Limited Automata and Descriptional Complexity

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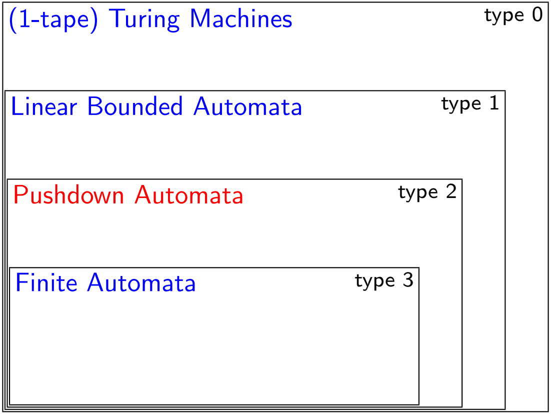
The Chomsky Hierarchy



Limited Automata [Hibbard'67]

One-tape Turing machines with restricted rewritings

Definition

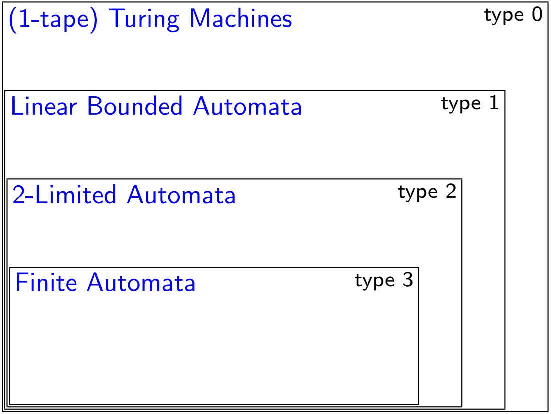
Fixed an integer $d \geq 1$, a *d-limited automaton* is

- a one-tape Turing machine
- which is allowed to rewrite the content of each tape cell *only in the first d visits*

Computational power

- For each $d \geq 2$, d-limited automata characterize context-free languages [Hibbard'67]
- 1-limited automata characterize regular languages [Wagner&Wechsung'86]

The Chomsky Hierarchy



Example: Balanced Parentheses

$\triangleright$	(	)	(	(	(	)	)	)	$\triangleleft$

(i) Move to the right to search a closed parenthesis

(ii) Rewrite it by $\#$

(iii) Move to the left to search an open parenthesis

(iv) Rewrite it by $\#$

(v) Repeat from the beginning

Special cases:

(i') If in (i) the right end of the tape is reached then scan all the tape and *accept* iff all tape cells contain $\#$

(iii') If in (iii) the left end of the tape is reached then *reject*

Each cell is rewritten only in the first 2 visits!

2-Limited Automata $\longrightarrow$ Pushdown Automata

Problem

How much it costs, in the description size, the simulation of 2-LAs *by* PDAs*?*

Our result

Exponential cost!
(optimal)

Deterministic case

- Determinism is preserved by the simulation *provided that the input of the* PDA *is right end-marked*
- Without end-marker: *double exponential* size for the simulation of D2-LAs by DPDA
- Conjecture: this cost cannot be reduced

CFLs $\longrightarrow$ 2-Limited Automata

New trasformation based on:

Theorem ([Chomsky&Schützenberger'63])

Every context-free language $L \subseteq \Sigma^*$ can be expressed as

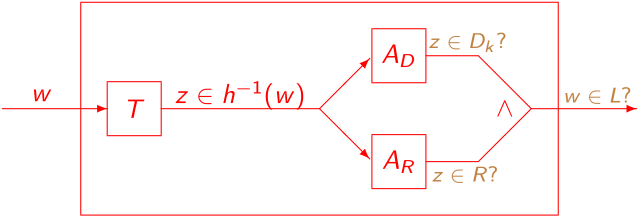
$$L = h(D_k \cap R)$$

where, for $\Omega_k = \{(_1,)_1, (_2,)_2, \ldots, (_k,)_k\}$:

- $D_k \subseteq \Omega_k^*$ *is a Dyck language*
- $R \subseteq \Omega_k^*$ *is a regular language*
- $h : \Omega_k \to \Sigma^*$ *is an homomorphism*

Furthermore, it is possible to restrict to *non-erasing* homomorphisms [Okhotin'12]

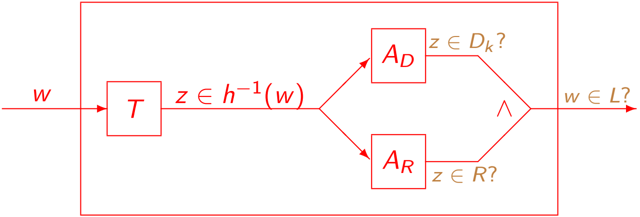
CFLs $\longrightarrow$ 2-Limited Automata



