Network Design and Optimization course

Lecture 2

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The problem

Given

- an existing network,
- a connection request between two nodes of the network,

I want to

- decide which links to use in the connection (route)
- maximizing the quality of service (e.g. minimizing delay time)

Assumptions

Some assumptions:

1. all packets must be routed on the same links,
2. the capacity of each link is enough for the whole connection request.

N.B. imposing (1), or assuming that link usage cost does not depend on the amount of traffic routed is the same.

Recognizing a known problem ...

We are facing a *Shortest path problem*!

Modeling the costs

Step 1: estimating link usage costs. How?

- Know your network;
- make suitable assumptions and simplifications!

Example 1: routing with TCP/IP

Let's assume to have a network using TCP/IP:

- packets are IP datagrams
- network congestion can cause some packets to be dropped (e.g. not enough buffer space), and therefore need of re-transmission
- a network designer can disregard link capacities, trying to minimize average packets delay.

Example 1: routing with TCP/IP

Statistical assumptions:

- Average packet size: K_p Kb.
- Average packet arrival time: λ_p packets per second.
- Link bandwidth: Q bits per second.
- Average service rate of the link: $\mu_p = Q/K_p$.
- Packet arrival is a Poisson process.
- Packet size (and therefore processing time) is exponentially distributed.
- We model the system as a M/M/1 queuing system.

$\rightarrow$ average delay of the link:

$$c = \frac{1}{\mu_p - \lambda_p}$$

Example 1: routing with TCP/IP

Numerical example:

- Average packet size: $K_p = 8$ Kb.
- Average packet arrival time: $\lambda_p = 100$ pps.
- Link bandwidth: $Q = 1.54$ Mbps.
- Average service rate of the link: $\mu_p = Q/K_p = 190$ pps.

$\rightarrow$ average delay of the link:

$$c = \frac{1}{\mu_p - \lambda_p} = 11.11ms$$

Example 1: routing with TCP/IP

So ... given any link e of the network, one can compute c_e, and the overall expected delay of a packet routed on links $(e(1) \ldots e(n))$ is $\sum_{i=1}^{n} c_{e(i)}$.

Network model

Given a network, build a graph $G = (V, E)$ having

- one vertex $i \in V$ for each node of the network
- one edge $e \in E$ for each link of the network
- costs c_e on each arc $e \in E$
- a special vertex $s \in V$ representing the origin of packets
- a special vertex $t \in V$ representing the destination of packets

And solve a Shortest Path Problem!

Dijktra's algorithm

See Orlin's slides

Dijktra's correctness proof

See Orlin's slides

Dijktra's algorithm implementation

Let's implement Dijkstra in AMPL ...