A Methodology for Surface Reconstruction Based on Hierarchical Models

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Abstract

In this paper we present the formalization of a methodology for producing three-dimensional models of real objects from sampled surface data.

We provide a high abstraction view of the process, depicting a common strategy to solve a class of problems related to the surface reconstruction and show the effectiveness of the use of hierarchical paradigm in this context.

1. Introduction

Production of digital copies of real objects is composed of three main stages: data acquisition, model reconstruction, and model optimization. Each stage requires the solution of several interdependent problems.

In this paper the formalization of a methodology for producing three-dimensional models of real objects from sampled surface data is presented.

A critical review of the knowledge available in the literature is useful since it provides the basis for categorizing the main problems and the techniques used to tackle them, but it does not provide a general schema which encompasses a class of problems and the corresponding solving techniques. Hence, we do not aim to present any implementative suggestion, but a higher abstraction view, disregarding the implementative details related to a specific application and depicting a common strategy to solve a class of problems. Besides, the use of a methodology allows for evaluating functionalities and explicitating relationships among the processing stages rather than assessing the performance of a specific technique.

An overview of the whole digitalization procedure will be presented in section 2, while section 3 will be focused on the methodology for the model reconstruction. Section 4 will be devote to illustrate how the use of hierarchical paradigms can positively affects the methodology.

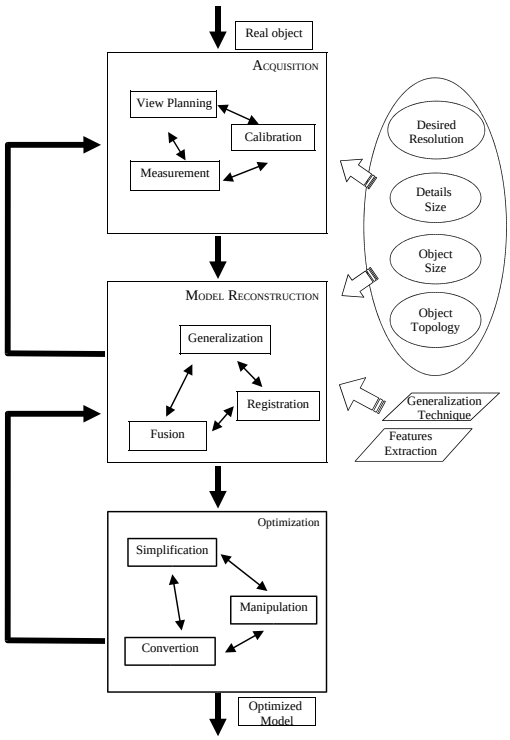

Figure 1. High level schema of the digitalization methodology.

2. Digitalization of real objects

The digitalization process and its various activities is depicted in fig. 1. The core of the digitalization of a real object is the transformation of an ensemble of measurement of the real object into a 3D model (i.e., the reconstruction). However, the digitalization process contains also two stages: the acquisition and the optimization. We will focus on the reconstruction phase, but a brief description of the other stages will be provided in the following sections.

0-7803-8108-4/03/$17.00

2.1. Acquisition

Acquisition provides a collection of information regarding the surface of a real object. It is composed of three activities: planning, calibration, and measurement. In *planning* the sensors of the measurement instrument are positioned so that the region of interest can be observed. The *calibration* finds out the transformation which make the expected values to be measured corresponding to the quantities captured by the sensors. The *measurement* activity uses the calibration information to transform the data collected by the sensors into the data that will be used in the reconstruction phase.

This stage provides a set of measurement of the surface (and, possibly, of external references) and the calibration information to the *reconstruction*. On the other hand, it receives from the reconstruction indications on the regions where new measurements are needed to achieve the desired quality of the reconstruction and an approximated model, which can be used to improve the measurement accuracy.

2.2. Reconstruction

The *reconstruction* stage processes the acquisition data to provide a 3D digital model of the measured object. The reconstruction stage is composed of three activities: generalization, registration, and fusion.

