Modeling, Analysis and Optimization of Networks

Alberto Ceselli
alberto.ceselli@unimi.it

Università degli Studi di Milano
Dipartimento di Informatica

Doctoral School in Computer Science

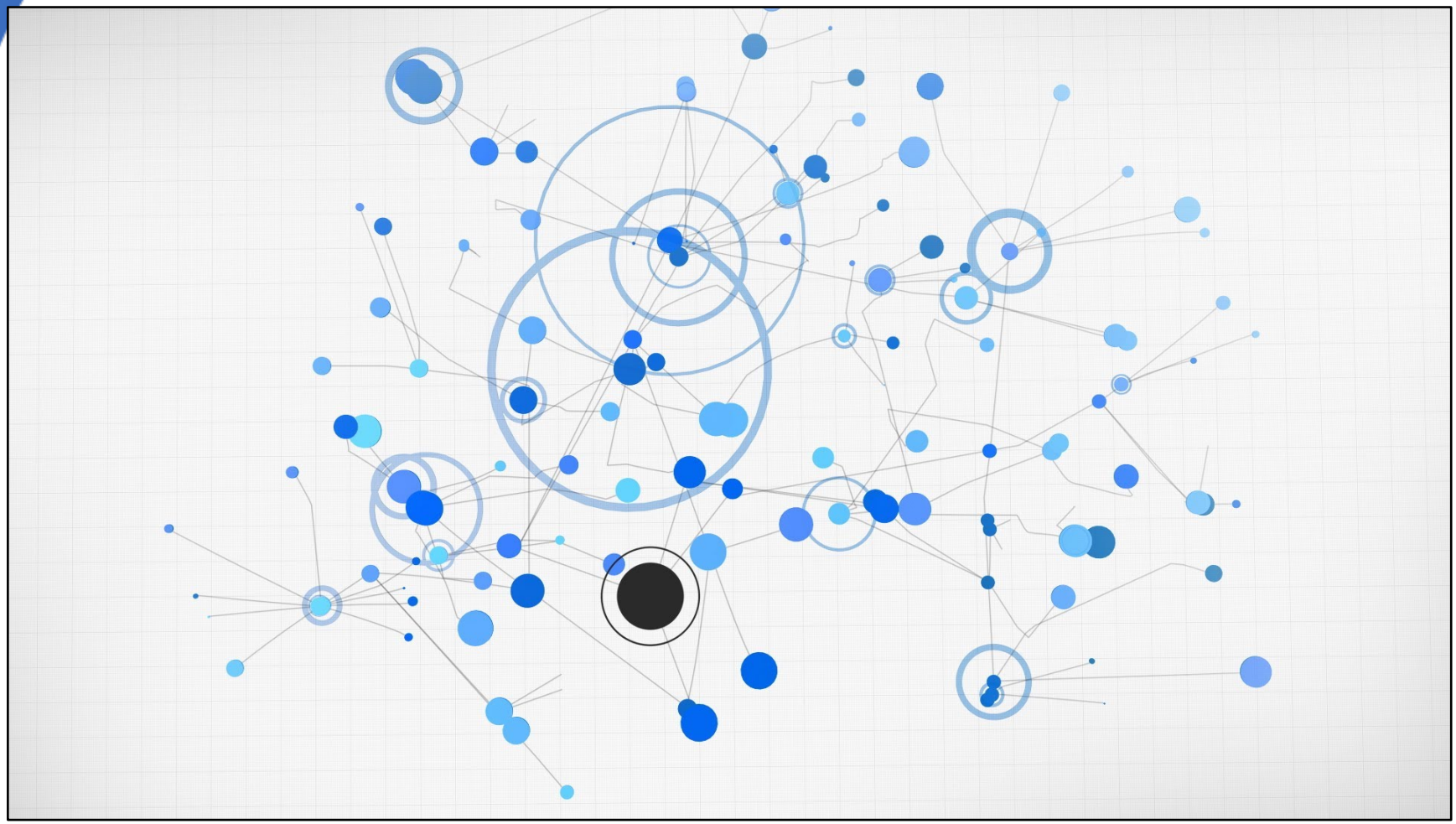
A.A. 2015/2016

What's a Network ?