L context-free language, with $L = h(D_k \cap R)$

- T nondeterministic transducer computing h^{-1}
- A_D 2-LA accepting the Dyck language D_k
- A_R finite automaton accepting R

CFLs $\longrightarrow$ 2-Limited Automata



u_1	u_2	$\cdots$	u_k

input of T

$z = \sigma_1\sigma_2\cdots\sigma_k \in h^{-1}(w)$

$h(\sigma_i) = u_i$

$\#\#\#\#\sigma_1$	$\#\#\sigma_2$	$\cdots$	$\#\#\#\sigma_k$

(padded) input of A_D and A_R

Non erasing homomorphism!

Not stored into the tape!

$\boxed{\#\#\#\#\gamma_1}$ $\ldots$

Each σ_i is produced "on the fly" and replaced by γ_i, its first rewriting by A_D

Pushdown Automata $\longrightarrow$ 2-Limited Automata

PDAs $\longrightarrow$ 2-LAs

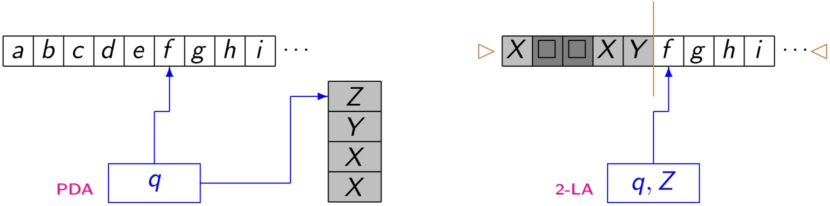
Polynomial cost!

DPDAs $\longrightarrow$ D2-LAs

Polynomial cost!

(in the description size)

Pushdown Automata $\to$ 2-Limited Automata



Normal form for (D)PDAs:

- at each step, the stack height increases at most by 1
- ϵ-moves cannot push on the stack

Each (D)PDA can be simulated by an equivalent (D)2-LA of polynomial size

2-Limited Automata $\equiv$ Pushdown Automata

Summing up...

- Descriptional complexity point of view

2-LAs $\rightarrow$ PDAs

Exponential gap

PDAs $\rightarrow$ 2-LAs

Polynomial upper bound

- Determinism vs Nondeterminism

Deterministic Context-Free Languages $\equiv$ Deterministic 2-LAs

On the other hand:

$$L = \{a^n b^n c \mid n \geq 0\} \cup \{a^n b^{2n} d \mid n \geq 0\} \in \mathit{det}\text{-3-LA} - \text{DCFL}$$

Infinite hierarchy [Hibbard'67]:

$$\mathit{det}\text{-}d\text{-LA} \supset \mathit{det}\text{-}(d-1)\text{-LA}, \text{ for each } d \geq 2$$

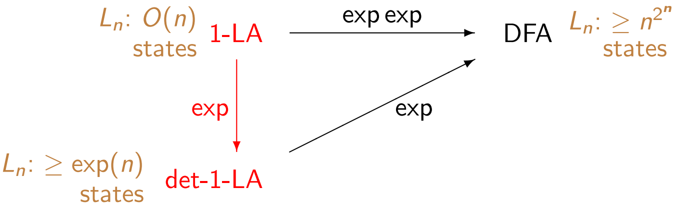
1-Limited Automata $\longrightarrow$ Finite Automata

Costs in states of the optimal simulations of n-state 1-LAs by finite automata:

	DFA	NFA
nondet. 1-LA	$2^{n \cdot 2^{n^2}}$	$n \cdot 2^{n^2}$
det. 1-LA	$n \cdot (n+1)^n$	$n \cdot (n+1)^n$

These upper bounds do not depend on the alphabet size of M!

Nondetermism vs Determinism in 1-LAs



Corollary

Removing nondeterminism from 1-LAs requires exponentially many states

Cfr. Sakoda and Sipser question [Sakoda&Sipser'78]:

How much it costs in states to remove nondeterminism from two-way finite automata?

Futher Investigations

- Descriptional complexity aspects for $d > 2$

  We conjecture that for $d > 2$ the size gap from d-limited automata to PDAs remains exponential

- Descriptional complexity aspects in the unary case
  - Unary context-free language are regular [Ginbsurg&Rice'62]
  - Ex: $L_n = (a^{2^n})^*$

	size
2-LA	$O(n)$
DPDA	$O(n)$
minimal DFA	2^n
minimal 2NFA	2^n