Possible variables of this stage are: *a priori* knowledge on the object characteristics, the membership of the object to a given class of objects, the reconstruction paradigm to be used and its related configuration algorithm, resolution and accuracy of the reconstruction.

The acquisition stage provides a collection of (partial) *views* of the objects from which the reconstruction is performed, usually by an iterative schema. Data obtained from the N views can be formalized as $S^0 = \{S_1^0, \ldots, S_N^0\}$, where the superscript index indicates the number of iterations of the reconstruction process — initially 0.

Two reconstruction strategies can be devised:

- for each set, S_i^0, the data are used to produce a model M_i^0 by generalization. The models of the *views* are parted in groups of adjacent *views* and transformed in order to refer them to the same group reference system (*registration*). Then, the geometry of the models in the overlapping regions is modified, in order to obtain a unique 3D model for each group (*fusion*). At the end of the process described above, there are N^1 models, $M^1 = \{M_1^1, \ldots, M_{N^1}^1\}$, where N^1 is the number of groups obtained during the registration step. The procedure of registration and fusion can be iterated until no more adjacent model can be found (in particular, when only one 3D model is obtained). These operations are schematized in fig. 2a.

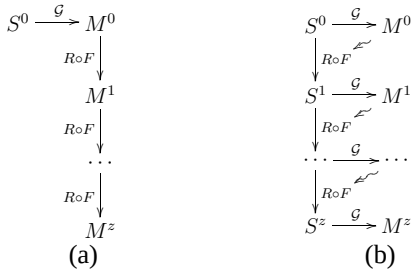

Figure 2. The two reconstruction strategies.

- For each set, S_i^0, the data are used to produce a model M_i^0 by generalization. As in the previous case, the models of the *views* are parted in groups of adjacent *views* and for each model a suitable operation of *registration* and *fusion* is devised. Differently to the previous case, these transformations are applied to the data, producing the collection of sets $S^1 = \{S_1^1, \ldots, S_{N^1}^1\}$. These sets can be used to obtain another collection of 3D models, $M^1 = \{M_1^1, \ldots, M_{N^1}^1\}$. This schema can be applied until no adjacent models can be found. This strategy is schematized in fig. 2b.

The two approaches differ mainly for the role of the generalization. In the first case, the generalization ability of the chosen approximation technique is used only on the data provided by the acquisition: the *views* data are no longer used. The generalization has to extract all the details that it can directly from the data. On the contrary, in the second case, the generalization is instrumental to the registration and fusion transformations search: the data changes at each iteration.

Then, the second strategy is computationally more expensive, but is more scrupulous: the modes can be reconstructed from bigger (possibly redundant) dataset with less influence of the border effects. Besides, the first strategy can be seen as a particular case of the second one. These approaches can be further generalized backpropagating to the acquisition stage the information provided by the integration (registration + fusion). However, the opportunity of using this information to diminishing the uncertainty on the data depends on the devices and the techniques used.

For these reasons in the following we will consider only the second strategy.

2.3. Optimization

Optimization processes the reconstructed model to obtain a model suitable for a specific target application. Generally, the reconstructed model is converted in another representational paradigm and is optimized.