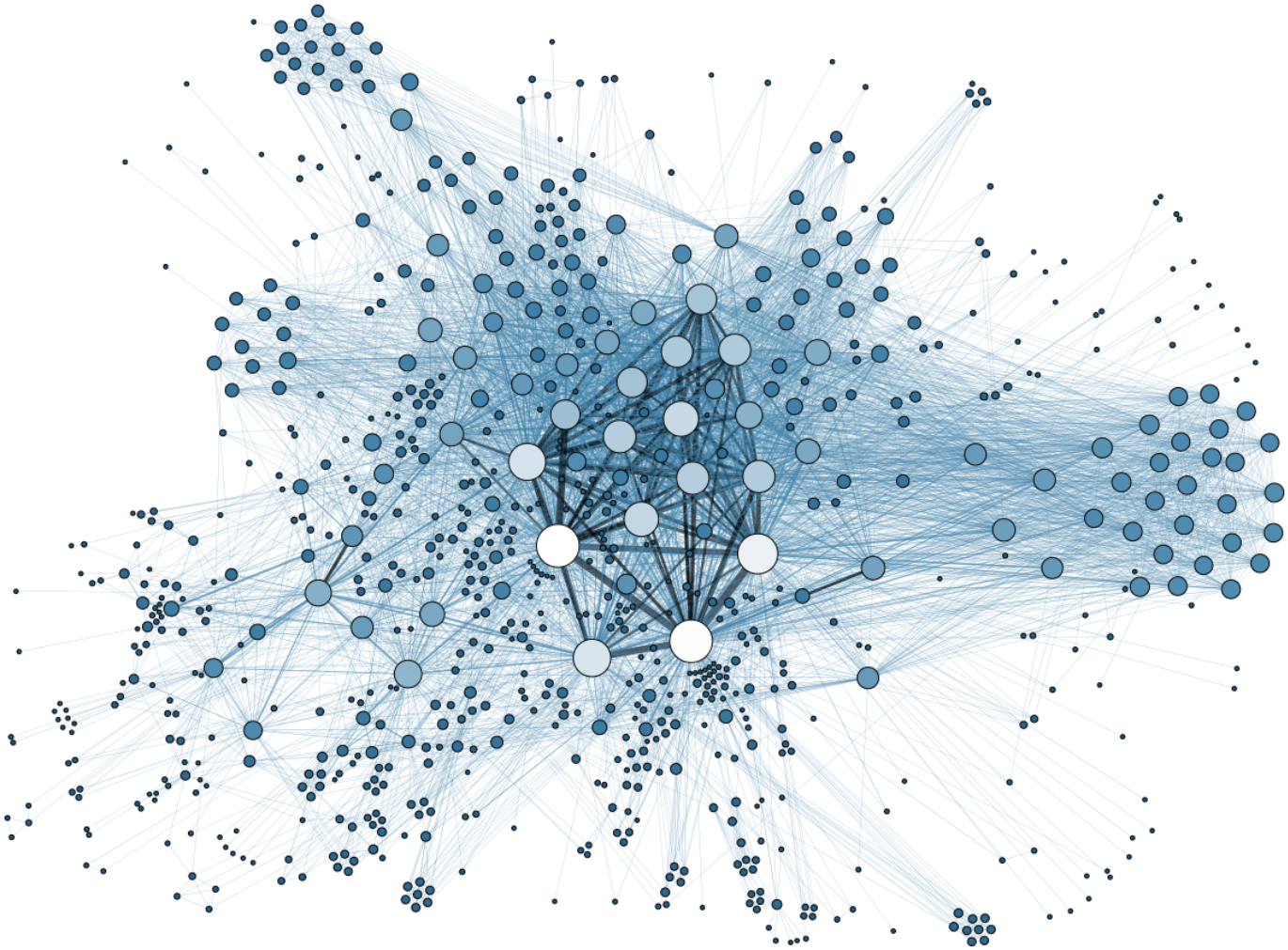
- Dictionary.com:
  any netlike combination of filaments, lines, veins, passages, or the like
- Wikipedia:
  disambiguation page with 30 entries in 4 categories
- Let's try images !

First image in Google :

