Once-Marking and Always-Marking 1-Limited Automata

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UNIVERSITÀ DEGLI STUDI DI MILANO

Loughborough University

Introduction to Limited Automata

Limited Automata [Hibbard '67, *scan limited automata*]

One-tape Turing machines with restricted rewritings

Definition

Fixed an integer $d \geq 1$, a *d-limited automaton* is

- a one-tape Turing machine
- which is allowed to replace the content of each tape cell *only in the first d visits*

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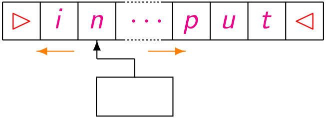

Technical details:

- Input surrounded by two end-markers
- End-markers are never changed
- The head cannot exceed the end-markers

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Computational power

- For each $d \geq 2$, d-limited automata characterize context-free languages [Hibbard '67]

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Computational power

- For each $d \geq 2$, d-limited automata characterize context-free languages [Hibbard '67]
- 1-limited automata characterize regular languages [Wagner&Wechsung '86]

Simulation of 1-Limited Automata by Finite Automata

Derived from the simulation of 2NFAs by 1DFAs [Shepherdson '59]:

- First visit to a cell: direct simulation
- Further visits: *transition tables*

- Finite control of the simulating automaton: 2^{n^2+n} states
  - *transition table* τ_x — 2^{n^2} possible tables
  - *set* of possible current states — 2^n possible sets

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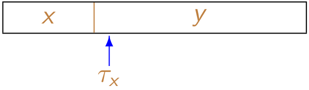

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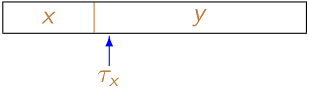

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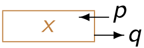
$(p, q) \in \tau_x$ iff 

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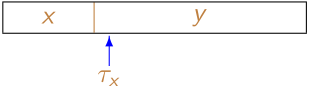

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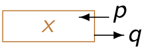
$(p, q) \in \tau_x$ iff 

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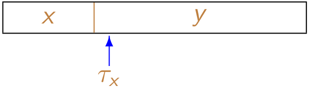

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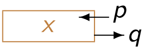
$(p, q) \in \tau_x$ iff 

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Simulation of 1-LAs:

τ_x depends on the choices made while reading x

$\Rightarrow$ The resulting automaton is *nondeterministic!*

Size Costs of Simulations

1-LAs versus Finite Automata [P.&Pisoni '14]

- 1-LA*s* → 1NFA*s*
  exponential
- 1-LA*s* → 1DFA*s*
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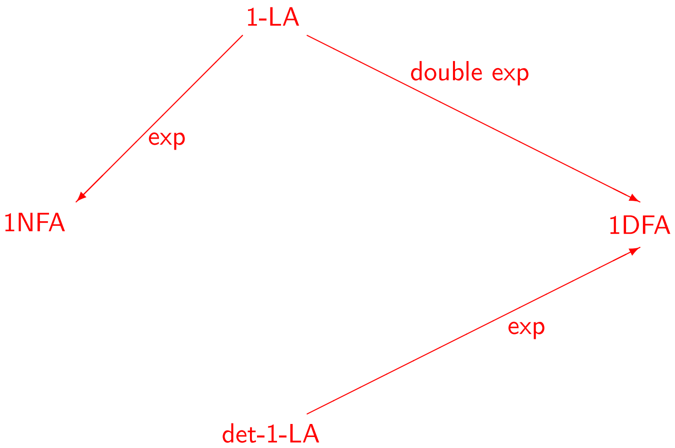

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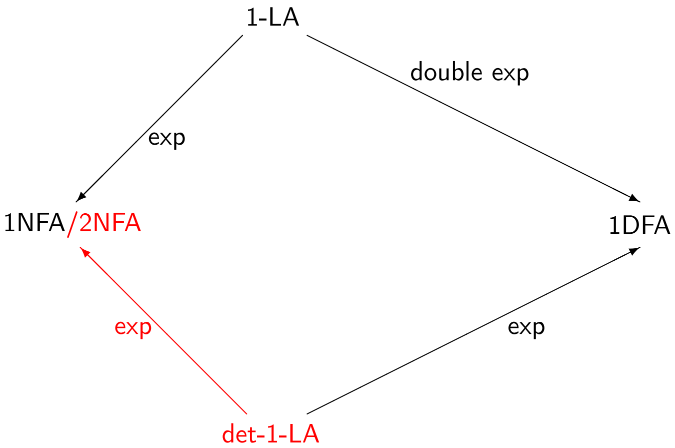