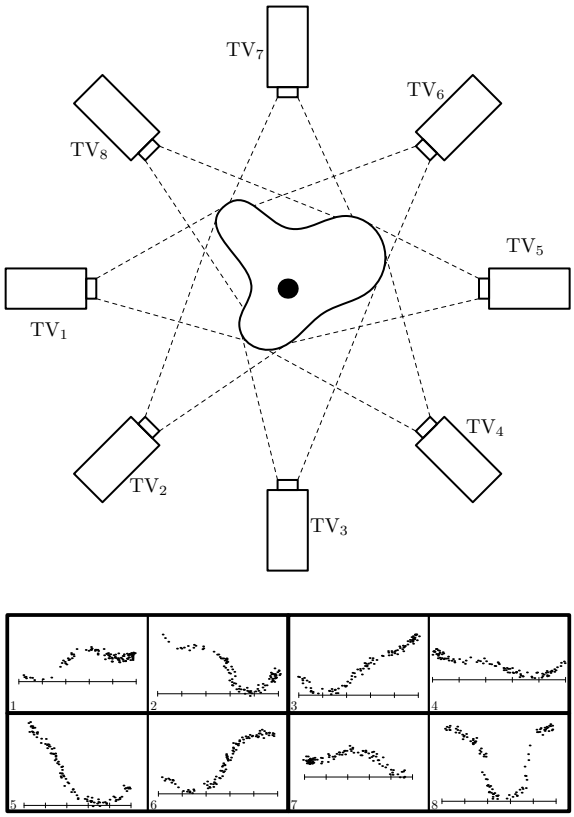

Figure 3. Multi-view acquisition.

This stage receives from the *reconstruction* a 3D model of the real object and may require a more detailed model (however, in real cases, this feedback is rarely needed, as acquiring devices can provide a higher resolution than the visualization devices can usually support).

3. The methodology for reconstruction

The reconstruction stage starts from a collection of N^0 data sets $S^0 = \{S_1^0, \ldots, S_{N^0}^0\}$ and produces a sequence of collections $\{S^j \mid j = 1, \ldots, z\}$, where N^j is the number of elements of S^j $N^j > N^{j+1} >= 1$, (possibly, $N^z = 1$). At each iteration step, each data set is represented in an independent reference system. At the j-th iteration, the $M^j = \{M_1^j, \ldots, M_{N^j}^j\}$ (partial) surface are reconstructed, where their reference systems are the same of the corresponding data sets. Then M^j is partitioned in N^{j+1} groups, where each group is composed by surfaces that correspond to partially overlapping region of the real object.

In order to make the formalization as light as possible, in the following the superscript indices will be dropped out, when explicitating the number of iterations will be superfluous.

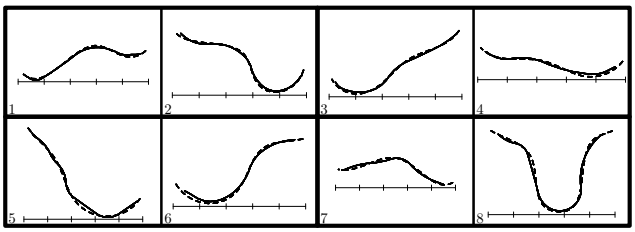

Figure 4. The data sets acquired in fig. 3 are generalized (the real profile is dashed).

The partitioning can be described by the collection of the set of indices of the models which belong to the same cluster:

$$\left\{C_k^j \,|\, C_k^j \subseteq \{1, \ldots, N^j\},\, k = 1, \ldots, N^{j+1}\right\}$$

such that $\bigcup_k C_k = \{1, \ldots, N\}$ and $C_k \cap C_h = \emptyset$, $\forall k, h\ k \neq h$.

Every processing applied to the models M_i in order to find the registration and fusion transformations (also called integration) is independently performed on each cluster. For each model which belong to the k-th cluster, $M_{C_k} = \{M_h \,|\, h \in C_k\}$, a transformation R_h, $h \in C_k$, is found, so that the surfaces which belong to the set $\{R_h(M_h) \,|\, h \in C_k\}$ are represented in the same reference system. Then, a suitable fusion function F_k should be devised: its role is to improve the correspondence of the overlapping regions of the registered surfaces $\{R_h(M_h)\}$. By applying these transformations to the data sets used to produce the models, a unique data set (consistent with the real object geometry) can be obtained. Hence, at the end of the j-th integration step, the data set S_k^{j+i} will be obtained as integration of the data sets $S_{C_k} = \{S_h^j \,|\, h \in C_k^j\}$ as $S_k^{j+i} = F\left(\{R_h(S_h)\}_{h \in C_k}\right)$.

In the following sections, a more elaborated description of the activities here delineated is given.