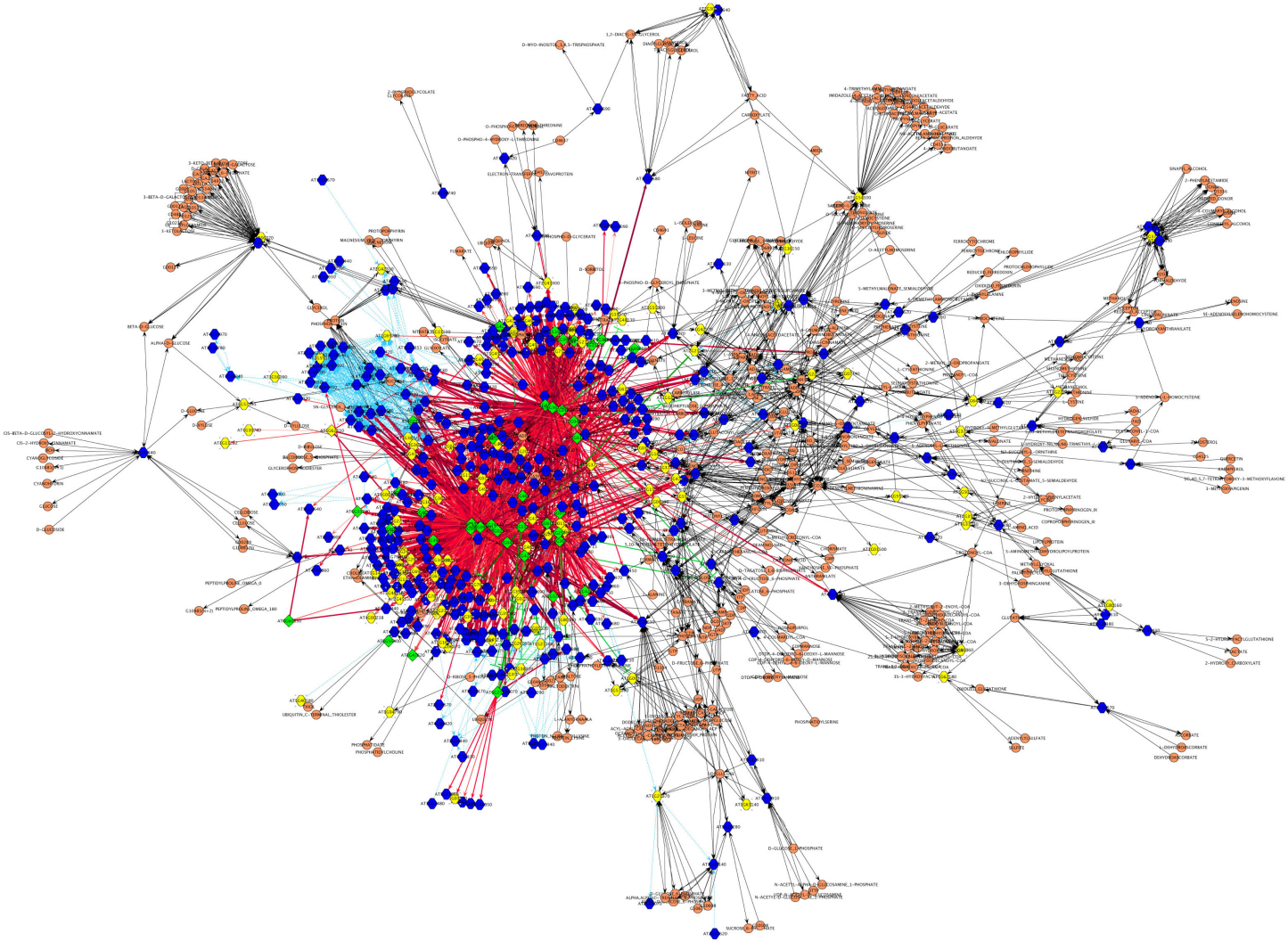
A few images after: artistic ones



A few images after: artistic ones



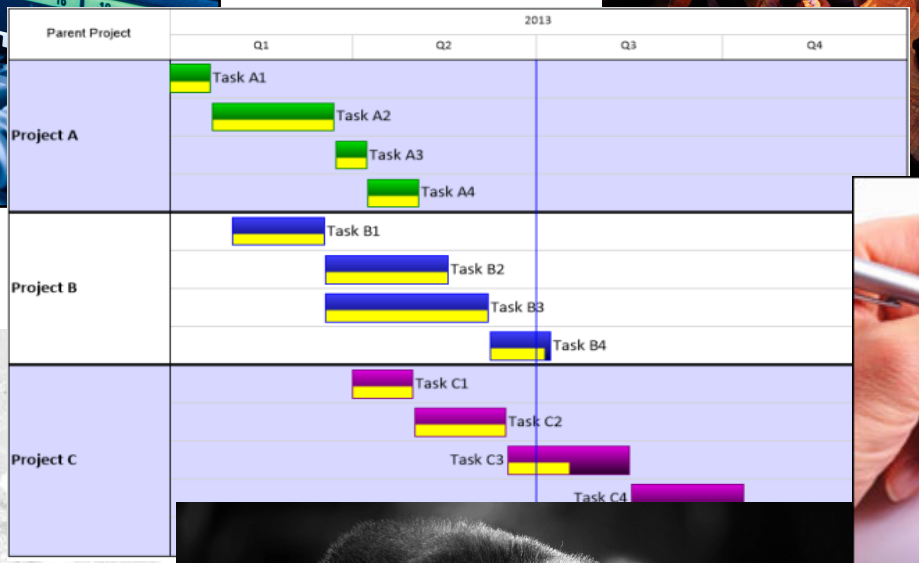
… and very different domains

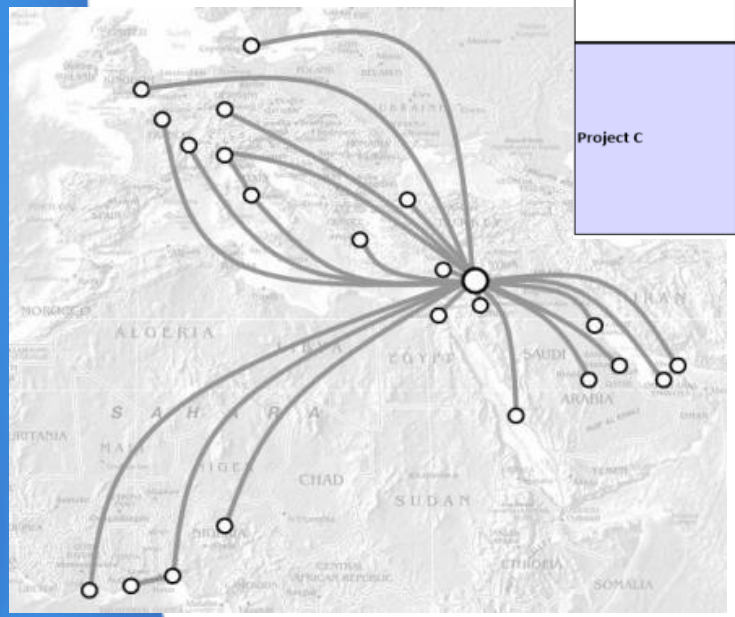


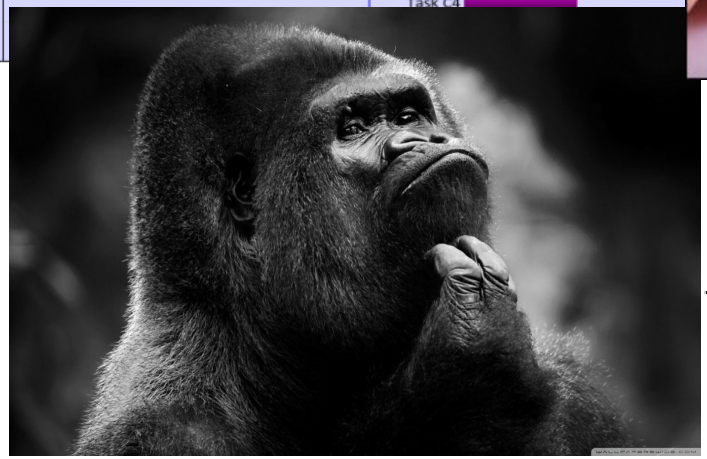
… only rank 26!


9 10 11 12 13 14 15 16 17 18 19
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
11X 13X

Trivia: how many networks do you see?