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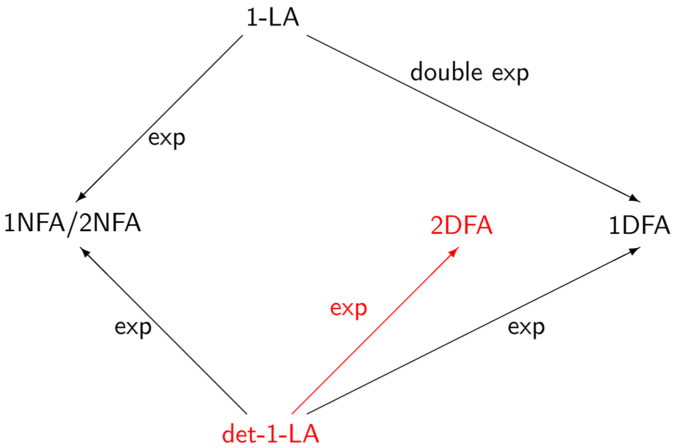

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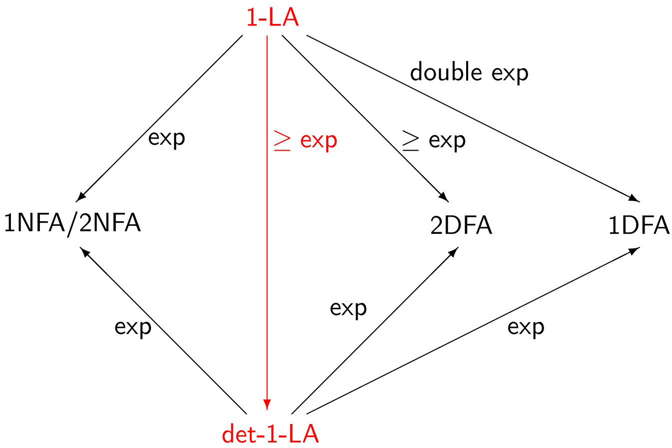

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Once-Marking
1-Limited Automata

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Once-Marking 1-Limited Automata

Always-Marking 1-Limited Automata

Keeping a Double Exponential Gap

Once-Marking 1-Limited Automata

The Language K_n $(n > 0)$

$$K_n = \{x_1\, x_2 \cdots x_k\, x \quad | \quad k > 0,\ x_1, \ldots, x_k, x \in \{\mathtt{a}, \mathtt{b}\}^n, \text{ and } \exists j \in \{1, \ldots, k\} \text{ s.t. } x_j = x\}$$

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Example $(n = 3)$:

a a b a b a b b a a b a b a a b b b b b a

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A Nondeterministic 1-Limited Automaton for K_n

$\triangleright$ a a b a b a b b a a b a b a a b b b b b a $\triangleleft$ $(n = 3)$

1. Scan all the tape from left to right:
   - check if the input length is a multiple of n
   - mark the rightmost cell of one nondeterministically chosen block
2. Compare symbol by symbol the last block and the one ending with the marked cell
3. Accept if the two blocks are equal

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- $O(n)$ states
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$\Rightarrow$ 1-LA of size $O(n)$

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Rewritings: to accept K_n it is enough to mark one tape cell during the first visit!

