenarios into account

Example

1. for each alternative $x \in X$, find the arithmetic mean of the impacts

$$\frac{\sum_{\omega \in \Omega} f(x, \omega)}{|\Omega|}$$

2. replace $f(x, \omega)$ with $\phi_{\text{Laplace}}(x)$
3. rank the alternatives based on $\phi_{\text{Laplace}}(x)$

$f(x, \omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{Laplace}}(x)$
$x^{(1)}$	2	2	4	3	$11/4 = 2.75$
$x^{(2)}$	3	3	3	3	$12/4 = 3$
$x^{(3)}$	4	0	4	6	$14/4 = 3.5$
$x^{(4)}$	3	1	4	4	$12/4 = 3$

The final ranking is

$$x^{(1)} \prec x^{(2)} \sim x^{(4)} \prec x^{(3)}$$

which vindicates alternative $x^{(1)}$, so far mistreated by the other methods

The regret criterium

The United States mathematician Leonard Savage
(whose Russian original surname was Ogashevitz) remarked that
a solution should be compared with alternative ones scenario by scenario

(*in particular with the best one*)

The idea is to

- introduce a regret function $\rho(x, \omega)$ to measure in each scenario the regret caused by the choice of a nonoptimal alternative

$$\rho(x, \omega) = f(x, \omega) - \min_{x' \in X} f(x', \omega) \qquad \text{when } f \text{ is a cost}$$

- apply the worst-case criterium to the regret function

$$\min_{x \in X} \phi_{\text{regret}}(x) = \min_{x \in X} \max_{\omega \in \Omega} \rho(x, \omega)$$

The regret criterium

In summary

1. find for each scenario the best alternative $x^* (\omega) = \arg\min_{x \in X} f (x, \omega)$

   (*when* f *is a cost; otherwise, it is the maximum*)

2. compute the regret of all alternatives as the distance from the best

$$\rho (x, \omega) = f (x, \omega) - f(x^*, \omega) \qquad \text{when } f \text{ is a cost}$$

   (*otherwise it is the opposite*)

3. for each alternative $x \in X$, find the worst scenario $\omega^\dagger (x)$
4. reduce $\rho (x, \omega)$ to $\phi_{\text{regret}} (x) = \rho \left(x, \omega^\dagger (x)\right)$
5. rank the alternatives based on $\phi_{\text{regret}} (x)$

It is a comparative approach: care only about unnecessary losses

Example

1. find for each scenario the best alternative $x^*(\omega) = \arg\min_{x \in X} f(x, \omega)$

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
$x^{(1)}$	2	2	4	3
$x^{(2)}$	3	3	3	3
$x^{(3)}$	4	0	4	6
$x^{(4)}$	3	1	4	4

$x^*(\omega)$	2	0	3	3

Example

2. compute the regret of all alternatives as the distance from the best

$$\rho(x,\omega) = f(x,\omega) - f(x^*,\omega)$$

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
$x^{(1)}$	2	2	4	3
$x^{(2)}$	3	3	3	3
$x^{(3)}$	4	0	4	6
$x^{(4)}$	3	1	4	4

$x^*(\omega)$	2	0	3	3

$\rho(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
$x^{(1)}$	0	2	1	0
$x^{(2)}$	1	3	0	0
$x^{(3)}$	2	0	1	3
$x^{(4)}$	1	1	1	1

There is at least a 0 in each column (best solution in each scenario)

Example

❸ apply the worst-case criterium to the regret function

$$\min_{x \in X} \phi_{\text{regret}}(x) = \min_{x \in X} \max_{\omega \in \Omega} \rho(x, \omega)$$

$\rho(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{regret}}(x)$	$\omega^{\dagger}(x)$
$x^{(1)}$	0	2	1	0	2	$\omega^{(2)}$
$x^{(2)}$	1	3	0	0	3	$\omega^{(2)}$
$x^{(3)}$	2	0	1	3	3	$\omega^{(4)}$
$x^{(4)}$	1	1	1	1	1	$\omega^{(1)}, \omega^{(2)}, \omega^{(3)}, \omega^{(4)}$