$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix}.$$

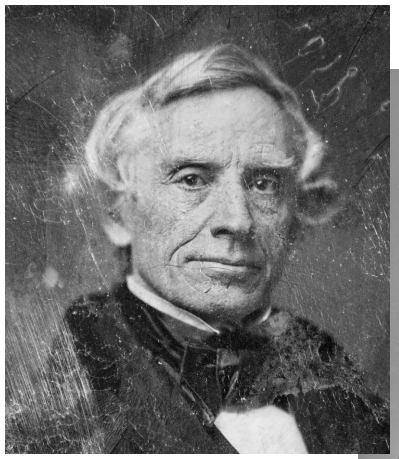
To remember:

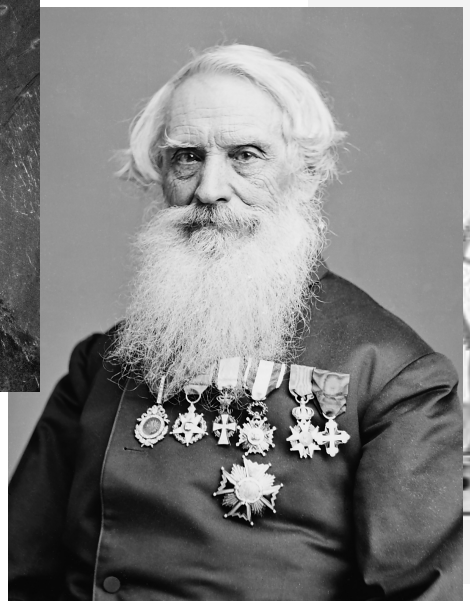
1 - Networks are pervasive !

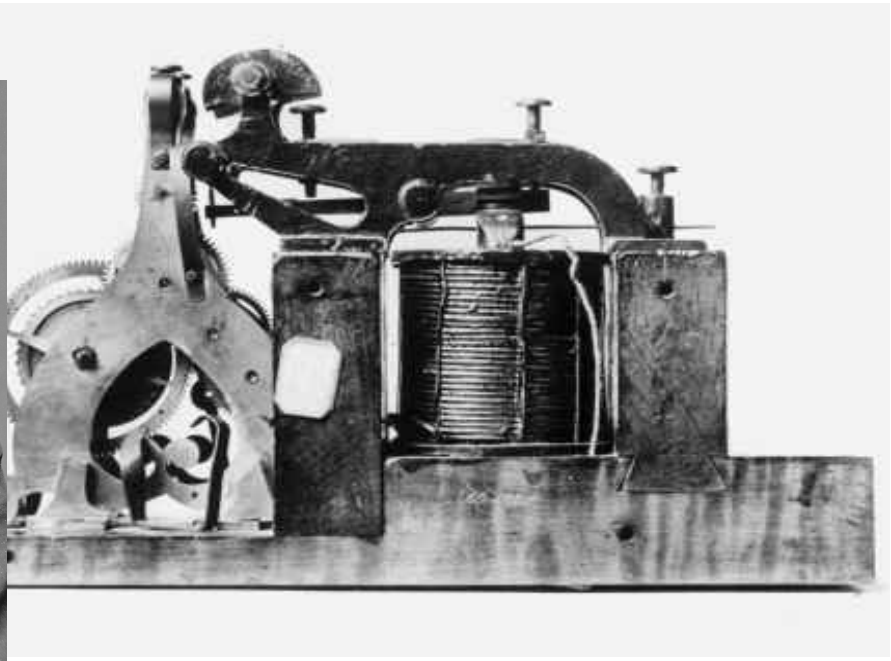
2 - by « network » we mean far more than computers connected by cables;

And what about computer networks?

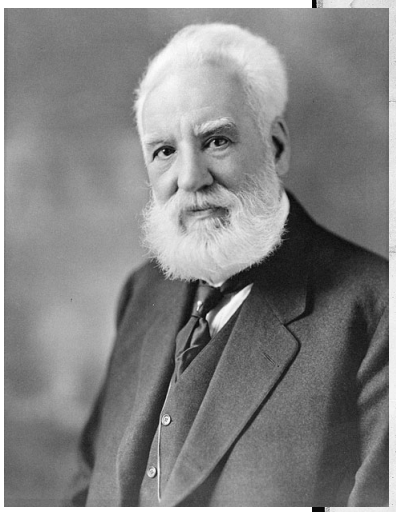
- Samuel Morse (1791 – 1872)
  - professor of arts and design at New York University
  - in 1835, proved that signals could be transmitted by wire …

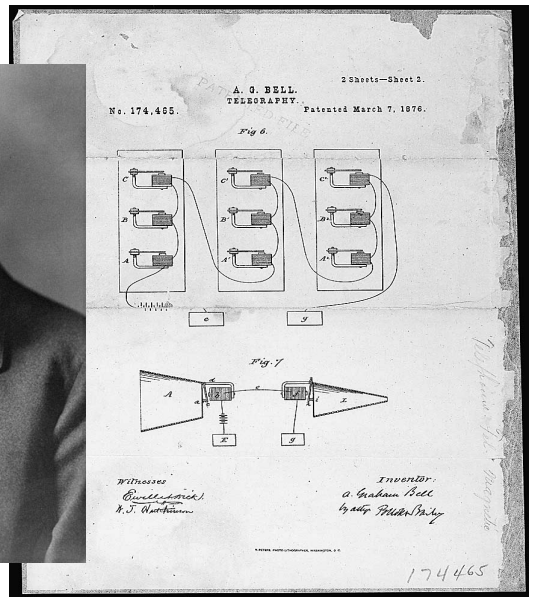






- Alexander Graham Bell (1847 - 1922), and Thomas Watson
  - initially working on multi-tone telegraphy (many signals on the same line at the same time)
  - multi-tone then became the telephone!
  - patented during 1876 ...

