Genetic algorithms

Exercise Consider a population of 4 individuals $X = \{x_1, x_2, x_3, x_4\}$, characterised by the following fitness values

$$\phi(x) = [\ 10\ 18\ 9\ 13\ ]$$

and assume that the pseudorandom number generator provides the following sequence: $r = (0.47, 0.33, 0.80, 0.71, 0.12, 0.93)$. Generate a new population of individuals with

a) the roulette wheel selection mechanism;

b) the rank selection mechanism;

c) the tournament selection mechanism on the following subsets of individuals: $(1, 3)$, $(1, 4)$, $(2, 3)$ and $(2, 4)$;

Solution

Part a) Roulette wheel selection assigns to each individual a probability proportional to the value of its fitness. Since the sum of all fitnesses is $10+18+9+13 = 50$, the corresponding probabilities and their partial sums are reported in the following table.

Individual i	x_1	x_2	x_3	x_4
$\phi(\xi(x))$	10	18	9	13
$\pi_i = \dfrac{\phi_i}{\sum_j \phi_j}$	0.20	0.36	0.18	0.26
$\sum_{j=1}^{i} \pi_j$	0.20	0.56	0.74	1.00

The sequence of extractions is the following (the extracted elements are not removed, so that they can be extracted any number of times):

1. $r = 0.47$ falls in the second interval, so that we select x_2;
2. $r = 0.33$ falls in the second interval, so that we select x_2;
3. $r = 0.80$ falls in the fourth interval, so that we select x_4;
4. $r = 0.71$ falls in the third interval, so that we select x_3.

The final population is $X = \{x_2, x_2, x_3, x_4\}$.

Part b) Rank selection assigns to each individual a probability proportional to its index in a fitness nondecreasing order. Since $\phi(\xi(x_3)) < \phi(\xi(x_1)) < \phi(\xi(x_2)) < \phi(\xi(x_4))$, solution x_3 has index 1, solution x_1 has index 2 and so on, producing the following table of indices, probabilities and partial sums of probabilities.

Individual i	x_1	x_2	x_3	x_4
Index k	2	4	1	3
$\pi_i = \dfrac{2k}{n(n+1)}$	0.20	0.40	0.10	0.30
$\sum_{j=1}^{i} \pi_j$	0.20	0.60	0.70	1.00

The sequence of extractions is the following (the extracted elements are not removed, so that they can be extracted any number of times):

1. $r = 0.47$ falls in the second interval, so that we select x_2;
2. $r = 0.33$ falls in the second interval, so that we select x_2;
3. $r = 0.80$ falls in the fourth interval, so that we select x_4;
4. $r = 0.71$ falls in the fourth interval, so that we select x_4.

The final population is $X = \{x_2, x_2, x_4, x_4\}$.

Part c) Tournament selection extracts $n_p = 4$ subsets of $\alpha = 2$ individuals, given in the text of the exercise, and selects the individual with the largest fitness in each subset.

The sequence of selections is:

1. from (x_1, x_3), select x_1;
2. from (x_1, x_4), select x_4;
3. from (x_2, x_3), select x_2;
4. from (x_2, x_4), select x_2.

The final population is $X = \{x_1, x_2, x_2, x_4\}$.