The final ranking is

$$x^{(4)} \prec x^{(1)} \prec x^{(2)} \sim x^{(3)}$$

The surplus criterium

Savage also proposed the surplus criterium, complementary to the regret, which compares a solution with the worst alternative scenario by scenario, to stress the relative gain obtained

The idea is to

- introduce a surplus function $\sigma(x, \omega)$ to measure in each scenario the extra gain obtained from the choice of a nonpessimal alternative

$$\sigma(x, \omega) = \max_{x' \in X} f(x', \omega) - f(x, \omega) \qquad \text{when } f \text{ is a cost}$$

- apply the worst-case criterium to the surplus function

$$\max_{x \in X} \phi_{\text{surplus}}(x) = \max_{x \in X} \min_{\omega \in \Omega} \sigma(x, \omega)$$

Notice that, contrary to the regret, the surplus is a benefit!

The surplus criterium

In summary

1. find for each scenario the worst alternative $x^\dagger(\omega) = \arg\max_{x \in X} f(x,\omega)$

   (*when* f *is a cost; otherwise, it is the minimum*)

2. compute the surplus of all alternatives as the distance from the worst

$$\sigma(x,\omega) = f(x^\dagger,\omega) - f(x,\omega) \qquad \text{when } f \text{ is a cost}$$

   (*otherwise it is the opposite*)

3. for each alternative $x \in X$, find the worst scenario $\omega^\dagger(x)$
4. reduce $\sigma(x,\omega)$ to $\phi_{\text{surplus}}(x) = \sigma(x,\omega^\dagger(x))$
5. rank the alternatives based on $\phi_{\text{surplus}}(x)$

It is a comparative approach: care only about nonguaranteed gains

Example

1. find for each scenario the worst alternative $x^{\dagger}(\omega) = \arg\min_{x \in X} f(x, \omega)$

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
$x^{(1)}$	2	2	4	3
$x^{(2)}$	3	3	3	3
$x^{(3)}$	4	0	4	6
$x^{(4)}$	3	1	4	4

$x^{\dagger}(\omega)$	4	3	4	6

Example

2. compute the surplus of all alternatives as the distance from the worst

$$\sigma(x,\omega) = f\left(x^{\dagger},\omega\right) - f(x,\omega)$$

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
$x^{(1)}$	2	2	4	3
$x^{(2)}$	3	3	3	3
$x^{(3)}$	4	0	4	6
$x^{(4)}$	3	1	4	4

$x^{\dagger}(\omega)$	4	3	4	6

$\sigma(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
$x^{(1)}$	2	1	0	3
$x^{(2)}$	1	0	1	3
$x^{(3)}$	0	3	0	0
$x^{(4)}$	1	2	0	2

There is at least a 0 in each column (worst solution in each scenario)

Example

3. apply the worst-case criterium to the surplus function

$$\min_{x \in X} \phi_{\text{surplus}}(x) = \max_{x \in X} \min_{\omega \in \Omega} \sigma(x, \omega)$$

$\sigma(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{surplus}}(x)$	$\omega^{\dagger}(x)$
$x^{(1)}$	2	1	0	3	0	$\omega^{(3)}$
$x^{(2)}$	1	0	1	3	0	$\omega^{(2)}$
$x^{(3)}$	0	3	0	0	0	$\omega^{(1)}, \omega^{(3)}, \omega^{(4)}$
$x^{(4)}$	1	2	0	2	0	$\omega^{(1)}, \omega^{(2)}, \omega^{(3)}, \omega^{(4)}$

The final ranking is

$$x^{(1)} \sim x^{(2)} \sim x^{(3)} \sim x^{(4)}$$

Each solution is the worst one in at least one scenario

An axiomatic approach

The six criteria considered give completely different rankings:
how to choose the most appropriate one?

