
Università degli Studi di Milano
Master Degree in Computer Science

Information Management course

Teacher: Alberto Ceselli

Lecture 15: 25/11/2014

Data Mining:

Concepts and Techniques

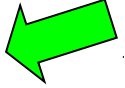
(3rd ed.)

— Chapter 6 —

Jiawei Han, Micheline Kamber, and Jian Pei
University of Illinois at Urbana-Champaign &
Simon Fraser University

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Chapter 5: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts 
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary

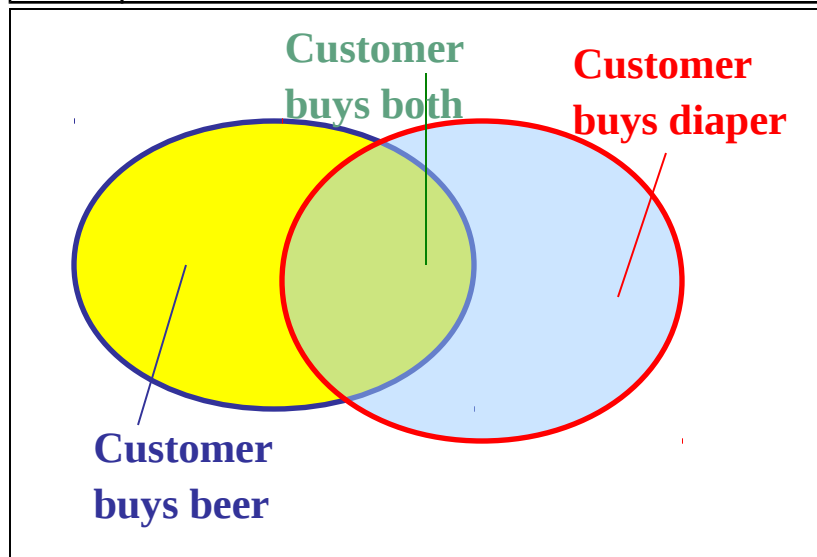
What Is Frequent Pattern Analysis?

- **Frequent pattern**: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal, Imielinski, and Swami (1993) in the context of **frequent itemsets** and **association rule mining**
- Motivation: Finding inherent regularities in data
 - What products were often purchased together? Beer and diapers?!
 - What are the subsequent purchases after buying a PC?
 - What kinds of DNA are sensitive to this new drug?
 - Can we automatically classify web documents?
- Applications
 - Basket data analysis, cross-marketing ..., Web log (click stream) analysis, and DNA sequence analysis.

Basic Concepts: Frequent Patterns

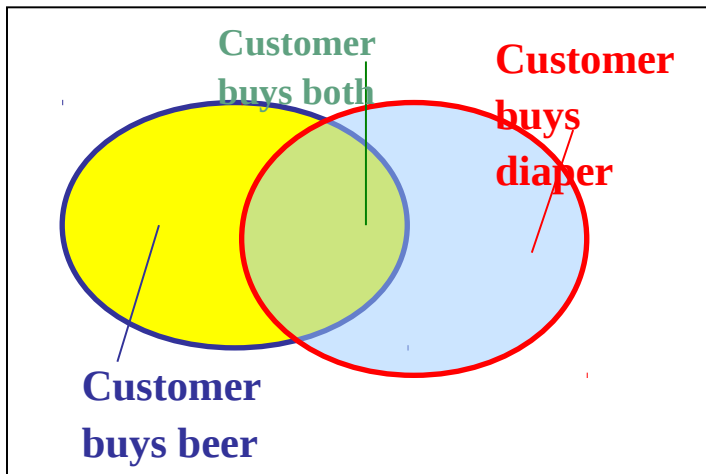
Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

- **itemset**: A set of one or more items
- **k-itemset** $X = \{x_1, \dots, x_k\}$
- **(absolute) support**, or, **support count** of X : number of occurrences of an itemset X in the dataset
- **(relative) support**, s , is the fraction of transactions that contains X (i.e., the **probability** that a transaction contains X)
- An itemset X is **frequent** if X 's support is no less than a **minsup** threshold



Basic Concepts: Association Rules

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
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- Find all the rules $X \rightarrow Y$ fixing a minimum support and confidence
 - support**, s , **probability** that a transaction contains $X \cup Y$
 - confidence**, c , **conditional probability** that a transaction having X also contains Y

Let $minsup = 50\%$, $minconf = 50\%$
 Freq. Pat.: Beer:3, Nuts:3, Diaper:4,
 Eggs:3, {Beer, Diaper}:3

Association rules: (many more!)