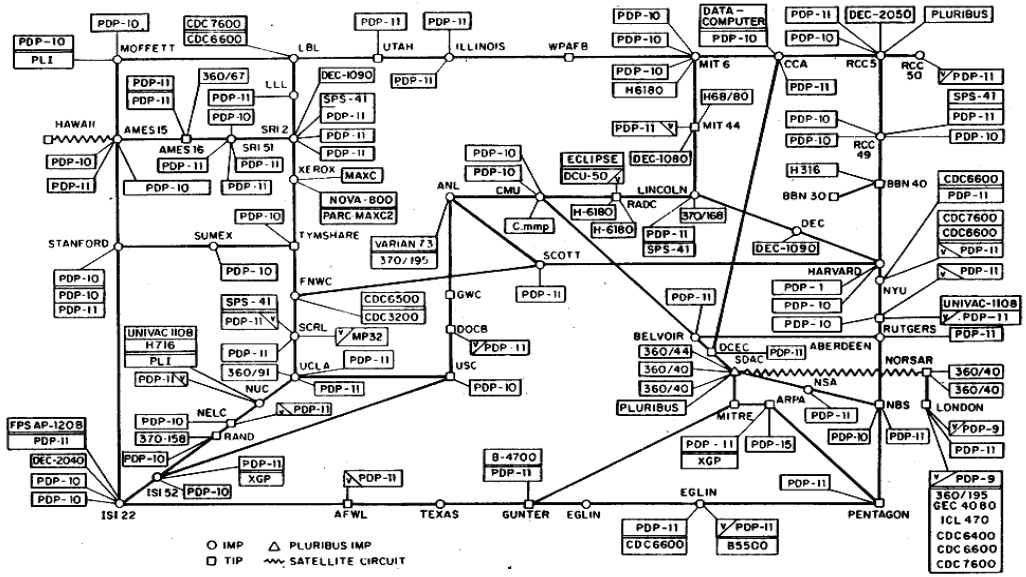




- The Advanced Research Projects Agency Network (ARPANet),
  - J. C. R. Licklider, articulated the ideas in his January 1960 paper, Man-Computer Symbiosis,
  - First operational packet switching network between computers
  - actually deployed in 1969
  - the first message « LO(G) » yielded a system crash!

29OCT69	2100	LOADED OP. PROGRAM EOI2 BBN BARKER BBN	CSK
	22:30	Talked to SRI Host to Host	CSK
		Left imp program running after sending a host dead message to imp.	CSK

ARPANET LOGICAL MAP, MARCH 1977



(PLEASE NOTE THAT WHILE THIS MAP SHOWS THE HOST POPULATION OF THE NETWORK ACCORDING TO THE BEST INFORMATION OBTAINABLE, NO CLAIM CAN BE MADE FOR ITS ACCURACY)

NAMES SHOWN ARE IMP NAMES, NOT (NECESSARILY) HOST NAMES

- WWW: a system of interlinked hypertext documents
  - Tim Berners-Lee wrote a proposal in March 1989
  - Robert Cailliau proposed in 1990 to use hypertexts …
- Google (Larry Page and Sergey Brin in 1998):
  - over 1 million servers, at least 12 data centers located only in the U.S.A. (in 2012);
  - internet search: about 24 PB of user data daily (in 2012);
  - cloud computing: managing and balancing distributed
  - resources.
- Facebook (Mark Zuckerberg et al. in 2004):
  - handling social networks of billion users (1.6×10^9?)

To remember:

1 - Networks are pervasive !

2 - by « network » we mean far more than computers connected by cables;

3 - network problems moved from technologies to applications, and now to services;

4 - networks are in general too complex to be managed by humans without decision support systems.

Modeling, Analysis and Optimization of Networks

- A « transverse » course offered by D.I.
- (Far) More on modeling and structural properties than on specific techniques and technologies
- Different editions cover different sub-topics

