1 Minimum spanning tree problem

1. Given a graph $G = (V, E)$, what is a spanning tree?
2. How many edges does a spanning tree with n nodes contain, and why?
3. Briefly describe Kruskal's algorithm.
4. Given a graph with n vertices and m edges, what is the best way to solve the minimum spanning tree problem depending on n and m, and why?
5. How can one check whether a given spanning tree is of minimum total cost without solving the problem from scratch?
6. Solve using Kruskal's algorithm the problem of connecting at minimum total cost five computing centres; the following table reports the connection cost for each pair of centres.

centre	1	2	3	4	5
1	-	17	16	18	17
2	17	-	12	10	16
3	16	12	-	13	15
4	18	10	13	-	15
5	17	16	15	15	-

7. Determine a minimum cost tree spanning the graph with the following cost matrix:

centre	1	2	3	4	5	6
1	-	6	8	3	-	-
2	6	-	5	1	2	-
3	8	5	-	-	2	5
4	3	1	-	-	2	8
5	-	2	2	2	-	6
6	-	-	5	8	6	-

2 Shortest path problem

1. Find the shortest paths between node 1 and all the other nodes in the graph with the following cost matrix:

	1	2	3	4	5	6
1	-	6	8	3	-	-
2	6	-	5	1	2	-
3	8	5	-	-	2	5
4	3	1	-	-	2	8
5	-	2	2	2	-	6
6	-	-	5	8	6	-

2. Briefly describe the method used and discuss its computational complexity.

3. Find the shortest paths between node 1 and all the other nodes in the following graph (SEE PROBLEM 6 IN TEMIESAME1B.PDF)

3 Maximum flow problem

Consider the following network, where the values on the arcs indicate the corresponding capacities.

SEE PROBLEM 7 IN TEMIESAME1B.PDF

Determine the value of a maximum flow that can be sent from s to t, starting from the feasible flow where 10 units are sent over the path $(s, 1, 2, 3, 4, t)$.

What property of maximum flows guarantees that the obtained flow is of maximum value? Explain why.

4 Maximum flow problem

Consider an undirected graph where the capacities are reported in the following table:

	a	b	c	d	e	f	t
s	20	15					
a		5	8		7		
b				17	9		
c		5					
d			3				18
e				1		11	
f	9						14

1. determine with Ford-Fulkerson's method a maximum flow between s and t, reporting all steps;
2. find a minimum capacity cut and indicate its relationship with a maximum flow;
3. assume that all capacities are integer. Indicate an upper bound on the value of the max flow between s and t as a function of the max capacity K_max. Deduce an upper bound on the complexity of the version of the Ford-Fulkerson's method that explicitly uses auxiliary (incremental) networks (as a function of the number of arcs m and of K_max). Explain why.

5 Project scheduling

A company is starting the production of a new product P. Each unit of P is obtained by assembling one unit of product $P1$ and one of product $P2$. Before starting the production of any product, materials must be bought and workers must be trained. Training is different for workers involved in the production of $P1$ and $P2$ (the assembling phase requires no training), while materials needed for the two production lines are bought at the same time. We can train and buy materials in parallel. Before products $P1$ and $P2$ can be assembled, each unit of $P2$ must be inspected. After the assembling phase, products P must be tested and then stocked in a suitable area. The list of activities with the corresponding durations are reported:

Index	Activity	Duration
A	buying materials	9
B	training line P1	10
C	training line P2	5
D	production of P1	8
E	production of P2	7
F	inspection of P2	4
G	assembling phase	6
H	testing product P2	5
I	stocking	2

1. report for each activity the set of predecessors (if any). Define the graph corresponding to the project.
2. Determine via the CPM method the earliest completion time for the project and the slack of each activity.
3. Report the order of complexity of the CPM algorithm, as a function of the number of arcs in the graph. Explain why.

6 Project scheduling

A company is starting a project composed of 8 activities, denoted by letters from A to H. The following table reports the duration of each activity and the precedence relations between them:

Activity	Duration	Predecessors
A	2	-
B	8	A
C	4	B
D	4	B
E	3	C
F	8	B,E
G	2	E
H	8	D,F

1. Draw the directed graph corresponding to the project, reporting $T_{\min}$ and $T_{\max}$ corresponding to the occurrence of each event corresponding to a node.

2. Determine the total minimum completion time for the project and the slack of each activity.

3. Indicate the critical activities.

4. Report the Gantt diagram at the latest (each activity is completed as late as possible) and at the earliest (each activity starts as early as possible).

7 Linear programming

Given the following linear programming problem: ...

1. Rewrite the problem in standard form.
2. Give an upper bound on the number of basic feasible solutions of the problem, as a function of the number of constraints and variables.
3. Solve the problem via the Simplex method (applying Bland's rule). Indicate the optimal solution found and the objective function value, reporting all the iterations on the back of the previous page.
4. What is the reduced cost of a nonbasic variable from the objective function point of view? Report the reduced costs for the nonbasic variables on the optimal solutions that has been found.
5. Write the dual problem.
6. Deduce an optimal dual solution from the optimal primal one, clearly stating how it has been obtained.

8 Linear programming

Given the following linear programming problem: ...

1. Write the dual.
2. Solved the dual geometrically.
3. Deduce an optimal primal solution from an optimal dual one. State and explain on which property or theorem this deduction is based.

9 Linear programming

Given the following linear programming problem: ...

1. Rewrite the problem in standard form.
2. Carry out *two* iterations of the simplex method using Bland's rule. Only indicate the solution and the value of the objective function, reporting all the iterations on the back of the previous page.
3. Explain why the solution is optimal. Are there any other optimal solutions? Motivate the answer.

4. Write the dual problem.
5. State the complementary slackness conditions for this pair of primal and dual problems. Determine an optimal dual solution.

10 Integer linear programming

Solve the following integer linear programming problem ... using the Branch-and-Bound method, solving the linear relaxations graphically. Explore first the subproblem with the most promising bound. If the solution of a relaxation has more than a single fractional component, branch on the component with the fractional part closest to 0.5.

1. Indicate the optimal solution that was found and the corresponding objective function value, reporting all the iterations (solutions of the subproblems) on the back of the previous page.
2. Report the enumeration tree.

11 Integer linear programming

Consider the following integer linear programming problem: ... with the optimal tableau of the continuous relaxation -31/4 — 0 0 1/2 0 1/4 ——————- 11/4 — 1 0 -1/2 0 1/4 9/4 — 0 1 3/2 0 -1/4 1/2 — 0 0 -2 1 -1/2

1. Briefly describe the idea of cutting planes.
2. Determine the Gomory cut associated with the basic variable with the fractional part closest to 0.5.
3. Express the Gomory cut as a function of the decision variables x1 and x2 and represent it graphically with the other constraints that determine the feasible region of the integer linear program.
4. Which algorithm should we use to find an optimal solution of the continuous relaxation of the problem when adding the new constraint? Why?