3.1. Generalization

Let $\mathcal{M}$ be the paradigm used to represent the surface. In the context of this work, an instance of this paradigm, $M \in \mathcal{M}$, can be characterized univocally by the set $\{\theta, K, P\}$, where θ is a vector of parameters which represent the geometrical and photometrical characteristics of the M surface, K is a description of the M topology, and P is the parameterization of the surface. Th generalization procedure of the data set S can be formalized as a function $\mathcal{G} : \mathcal{S} \rightarrow \mathcal{M}$, which has to choose, given a paradigm $\mathcal{M}$, the most suitable element M_i to represent the surface from which S has been sampled. Given a suitable function $\mathcal{D}$, which measures the

distance between a data set and a surface, the optimal model is:

$$\mathcal{G}(S) = \arg \min_{M_i \in \mathcal{M}} = \mathcal{D}(M_i,\, S)$$

Theoretically, the search should cover all the set of characteristics of the model, $\{\theta_i,\, K_i,\, P_i\}$. However, in real application, this does not happen, as the topology and the parameterization are given (usually, at the begin, they are homeomorphic to the plane and in the following iterations they are constructed). The search is usually performed to θ_i. Moreover, in practice, the photometrical characteristics are reconstructed after the geometry is recovered, but at the level of abstraction of this context, it can be considered a unique process of adaptation to the data. The parameterization, P_i, can be (*a posteriori*) adapted onto the topology and the geometry, as the target application requires. Besides, the search is usually constrained by the formalization of some *a priori* known characteristics (e.g., accuracy, size of a model description). Hence, a more realistic formalization of the generalization procedure is a function $\mathcal{G} : \mathcal{S} \times \mathcal{K} \times \mathcal{P} \times \mathcal{V} \rightarrow \mathcal{M}$, where $\mathcal{K}$ and $\mathcal{P}$ are the set of the topologies and the parameterizations of the elements of $\mathcal{M}$, and $\mathcal{V}$ is the class of the set of constraints which can be imposed on the geometrical and photometrical characteristics vector, θ_i. Parameterization and topology are usually correlated, hence the function $\mathcal{G}$ can be not surjective.

3.2. Registration

The partitioning of the models $\{M_i\}$ into cluster is a critical step, as all the integration processes are applied only on the models which belong to the same cluster. If calibration information is available, it can be exploited for the partitioning, as it may provide the position of the sensors with respect to some common reference system. Otherwise, the geometrical (and photometrical) properties of the models can be exploited by a feature extraction and matching procedures. This process is similar to those adopted in the registration transformation search, and, hence, the information gathered can be reused later.

The registration transformations are generally affine transformations (rototranslations, in particular) to be applied to the models which belong to the same cluster, such that the resulting models are represented in the same reference system and the overlapping region of the models are themselves consistent. In real applications, the data from which the models are produced are affected by noise and (non linear) distortions that are different from *view* to *view*. Hence, the perfect alignment cannot be found and some approximation must be considered: given a suitable distance function, the registration transformations will be the ones that minimize the distance between the models in the overlapping regions. This optimization cannot usually

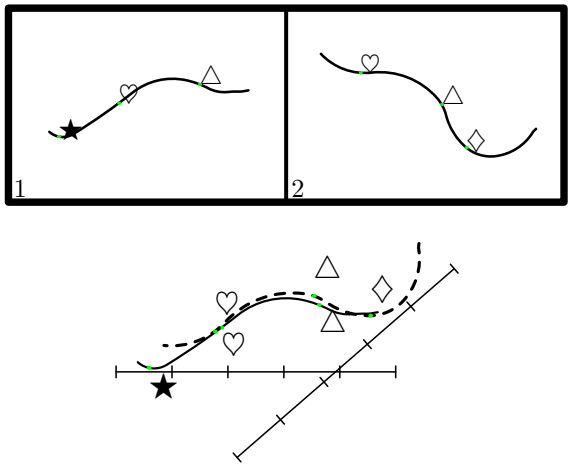

Figure 5. Features extraction and alignment.

be achieved directly by using the surfaces because the required computational cost would be unaffordable and this approach is sensitive to local minima. For these reasons, the registration is usually achieved in two steps: *features* extraction followed by features alignment. The goal of the first operation is the characterization of the common regions of the models belonging to the same cluster. For instance, the *features* can be points or lines lying on the surface. It is important that their estimation would be robust, as the reliability of the registration depends on them. The *features* can be formalized as the result of the following procedure:

$$f_i^j = \mathcal{E}(M_i^j).$$

and can be defined as properties of the object in a given region, and the terms f_i's identify their representation in M_i.