- $Beer \rightarrow Diaper$ (60%, 100%)
- $Diaper \rightarrow Beer$ (60%, 75%)

Closed Patterns and Max-Patterns

- A (long) pattern contains a combinatorial number of sub-patterns, e.g., $\{a_1, \dots, a_{100}\}$ contains

$$\binom{100}{1} + \binom{100}{2} + \dots + \binom{100}{100} = 2^{100} - 1 = 1.27 \cdot 10^{30}$$

sub-patterns!

- Idea: restrict to *closed* and *maximal* patterns
 - An itemset X is a **closed p.** if X is *frequent* and there exists *no super-pattern* $Y \supset X$, with the *same support* as X
 - An itemset X is a **maximal p.** if X is frequent and there exists *no super-pattern* $Y \supset X$, which is *also frequent*
- Closed pattern is a lossless compression of freq. Patterns: reducing the # of patterns and rules

Closed Patterns and Max-Patterns

- Exercise.

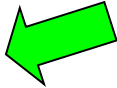
DB = { $\langle a_1, \dots, a_{100} \rangle$, $\langle a_1, \dots, a_{50} \rangle$ }

- Min_sup = 1.
- What is the set of **closed itemset**?
 - $\langle a_1, \dots, a_{100} \rangle$: 1
 - $\langle a_1, \dots, a_{50} \rangle$: 2
- What is the set of **maximal pattern**?
 - $\langle a_1, \dots, a_{100} \rangle$: 1
- What is the set of **all patterns**? !!

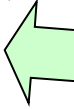
Computational Complexity of Frequent Itemset Mining

- How many itemsets are potentially to be generated in the worst case?
 - The number of frequent itemsets to be generated is sensitive to the minsup threshold
 - When minsup is low, there exist potentially an exponential number of frequent itemsets
 - The worst case: M^N where M : # distinct items, and N : max length of transactions
- The worst case complexity vs. the expected probability
 - Ex. Suppose Amazon has 10^4 kinds of products
 - The chance to pick up one product 10^{-4}
 - The chance to pick up a particular set of 10 products: $\sim 10^{-40}$
 - What is the chance this particular set of 10 products to be frequent (e.g. 10^3 times in 10^9 transactions)?

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Scalable Frequent Itemset Mining Methods

- Apriori: Candidate Generate&Test Approach
- Improving the Efficiency of Apriori 
- FPGrowth: A Frequent Pattern-Growth Approach
- ECLAT: Frequent Pattern Mining with Vertical Data Format

