Exercises on Lagrangean relaxation

1. Consider the formulation of the transportation problem already introduced in the exercises of the first lesson:

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m} F_i y_i$$

$$\sum_{i=1}^{m} x_{ij} = d_j \quad \text{for } j=1,..,n \qquad (3.1)$$

$$\sum_{j=1}^{n} x_{ij} \le D y_i \quad \text{for } i=1,..,m \qquad (3.2)$$

$$x_{ij} \ge 0 \quad \text{for } i=1,..,m, \text{ for } j=1,..,n$$

$$y_i \in \{0,1\} \quad \text{for } i=1,..,m$$

Consider separately the lagrangean relaxation of constraints (3.1) and that of constraints (3.2) explaining how to solve them.
Is it stronger the Lagrangean relaxation of constraints (3.1) or of constraints (3.2)? Justify your answer.

2. The following ILP is given:

$$\begin{aligned} \max\ & x1 + x2 \\ & 2x1 + 2x2 \ge 5 \\ & x1 + 2x2 \ge 9 \\ & x2 \le 4 \\ & 2x1 - 2x2 \le 5 \\ & x1 \ge 0,\ x2 \ge 0 \text{ integer} \end{aligned}$$

Solve in graphical way the Lagrangean dual problem of the Lagrangean relaxation of constraint $2x1 - 2x2 \le 5$ and show that the same result can be also obtained with the Geoffrion's Lemma. Compare the bound obtained with that of linear relaxation explaining why can we expect that in this case the former is better than the latter.

3. Consider the ILP formulation of the set covering and the Lagrangean relaxation of the covering constraints. Build a matheuristic that exploits this relaxation. Apply it to the numerical example of slide 12 of Lesson 2.