Recognizing K_n with Finite Automata

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- in each computation there is a unique tape cell whose input symbol σ is replaced with its marked version $\mathring{\sigma}$
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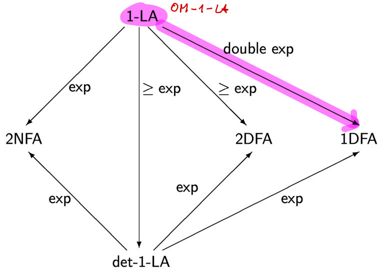

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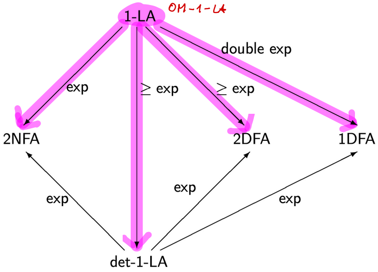
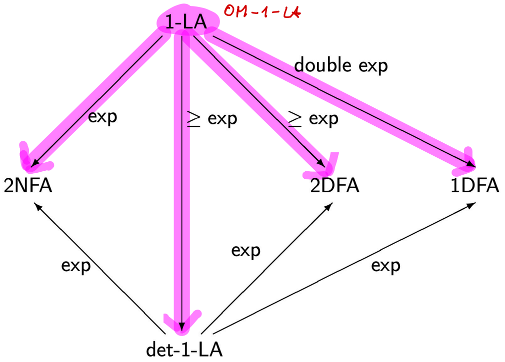


Conversions from Once-Marking 1-Limited Automata

Costs from *nondeterministic* OM-1-LAs:



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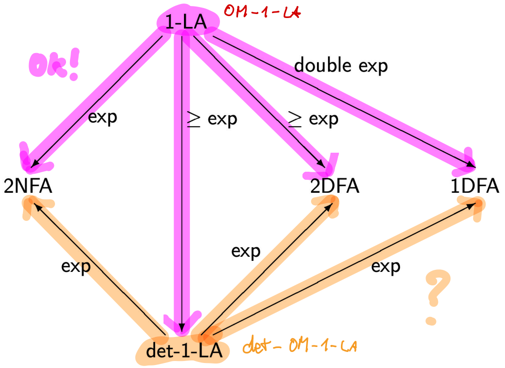

Costs from *deterministic* OM-1-LAs?

Conversions of *deterministic* OM-1-LAs

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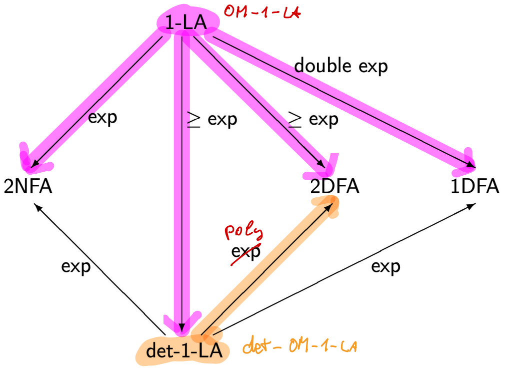

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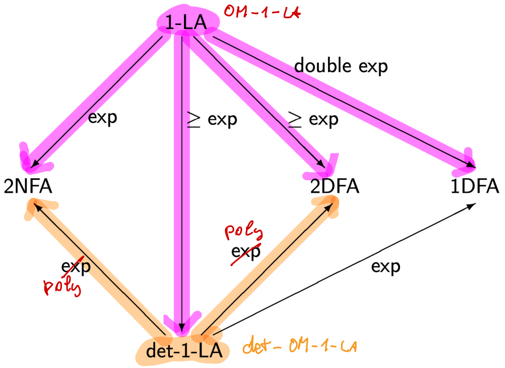

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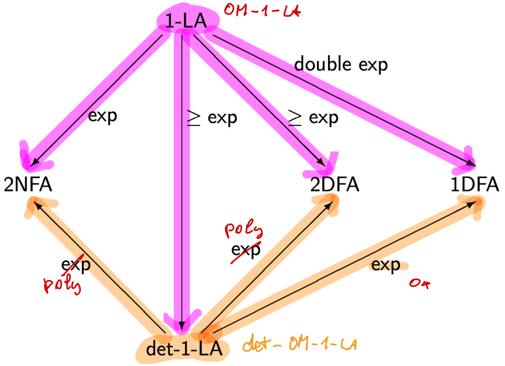

Reducing the Gap to a Single Exponential

Always-Marking 1-Limited Automata

Reducing the Gap: First Attempts

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Two "naif" restrictions of 1-LAs:

1. *Deterministic choice* for rewritings
   Nondeterministic choice for next state and head movement
2. *Nondeterministic choice* for rewritings
   Deterministic choice for next state and head movement

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Two "naif" restrictions of 1-LAs:

1. *Deterministic choice* for rewritings
   Nondeterministic choice for next state and head movement
2. *Nondeterministic choice* for rewritings
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Reducing the Gap: First Attempts

Double role of nondeterminism in 1-LA*s*

On a tape cell:

First visit: To replace the content
by a nondeterministically chosen symbol γ

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the set of available transitions depends on γ!

Two "naif" restrictions of 1-LAs:

1. *Deterministic choice* for rewritings
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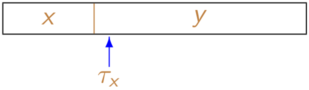
For both restrictions, in the worst case the size gap to 1DFAs remains double exponential!

Simulation of 1-Limited Automata by Finite Automata

- First visit to a cell: direct simulation
- Further visits: *transition tables*



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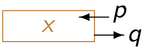
$(p, q) \in \tau_x$ iff 

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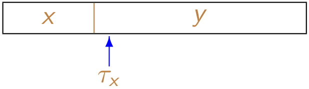

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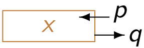
$(p, q) \in \tau_x$ iff 

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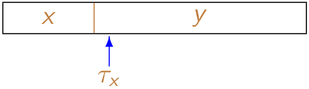

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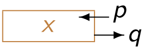
$(p, q) \in \tau_x$ iff 

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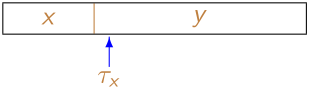

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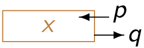
$(p, q) \in \tau_x$ iff 

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$\Rightarrow$ Always-Marking 1-Limited Automata

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A 1-LA is said to be *always marking* if

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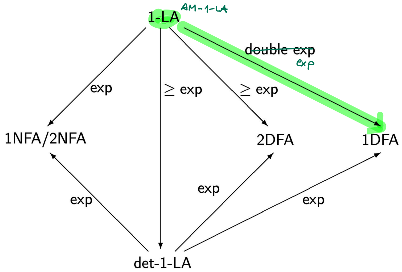

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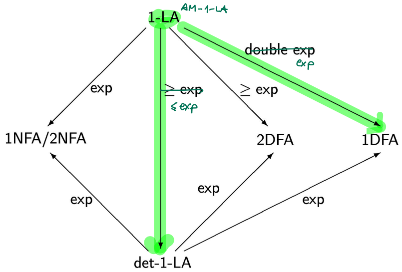

The Language J_n $(n > 0)$

$$J_n = \{x\, x_1\, x_2 \cdots x_k \quad | \quad k > 0,\ x_1, \ldots, x_k, x \in \{\mathtt{a}, \mathtt{b}\}^n, \text{ and } \exists j \in \{1, \ldots, k\} \text{ s.t. } x_j = x\}$$

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Then:

- $J_n = (K_n)^R$
- Each 2NFA accepting J_n has a number of states at least exponential in n
  - 2NFAs $\to$ 1DFAs costs exponential
  - K_n needs at least 2^{2^n} states to be accepted by 1DFAs
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Recognizing J_n with AM-1-LAs

▷ a b a a b b a b b a b a b a a b b b b b a ◁ $(n = 3)$

- Visit and mark the first n tape cells

  Then inspect the following blocks as follows:

- When the head reaches a cell for the first time:
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  - If *the symbols in the two cells do not match*

    then skip the remaining symbols of the current block

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    otherwise *accepts* if the length of the remaining part of the input is a multiple of n

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Implementation with $O(n)$ states

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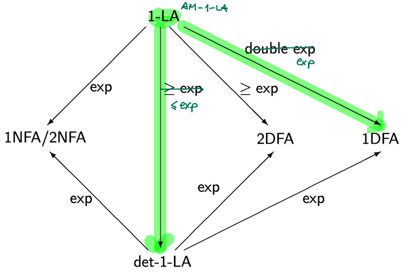
Implementation with $O(n)$ states

Only deterministic transitions!