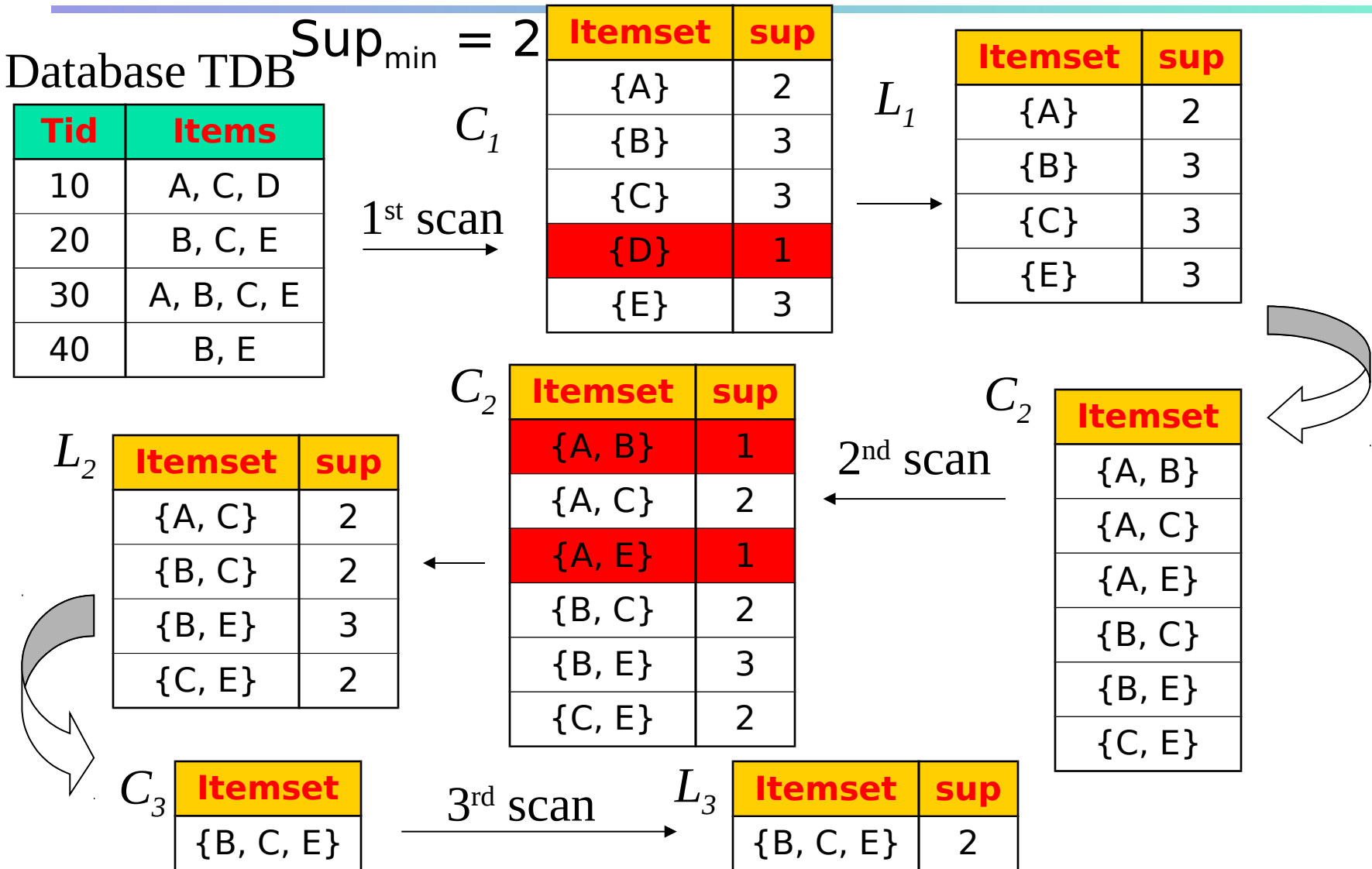
The Downward Closure Property and Scalable Mining Methods

- The **downward closure** property of frequent patterns
 - Any subset of a frequent itemset is frequent
 - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
 - i.e., every transaction having {beer, diaper, nuts} also contains {beer, diaper}
- Scalable mining methods: Three major approaches
 - Apriori (Agrawal & Srikant@VLDB'94)
 - Freq. pattern growth (FPgrowth—Han, Pei & Yin @SIGMOD'00)
 - Vertical data format approach (Charm—Zaki & Hsiao @SDM'02)

Apriori: A Candidate Generate & Test Approach

- Apriori pruning principle: If there is **any** itemset which is infrequent, its superset should not be generated/tested! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Method:
 - Initially, scan DB once to get frequent 1-itemset
 - **Generate** length $(k+1)$ **candidate** itemsets from length k **frequent** itemsets
 - **Test** the candidates against DB
 - Terminate when no frequent or candidate set can be generated

The Apriori Algorithm—An Example



The Apriori Algorithm (Pseudo-Code)

C_k : Candidate itemset of size k

L_k : frequent itemset of size k

$L_1 = \{\text{frequent items}\};$

for ($k = 1; L_k \neq \emptyset; k++$) **do begin**

$C_{k+1} =$ candidates generated from L_k ;

for each transaction t in database do

increment the count of all candidates in C_{k+1}
that are contained in t

$L_{k+1} =$ candidates in C_{k+1} with enough support

end

return $\cup_k L_k$;