- This year: **network flows**

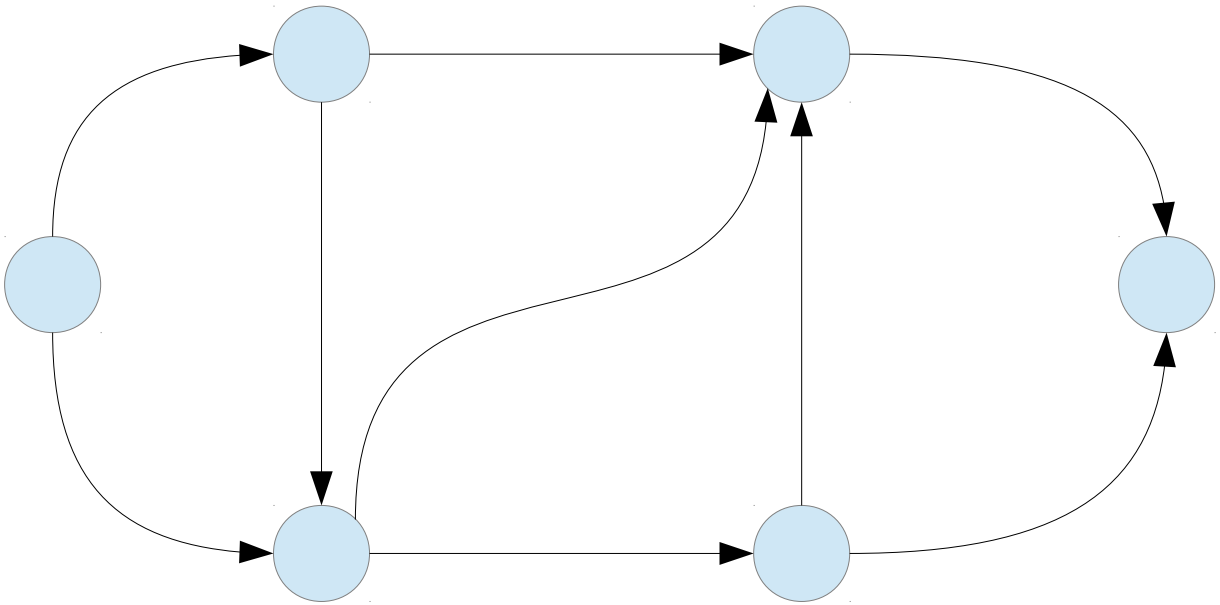
Course objectives

- finding network (flow) structures in applications
- modeling as flow problems on networks
- main theoretical results on flows and flow algorithms
- overview of tools for solving network flow problems

PART I: Modeling *with* network flows

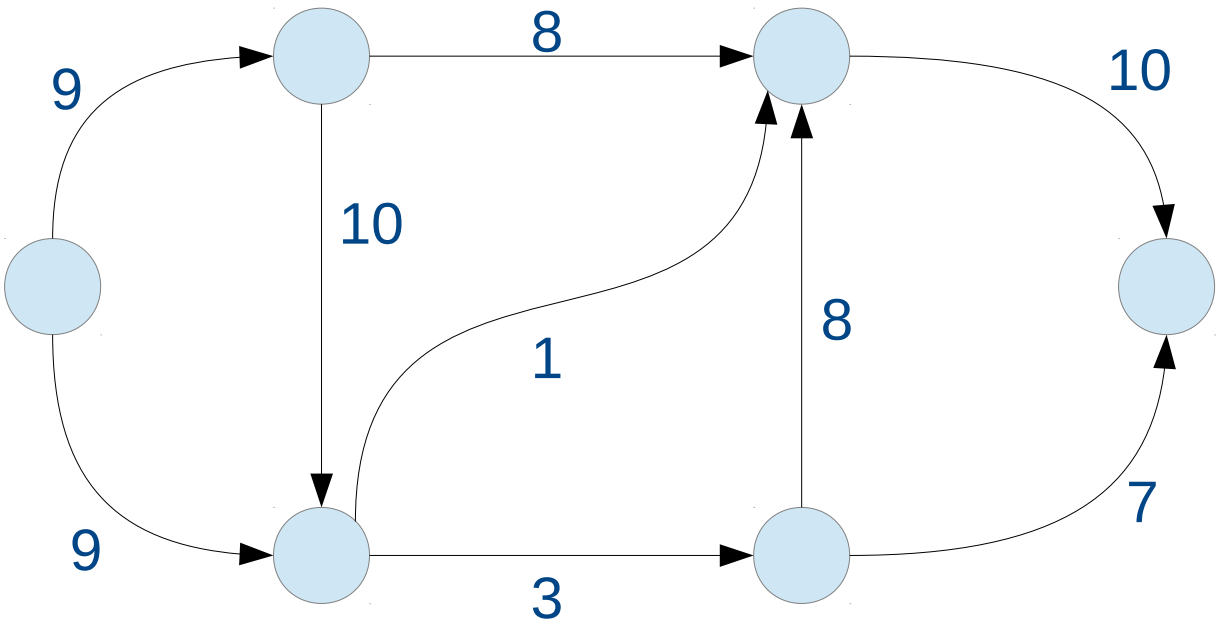
Notation

- A directed graph G(V,A)



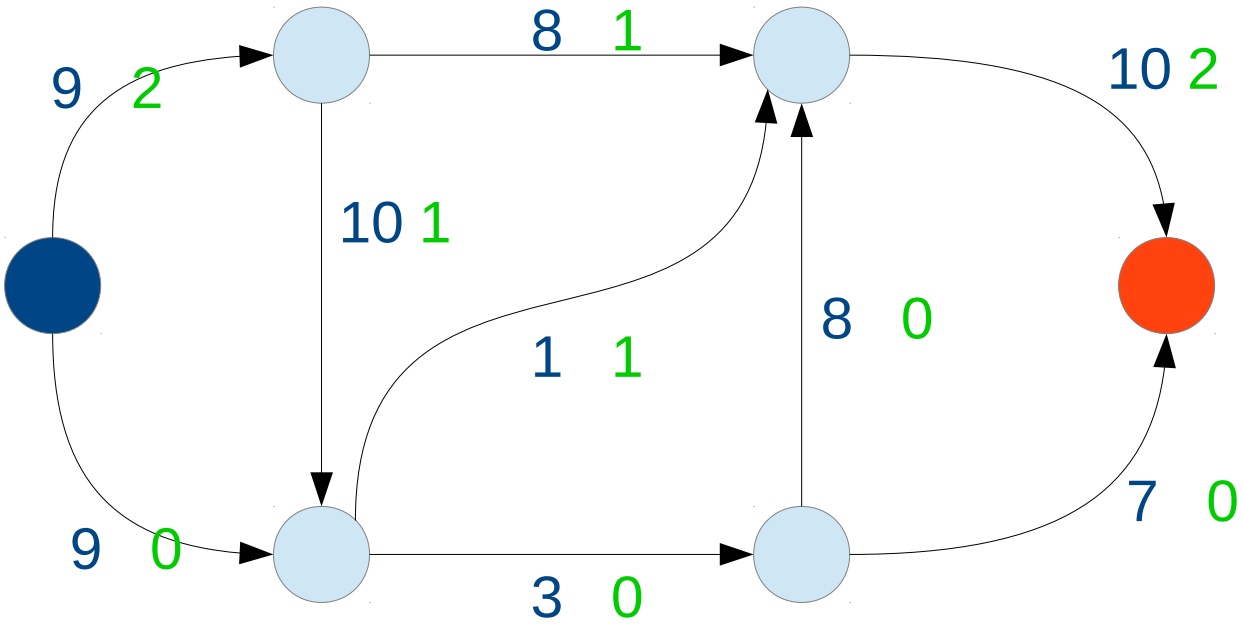
Notation

- A directed graph G(V,A)
- A function c: A → ℝ (arc capacities)