We want an algorithm to build a dominance relation on X (pairs (x, x')) from scenario set Ω and impacts $f(x, \omega)$ and $f(x', \omega)$ (functions on Ω)

The axiomatic approach consists in

- listing the formal properties of the desired algorithm
- building an algorithm that satisfies them, or prove that none exists

Unfortunately, we are in the second case

We will introduce 7 reasonable properties and show that

- all previous criteria satisfy the first 4 properties
- all previous criteria violate at least one of the last 3 properties

Basic desirable properties for choice criteria

1. weak ordering: the dominance relation is a weak order

   All criteria generate a choice criterium $\phi_\Omega(x)$, implying a weak order

2. labelling independence: the dominance relation is independent from the names and order of alternatives and scenarios

   Examples of dependent criteria:

   - choose the first alternative
   - choose the best alternative in the first scenario

   All criteria satisfy this property

3. scale invariance: impacts f and $f' = \alpha f + \beta$ yield the same dominance relation for every $\alpha > 0$ and every $\beta \in \mathbb{R}$, meaning that the result is independent from unit of measure and offset

   The six criteria are scale-invariant because $\phi'_\Omega(x) = \alpha\phi_\Omega(x) + \beta$

4. strong dominance: the dominance relation includes the strong dominance relation

$$f(x, \omega) \leq f(x', \omega) \; \forall \omega \in \Omega \Rightarrow x \preceq x'$$

   The six criteria preserve strong dominance

Prove it by contradiction

Basic desirable properties for choice criteria

The other three properties, however, are not always respected

5. independence from irrelevant alternatives: rank reversal never occurs
   Adding or removing alternatives never modifies the other ranks
   - worst-case, best-case, Hurwicz and Laplace satisfy the property:
     $\phi_\Omega(x)$ depends only on a single row $f(x,\cdot)$, the others are ininfluent
   - regret and surplus can violate the property:
     $\phi_\Omega(x)$ depends on row $f(x,\cdot)$, but also on $f(x^*(\cdot),\cdot)$ or $f(x^\dagger,\cdot)$, that belong to other rows

Example

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{worst}}(x)$	$\phi_{\text{best}}(x)$	$\phi_{\text{Laplace}}(x)$
$x^{(1)}$	2	2	4	3	4	2	2.75
$x^{(2)}$	3	3	3	3	3	3	3
$x^{(3)}$	4	0	4	6	6	0	3.5
$x^{(4)}$	3	1	4	4	4	1	3
$x^{(5)}$	6	4	0	4	6	0	4

The new alternative $x^{(5)}$ is the worst in $\omega^{(1)}$ and $\omega^{(2)}$, the best in $\omega^{(3)}$

Example: the regret criterium suffers from rank reversal

The improved best cases increase the regret in a nonuniform way

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
$x^{(1)}$	2	2	4	3
$x^{(2)}$	3	3	3	3
$x^{(3)}$	4	0	4	6
$x^{(4)}$	3	1	4	4
$x^{(5)}$	6	4	0	4

$x^*(\omega)$	2	0	0	3
	-	-	-3	-

$\sigma(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{surplus}}(x)$
$x^{(1)}$	0	2	4	0	4
$x^{(2)}$	1	3	3	0	3
$x^{(3)}$	2	0	4	3	4
$x^{(4)}$	1	1	4	1	4
$x^{(5)}$	4	1	0	1	4

The final ranking is $x^{(2)} \prec x^{(1)} \sim x^{(3)} \sim x^{(4)} \sim x^{(5)}$,
instead of $x^{(4)} \prec x^{(1)} \prec x^{(2)} \prec x^{(3)}$

Example: the surplus criterium suffers from rank reversal

The worsened worst cases increase the surplus in a nonuniform way

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
$x^{(1)}$	2	2	4	3
$x^{(2)}$	3	3	3	3
$x^{(3)}$	4	0	4	6
$x^{(4)}$	3	1	4	4
$x^{(5)}$	6	4	0	4