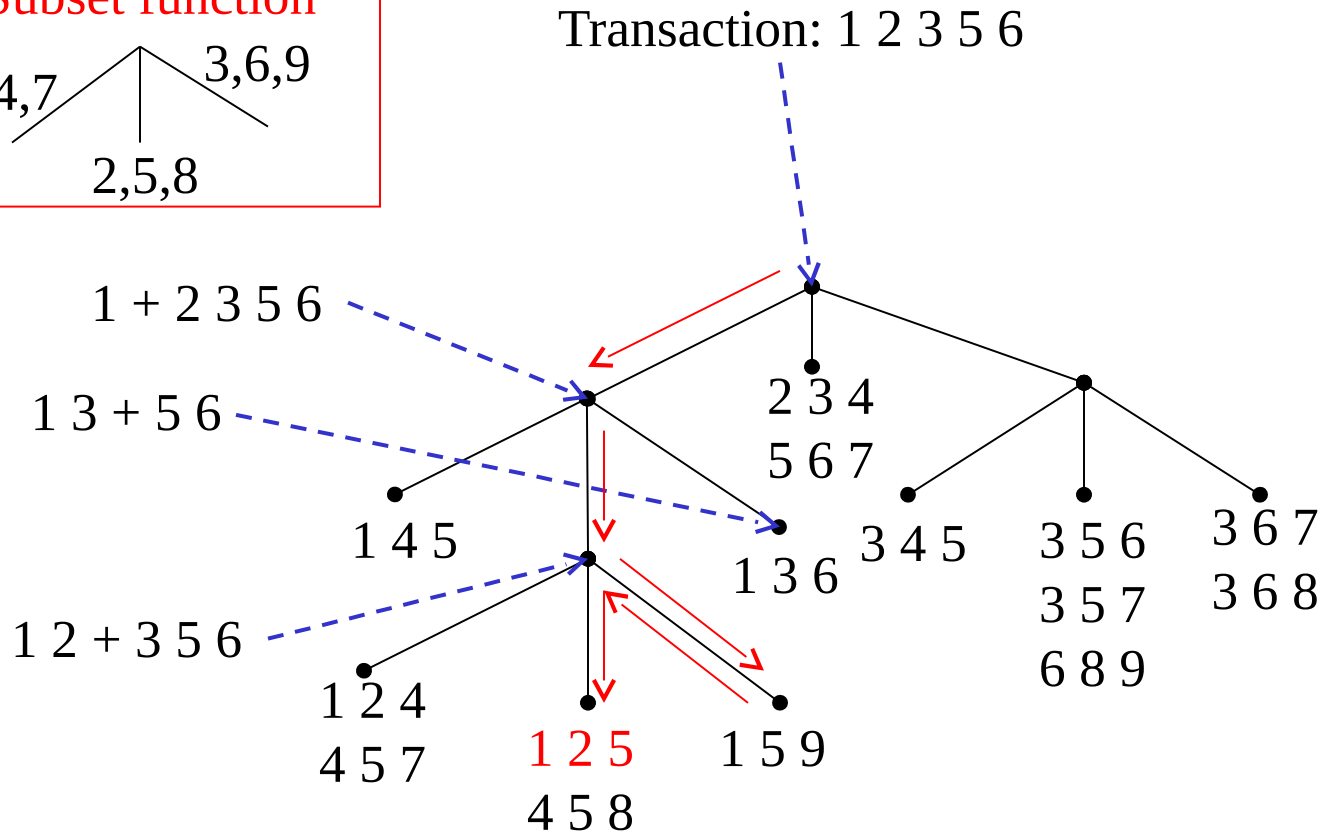
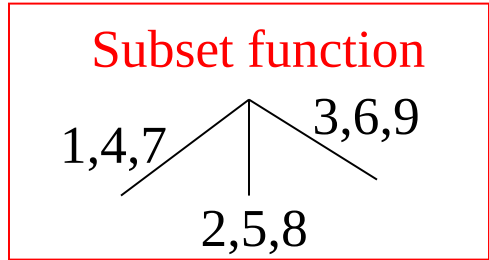
Implementation of Apriori

- How to generate candidates?
 - Step 1: self-joining L_k
 - Step 2: pruning
- Example of Candidate-generation
 - $L_3 = \{abc, abd, acd, ace, bcd\}$
 - Self-joining: $L_3 * L_3$
 - $abcd$ from abc and abd
 - $acde$ from acd and ace
 - Pruning:
 - $acde$ is removed because ade is not in L_3
 - $C_4 = \{abcd\}$

How to Count Supports of Candidates?