The aligning transformation is computed as the affine transformation that best matches the *features* of different models. Hence, for each cluster C_k, the alignment can be formalized as:

$$\arg \min_{\{R_{h_i}\}_{h_i \in C_k}} Dist_f \left(\{R_{h_i}(f_{h_i})\}_{h_i \in C_k} \right),$$

where $\{R_{h_i}\}_{h_i \in C_k}$ identifies the ensemble of the registration transformations for the models which belong to the k-th cluster, and the function $Dist_f$ measures the difference between the features after the transformation have been applied.

If there is no external reference system to relate with, one of the registration transformation can be chosen equal to the identity and the other one can be computed using the reference system of the first model of the cluster as the reference system of the cluster.

Usually, the minimization of $Dist_f$ cannot be obtained by analytical solution, as the number of possible features

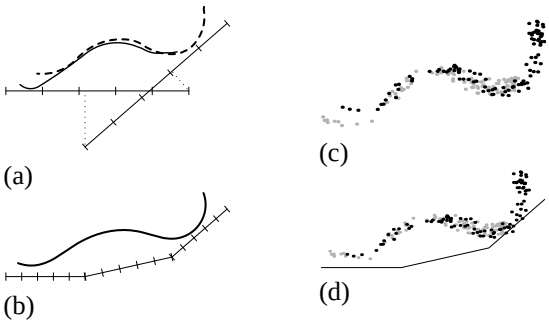

Figure 6. Integration (a)-(b). Back-propagation of the integration transformation to the data sets (c)-(d).

matched makes the exhaustive search of the solution space unfeasable. An ICP algorithm variant is generally used here [1],[2].

3.3. Fusion

A simplicistic definition of the fusion operator, F, could be the union of the data set related to each cluster:

$$F(R_{C_k}(S_{C_k})) = \bigcup_{i \in C_k} R_{C_k}(S_i)$$

This choice can lead to unreliable dataset, as the noise and the distortion of the acquired data set can vary even in different regions of the same *view*. Hence, the measurement error is not uniform. This problem can be overcome if the generalization procedure can process data which include a reliability attribute.

A more general solution, however, is to include in the fusion process a procedure to homogenize the data that belong to common regions, using the more reliable data to correct the ones with higher uncertainty. This can be done by exploiting the surface description of the models.

The fusion can be applied to the models first ($F_h(R_h(M_h)), \forall h \in C_k$), and then backpropagated to the data ($F_h(R_h(S_h)), \forall h \in C_k$), obtaining the fused data set $S^j_{C^j_k} = S^{j+1}_k$.

Besides, the fusion process applied to the cluster C^j_k induce a new topological description of the integrated data set, K^{j+1}_k and a new parameterization function, P^{j+1}_k which may be exploited in the generalization procedure to produce $M^{j+1}_k = \mathcal{G}(S^j_{C^j_k})$. Although there are techniques to devise the topology and the parameterization from the geometrical data (e.g., [3]), the knowledge gained during the fusion processing can be exploited, improving the efficiency of the methodology.

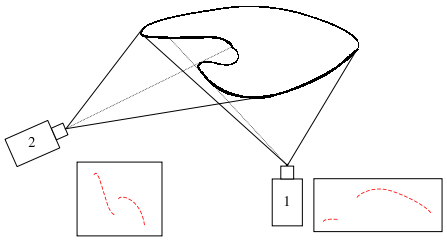

Figure 7. Regions can be hidden by the visible portion of the object.