Notation

- A directed graph G(V,A)
- A function c: A → $\mathbb{R}$ (arc capacities)
- Special nodes s (source) and t (sink)
- A flow is a function f: A → $\mathbb{R}_+$



Notation

- Let $\partial^-(i)$ be the set of incoming arcs in i
- Let $\partial^+(i)$ be the set of outgoing arcs from i
- A flow is **feasible** iff
  - $f(i,j) \leq c(i,j)$
    for each (i,j) in A (capacity constr.)
  - $f(i,j) \geq 0$
    for each (i,j) in A (non-negativity constr.)
  - $\sum_{j \in \partial^-(i)} f(j,i) = \sum_{j \in \partial^+(i)} f(i,j)$
    for each i in V \ {s,t} (flow conservation constr.)
- A flow is **maximum** if

  $\sum_{j \in \partial^+(s)} f(s,j) = \sum_{j \in \partial^-(t)} f(j,t)$

  is maximum

How to *model* with flows ?

- Given an application
- Design V, s and t
- Design A
- Design c()
- Find f()
- Give to f() an interpretation in the original application

How to *learn* modeling with flows ?

- As for math modeling in general, no specific recipe
  - A bit of theory
  - A few modeling tricks and gadgets
  - Training on a few examples
  - + Intuition :)

How to learn modeling with flows ?

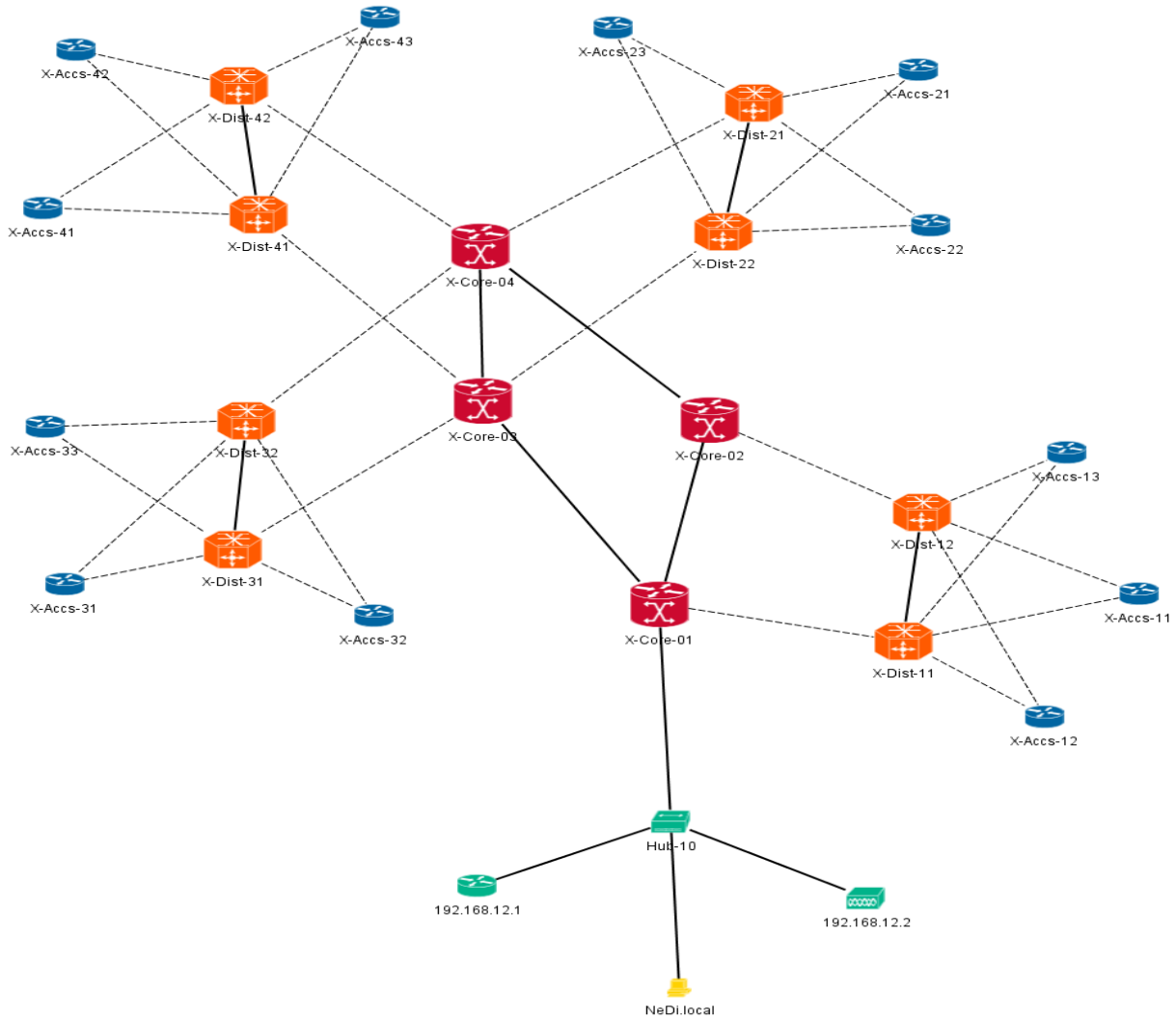
- As for math modeling in general, no specific recipe
  - A bit of theory
  - A few modeling tricks and gadgets
  - Training on a few examples
  - + Intuition :)