- Why counting supports of candidates is a problem?
 - The total number of candidates can be huge
 - Each transaction may contain many candidates
- Method:
 - Candidate itemsets are stored in a *hash-tree*
 - *Leaf nodes* of hash-tree contain a list of itemsets and counts
 - *Interior nodes* contain a hash table
 - *Subset function*: finds all the candidates contained in a transaction

Counting Supports of Candidates Using Hash Tree



Build: store only frequent candidates and their count; do it incrementally while building L_k

Query for a candidate: visit the tree;

Query for an itemset: perform a visit for each sub-itemset;

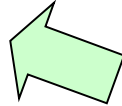
Generating Association Rules from frequent itemsets

- When all frequent itemsets are found, generate *strong* association rules:
 - Pick each frequent itemset F , generate all its nonempty subsets
 - For each such subset S , test the rule
 - $S \rightarrow (F \setminus S)$
 - $\text{support}(S \rightarrow (F \setminus S))$ is above the threshold (as F is frequent by construction)
 - $\text{confidence}(S \rightarrow (F \setminus S)) = P((F \setminus S) | S) = \text{count}(F) / \text{count}(S)$
 - $\text{count}(F)$ and $\text{count}(S)$ are known, and so checking is quick

Candidate Generation: An SQL Implementation

- SQL Implementation of candidate generation
 - Suppose the items in L_{k-1} are listed in an order
 - Step 1: self-joining L_{k-1}
insert into C_k
select **$p.item_1, p.item_2, \dots, p.item_{k-1}, q.item_{k-1}$**
from L_{k-1} **$p, L_{k-1} q$**
where **$p.item_1=q.item_1, \dots, p.item_{k-2}=q.item_{k-2}, p.item_{k-1} < q.item_{k-1}$**
 - Step 2: pruning
forall **$itemsets c$ in C_k** do
 forall **$(k-1)$ -subsets s of c** do
 if (s is not in L_{k-1}) **then delete** c **from** C_k
- Use object-relational extensions like UDFs, BLOBs, and Table functions for efficient implementation [See: S. Sarawagi, S. Thomas, and R. Agrawal. Integrating association rule mining with relational database systems: Alternatives and implications. SIGMOD'98]

Scalable Frequent Itemset Mining Methods

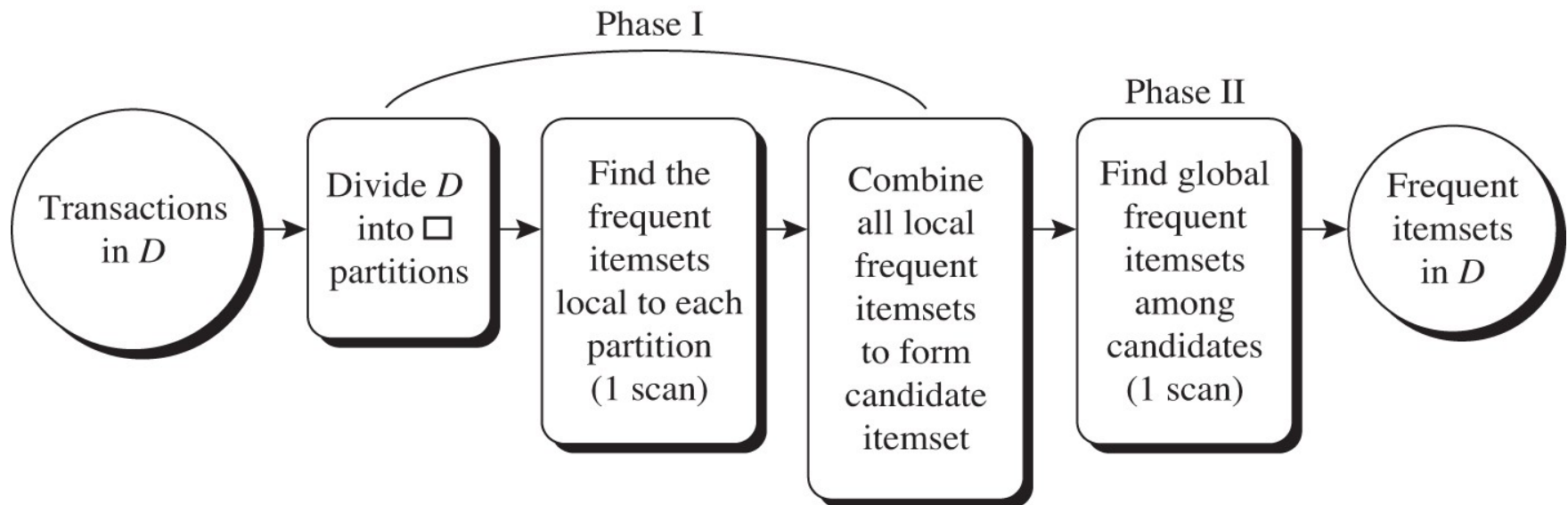
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- Mining Close Frequent Patterns and Maxpatterns