3.4. Problems to be considered

The above illustred methodology does not consider explicitly some problems that can occur in real applications, which, however, can be tackled by the processes of the methodology. If the shape of the object to be acquired is non convex, some regions hide other parts of the object (fig. 7), resulting in:

- ambiguity of the sampled data,
- missing information in certain regions of the object.

Data are *ambiguous* when they lead to conflicting reconstructions. In the methodology here presented, the ambiguity can be catched during the registration, when same regions match, but inner regions do not. This can happen because the generalization procedure is based on the hypothesis of spatial continuity of the data. If occlusions occur during the acquisition, this hypothesis is no longer verified.

Missing data happens because sensors do not reach some regions of the object. That can be a symptom of a bad *view-planning*, but can be caused by inaccessibility of the surface (for instance, the inner surface of an amphora). In this case, the methodology behavior is driven by the topology devising strategy and by the ability of the generalization procedure to address the absence of data in some region (fig. 8).

To tackle this situation, it is necessary to introduce an ancillary *segmentation* activity to subdivide the data of a single view in two or more views in which the spatial contiguity hypothesis holds. To perform this activity, auxiliary information gathered by the acquisition can be exploited (e.g., slope information [4] [5]).

Thus, if in the registration activity ambiguities are found, a *roll-back* procedure have to be applied, in order to change the views data sets (fig. 9).

4. Using a hierarchical paradigm

Hierarchical paradigms describe a model surface at different level of detail (LOD) and they can be used effectively

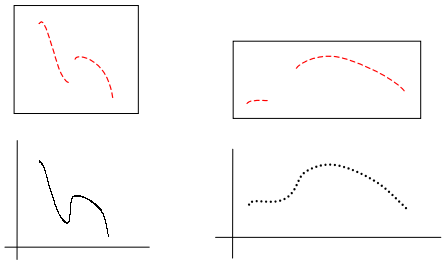

Figure 8. Generalization tackles missing data.

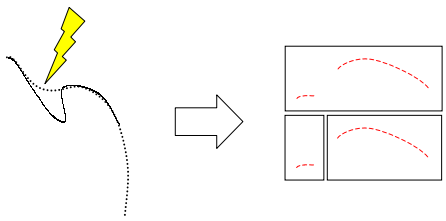

Figure 9. Segmentation.

to reduce the computational cost of the above described procedure and to make it more robust.

The hierarchical paradigms are characterized by the properties of multiresolution and spatial locality. The multiresolution allows to describe the properties of the surface at different level of detail, which is equivalent to locality in the frequency domain. The spatial locality allows to limit the use of computational resources only where the information is effectively located.

These characteristics can be exploited in several procedures of the methodology.

The most natural use is during generalization: spatial locality allows a cheap model configuration as it can use only a local subset of the data, while multiresolution allows to tune the level of detail required by the processing. In general, the use of a hierarchical structure allows to save computational resources (e.g., access to the data). Their ability to decorrelate the information at different level of detail provides spatial-frequency characterization of the information which can be exploited to extract the *features*. Moreover, the LOD description allows to perform the search for the integration transformation using a strategy based on successive approximations. Since a low resolution description includes only the main characteristics of a surface, it is possible to explore very large regions of the solution space in a robust way — although with low accuracy. This can lead to a little pool of approximated candidates solutions which can be explored more deeply by exploiting the detail information of the models. This procedure can be iterated several times using the different level of detail descriptions of the models. Besides, the use of details only in a refinement step diminish the risks of be trapped in local minima.

Besides, the spatial locality may be exploited during the fusion: only the common regions (and, hence, the subset of parameters that describe that regions) have to be modified.

Lastly, multiresolution representation can be exploited to refine the topology, since, usually, the representation of the model at a given resolution can be used to approximate the topology of the representation at higher level of details.

5. Conclusions

In this paper a general methodology for the digitization of real object is presented. It is focused mainly on the reconstruction stage, but its relationship with the acquisition and the optimization stages is described. Moreover, the effectiveness of the use of hierarchical paradigms in this context is shown.

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