Example 1: routing in communication networks

Given a communication network composed by nodes and links, and a special pair of nodes (source and destination) that need to transfer data

- A Path Problem
  - Find a path from source to destination in the network
- A Backup Path Problem
  - Find **two** paths from source to destination in the network having **no common arc**
- A network robustness Problem
  - find **how many** paths are there from source to destination, having **no common arc**

- From X-Accs-33 to X-Accs-41
- From X-Core-01 to X-Dist-21



Example 2: Tango Dancers problem

Taken from J. Kleinberg, E. Tardos, « Algorithm Design»

- Given :
  - a set G of gentlemen and a set L of ladies
  - a set of compatibilities
  - find how many couples can be on the dance floor at the same time, at most
- i.e. a max matching problem on a bipartite graph
- **Observation:** the number of couples equals the flow, that in turn equals the number of arcs whose capacity constraints are active!

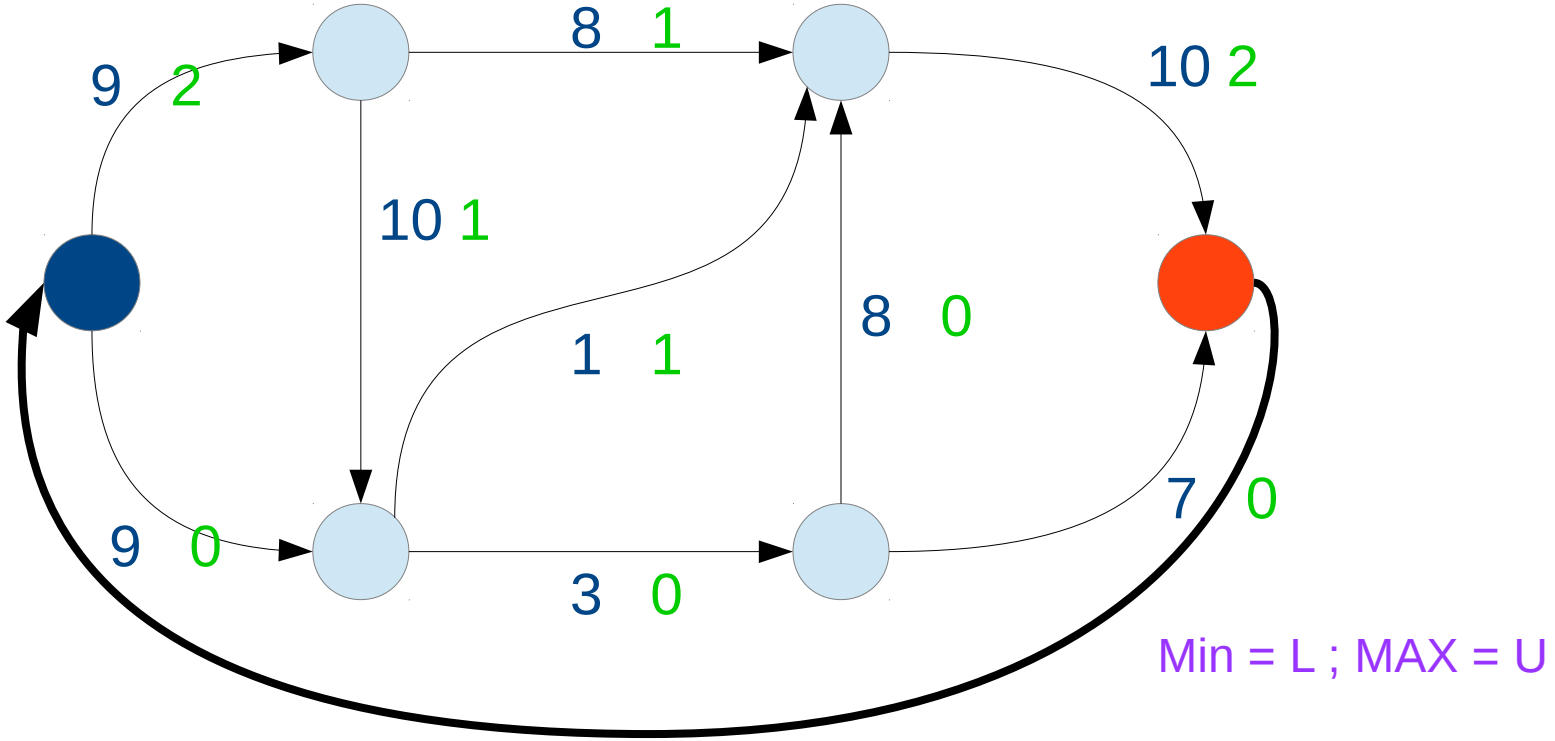
A few extensions (1)

- Capacity on nodes
- Multiple sources and multiple sinks
- Integral flows

(blackboard discussion)

A few extensions (2)

- Flows and Circulations



- Circulations with demands
- Lower bound on flow on each arc

(blackboard discussion)

Example 3: consistent rounding problem

- Did you ever compile tax forms?
- Given a pxq matrix D containing real values d_{ij} with row sums a_i and column sums b_j.
- You want to round **both** matrix coefficients **and** row/col sums in a **consistent** way
- The decision to round up or down is up to you

Example 3: consistent rounding problem

- From N.F., Application 6.3

3.1	6.8	7.3	17.2
9.6	2.4	0.7	12.7
3.6	1.2	6.5	11.3
16.3	10.4	14.5	