Further Improvement of the Apriori Method

- Major computational challenges
 - Multiple scans of transaction database
 - Huge number of candidates
 - Tedious workload of support counting for candidates
- Improving Apriori: general ideas
 - Reduce passes of transaction database scans
 - Shrink number of candidates
 - Facilitate support counting of candidates

Partition: Scan Database Only Twice

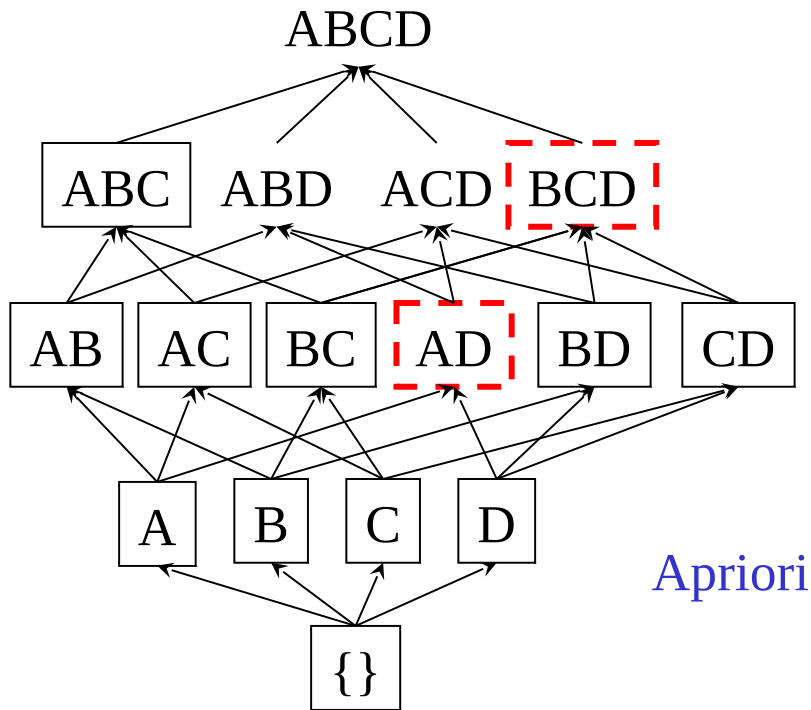
- Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
 - Scan 1: partition database and find local frequent patterns
 - Scan 2: consolidate global frequent patterns
- A. Savasere, E. Omiecinski and S. Navathe '95



Sampling for Frequent Patterns

- Select a sample of original database, mine frequent patterns within sample using Apriori
- Scan database once to verify frequent itemsets found in sample, only *borders* of closure of frequent patterns are checked
 - Example: check *abcd* instead of *ab, ac, ..., etc.*
- Scan database again to find missed frequent patterns
- H. Toivonen. *Sampling large databases for association rules*. In *VLDB'96*

Dynamic Itemset Counting: Reduce Number of Scans



Itemset lattice

S. Brin R. Motwani, J. Ullman,
and S. Tsur. **DIC** Dynamic itemset
counting and implication
rules for market basket data.
SIGMOD'97

- Once both A and D are determined frequent, the counting of AD begins
- Once all length-2 subsets of BCD are determined frequent, the counting of BCD begins

