# Network Design and Optimization course

## Lecture 3

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# The problem

Given

- a set of computers to be connected
- the set of links that can be installed

(that is, there is no network yet ... I want to build one).
I want to

- decide where to install connections (cables)
- maximizing the quality of service.

Any idea?

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Install all connections

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(that is, there is no network yet ... I want to build one).
I want to

- decide where to install connections (cables)
- *minimizing the cost of the network.*

... not so easy anymore ...

# Assumptions

Some assumptions:

1. no *dimensioning* (yet!): the capacity of each link will always be enough to satisfy connection request,
2. we do not care how demand will be *routed* on the network.

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Basic observation: to connect $n$ computers we only need $n-1$ cables!

# Recognizing a known problem ...

We are facing a *Minimum spanning tree problem*!

# Modeling the costs

Step 1: estimating link installation costs.

- Basic assumption: the link cost is a function $\mu(x)$ of a distance $x$ between its endpoints.
- Basic assumption: I cannot move nodes!

e.g. I have an existing facility (let's say a computer lab) and I want to connect all terminals using point-to-point links.

# Basic design example.

So ...

- given any pair $l = (i, j)$ of nodes, one can compute $x_l = dist(i, j)$, and therefore $c_l = \mu(x_l)$;
- let $L$ be the set of links to be installed in the network;
- the overall cost of the network is $\sum_{l \in L} c_l$.

# Network model

Given a set of nodes and potential connections, build a graph $G = (V, E)$ having

- one vertex $i \in V$ for each node of the network
- one edge $e \in E$ for each potential connection of the network
- costs $c_e$ on each edge $e \in E$

And solve a Minimum Spanning Tree problem!

# MCSTP algorithms

There is a suite of algorithms for the MCSTP:

- optimality conditions,
- algorithms,
- complexity,

(See Orlin's slides).

# MCSTP algorithm implementation

Lab session: implementing

- Kruskal,
- Prim,
- Sollin,

in AMPL.