Simulations of AM-1-LAs

Summing up:

- det-1-LAs → 2NFAs costs exponential
- J_n is accepted by a det-AM-1-LA with $O(n)$ states
- Each 2NFA accepting J_n has a number of states at least exponential in n



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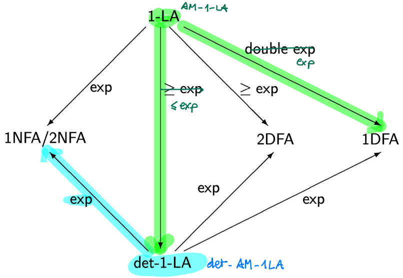
det-AM-1-LAs → 2NFAs

Exponential cost!



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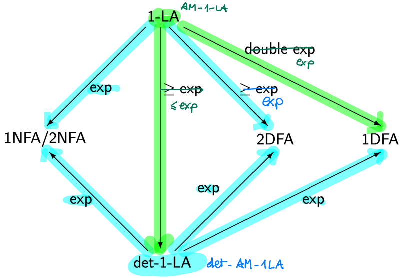
Then:

det-AM-1-LAs $\to$ 2NFAs

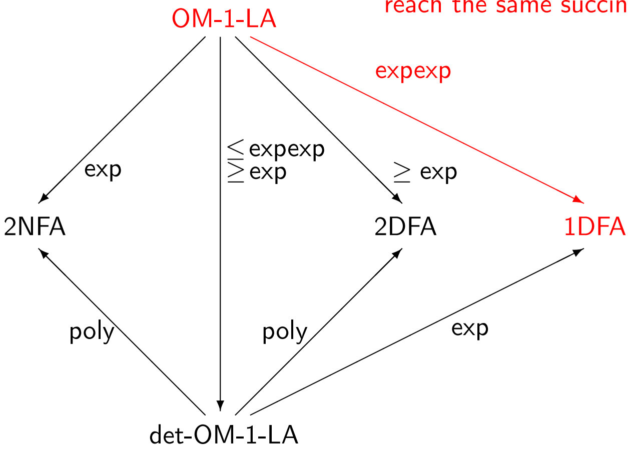
Exponential cost!

$\Rightarrow$ Even the costs of the remaining simulations are exponential



Conclusion: Once-Marking 1-Limited Automata

“One drop” of nondeterminism allows to reach the same succinctness as 1-LAs (language K_n)



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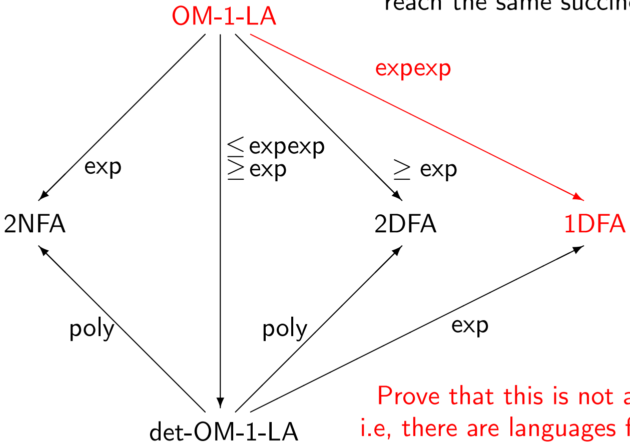

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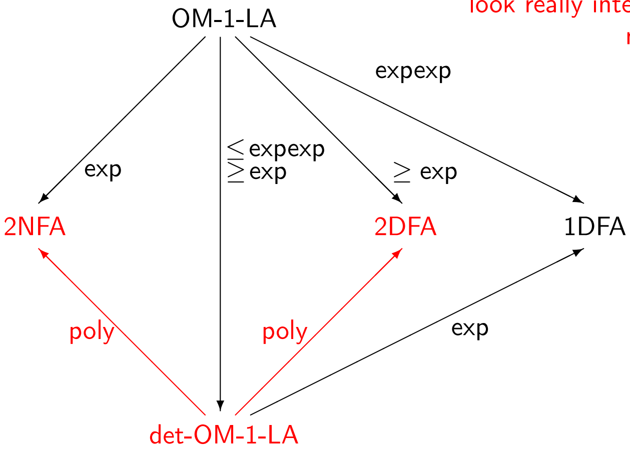
Problem