$x^{\dagger}(\omega)$	6	4	4	6
	+2	+1	-	-

$\sigma(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{surplus}}(x)$
$x^{(1)}$	4	2	0	3	0
$x^{(2)}$	3	1	1	3	1
$x^{(3)}$	2	4	0	0	0
$x^{(4)}$	3	3	0	2	0
$x^{(5)}$	0	0	4	2	0

The final ranking is $x^{(2)} \prec x^{(1)} \sim x^{(3)} \sim x^{(4)} \sim x^{(5)}$,
instead of $x^{(1)} \sim x^{(2)} \sim x^{(3)} \sim x^{(4)}$

Basic desirable properties for choice criteria

The other three properties, however, are not always respected

6. independence from scenario duplication: the dominance relation does not change adding scenarios with identical impacts
   Example: "day of the week" becomes { "Monday", ..., "Friday" }
   - worst-case, best-case, Hurwicz satisfy the property:
     the minimum and maximum impact over the scenarios are the same
   - regret and surplus satisfy the property: as minimum and maximum do not change, also the regret and surplus remain the same
   - Laplace in general violates the property:
     the weight of the duplicated scenario in the average increases

Example

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(2')}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{Laplace}}(x)$
$x^{(1)}$	2	2	2	4	3	~~2.75~~ 2.6
$x^{(2)}$	3	3	3	3	3	~~3~~ 3
$x^{(3)}$	4	0	0	4	6	~~3.5~~ 2.8
$x^{(4)}$	3	1	1	4	4	~~3~~ 2.6

The ranking changes from $x^{(1)} \prec x^{(2)} \sim x^{(4)} \prec x^{(3)}$ to
$x^{(1)} \sim x^{(4)} \prec x^{(3)} \prec x^{(2)}$

Basic desirable properties for choice criteria

The other three properties, however, are not always respected

7. uniform variations of a scenario: the dominance relation does not change if $f(x, \omega)$ (in scenario $\bar{\omega}$) varies by a uniform amount δf, i.e., the scenario becomes equally better or worse for all alternatives
   - Laplace satisfies the property: $\phi_{\text{Laplace}}(x)$ varies by $\dfrac{\delta f}{|\Omega|}$, $\forall x \in X$
   - regret and surplus satisfy the property: if column $\bar{\omega}$ changes by δf, minimum and maximum also do; regret and surplus do not change
   - worst-case, best-case, Hurwicz can violate the property: changing $f(x, \bar{\omega})$ can vary minimum and maximum of $f(x, \omega)$ on all scenarios

Example

$f(x,\omega)$	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$	$\phi_{\text{worst}}(x)$	$\phi_{\text{best}}(x)$
$x^{(1)}$	2	~~2~~ 6	4	3	~~4~~ 6	2
$x^{(2)}$	3	~~3~~ 7	3	3	~~3~~ 7	3
$x^{(3)}$	4	~~0~~ 4	4	6	6	~~0~~ 4
$x^{(4)}$	3	~~1~~ 5	4	4	~~4~~ 5	~~1~~ 3

$x^{(2)} \prec x^{(1)} \sim x^{(4)} \prec x^{(3)}$ turns into $x^{(4)} \prec x^{(1)} \sim x^{(3)} \prec x^{(2)}$

$x^{(3)} \prec x^{(4)} \prec x^{(1)} \prec x^{(2)}$ turns into $x^{(1)} \prec x^{(2)} \sim x^{(4)} \prec x^{(3)}$

A theoretical impossibility result

None of the six criteria satisfies all desired 7 properties

	Weak order	Labels	Scale invar.	Strong domin.	Rank reversal	Scenario duplic.	Uniform variation
Worst-case	OK	OK	OK	OK	OK	OK	NO
Best-case	OK	OK	OK	OK	OK	OK	NO
Hurwicz	OK	OK	OK	OK	OK	OK	NO
Laplace	OK	OK	OK	OK	OK	NO	OK
Regret	OK	OK	OK	OK	NO	OK	OK
Surplus	OK	OK	OK	OK	NO	OK	OK

Theorem:
The above mentioned properties are mutually exclusive:
no algorithm can satisfy all of them (without additional information)