Prove that this is not always the case, i.e, there are languages for which 1-LAs are more succinct than OM-1-LAs (e.g., variants of K_n)

Conclusion: Once-Marking 1-Limited Automata

The capability of once marking does not look really interesting without nondeterminism



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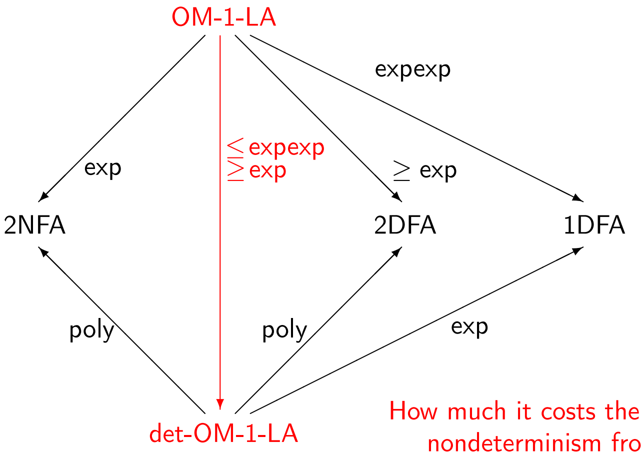
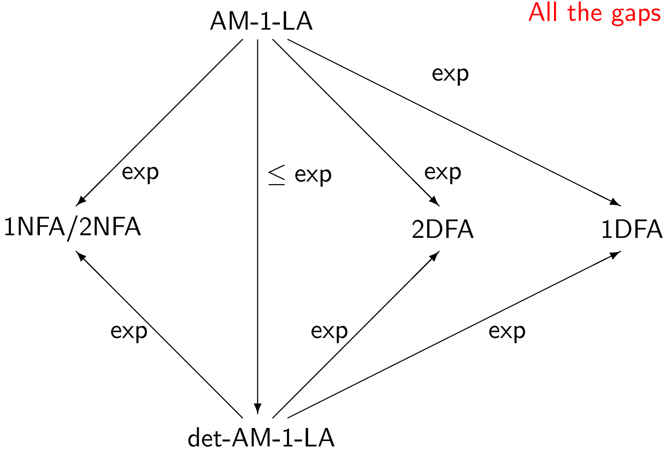


Problem

How much it costs the elimination of nondeterminism from OM-1-LAs?

Conclusion: Always-Marking 1-Limited Automata

All the gaps are exponential



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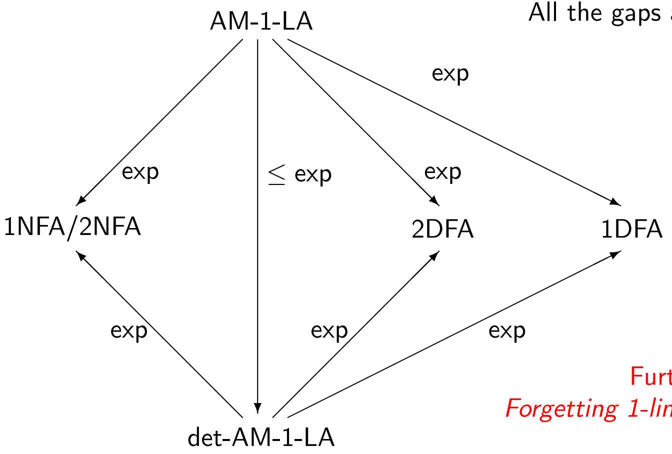

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Further restriction:
Forgetting 1-limited automata
(NCMA 2023)

Further possible investigation lines

- Unary case
- Connections with the Sakoda and Sisper question (costs of 2NFA$\rightarrow$ 2DFA and 1NFA$\rightarrow$ 2DFA)
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Thank you for your attention!