Limited Automata and Regular Languages

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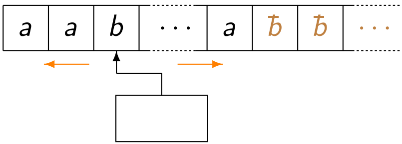
One-Tape Turing Machine



Very simple but powerful model!

Recursive enumerable languages

What about restricted versions?

- No rewritings: *two-way finite automata*
  Regular languages
- Linear space:
  Context-sensitive languages [Kuroda'64]
- Linear time:
  Regular languages [Hennie'65]

Limited Automata [Hibbard'67]

One-tape Turing machines with restricted rewritings

Definition

Fixed an integer $d \geq 1$, a *d-limited automaton* is

- a one-tape Turing machine
- which is allowed to rewrite the content of each tape cell *only in the first d visits*

- End-marked tape
- The space is bounded by the input length
  (this restriction can be removed without changing the computational power and the state upper bounds)

Example: Balanced Parentheses

$\triangleright$	(	)	(	(	(	)	)	)	$\triangleleft$

(i) Move to the right to search a closed parenthesis
(ii) Rewrite it by X
(iii) Move to the left to search an open parenthesis
(iv) Rewrite it by X
(v) Repeat from the beginning

Special cases:

(i') If in (i) the right end of the tape is reached then scan all the tape and *accept* iff all tape cells contain X
(iii') If in (iii) the left end of the tape is reached then *reject*

Cells can be rewritten only in the first 2 visits!

d-Limited Automata: Computational Power

$d = 1$: regular languages [Wagner&Wechsung'86]

$d \geq 2$: context-free languages [Hibbard'67]

Our Contributions

$d = 1$: regular languages [Wagner&Wechsung'86]
Descriptional complexity aspects

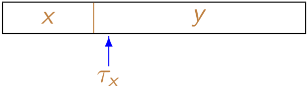
$d \geq 2$: context-free languages [Hibbard'67]
New transformation

context-free languages $\rightarrow$ 2-limited automata

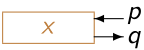
based on the Chomsky-Schützenberger Theorem

Simulation of 1-Limited Automata by Finite Automata

- Main idea:
  transformation of *two-way* NFAs into *one-way* DFAs:
  [Shepherdson'59]
  - First visit to a cell: direct simulation
  - Further visits: *transition tables*



$\tau_x \subseteq Q \times Q$

$(p, q) \in \tau_x$ iff 

  - Finite control of the simulating DFA:
    - transition table of the already scanned input prefix
    - set of possible current states

- Simulation of 1-LAs:
  - The scanned input prefix is rewritten by a *nondeterministically chosen string*
  - The simulating DFA keeps in its finite control a *sets of transition tables*

1-Limited Automata $\longrightarrow$ Finite Automata: Upper Bounds

Theorem

Let M be a 1-LA *with n states.*

- *There exists an equivalent* DFA *with $2^{n \cdot 2^{n^2}}$ states.*
- *There exists an equivalent* NFA *with $n \cdot 2^{n^2}$ states.*

If M is deterministic then there exists an equivalent DFA *with no more than $n \cdot (n+1)^n$ states.*

	DFA	NFA
nondet. 1-LA	$2^{n \cdot 2^{n^2}}$	$n \cdot 2^{n^2}$
det. 1-LA	$n \cdot (n+1)^n$	$n \cdot (n+1)^n$

These upper bounds do not depend on the alphabet size of M!

The gaps are optimal!

Optimality: the Witness Languages

Given $n \geq 1$:

$$\underbrace{a_1 \quad a_2 \quad \cdots \quad a_n}_{x_1} \; \underbrace{a_{n+1}\, a_{n+2} \; \cdots \; a_{2n}}_{x_2} \; \cdots \; \underbrace{a_{\ldots} \quad a_{\ldots} \quad \cdots \quad a_{kn}}_{x_k}$$

At least n of these blocks contain the same factor

$$\begin{aligned} L_n \quad = \{x_1 x_2 \cdots x_k \mid \quad & k \geq 0,\ x_1, x_2, \ldots, x_k \in \{0,1\}^n, \\ & \exists i_1 < i_2 < \cdots < i_n \in \{1, \ldots, k\}, \\ & x_{i_1} = x_{i_2} = \cdots = x_{i_n} \} \end{aligned}$$

Example ($n = 3$): 0 0 1 | 1 1 0 | 0 1 1 | 1 1 0 | 1 1 0 | 1 1 1 | 0 1 1

How to Recognize L_n: 1-Limited Automata

$$0\,0\,1 | \hat{1}\,1\,0 | 0\,1\,1 | \hat{1}\,1\,0 | \hat{1}\,1\,0 | 1\,1\,1 | 0\,1\,1 \qquad (n = 3)$$

- Nondeterministic strategy:
  Guess the leftmost positions of n input blocks containing the same factor and *Verify*

- Implementation:
  1. Mark n tape cells
  2. Count the tape modulo n to check whether or not:
     - the input length is a multiple of n, and
     - the marked cells correspond to the leftmost symbols of some blocks of length n
  3. Compare, symbol by symbol, each two consecutive blocks of length n that start from the marked positions

- $O(n)$ states

How to Recognize L_n: Deterministic Finite Automata

- Idea:
  - For each $x \in \{0,1\}^n$ count how many blocks coincide with x
  - Accept if and only if one of the counters reaches the value n
- State upper bound:
  - Finite control:
    a counter (up to n) for each possible block of length n
  - There are 2^n possible different blocks of length n
  - Number of states double exponential in n
    more precisely $(2^n - 1) \cdot n^{2^n} + n$
- State lower bound:
  - n^{2^n} (standard distinguishability arguments)

The state gap between 1-LAs and DFAs is double exponential!

Nondetermism vs. Determinism in 1-LAs

$$
\begin{tikzcd}
\begin{array}{c} L_n\text{: } O(n) \\ \text{states} \end{array} \quad \text{1-LA} \arrow[rr, "\text{exp exp}"] \arrow[d, "\text{exp}"'] & & \text{DFA} \quad \begin{array}{c} L_n\text{: } \geq n^{2^n} \\ \text{states} \end{array} \\
\begin{array}{c} L_n\text{: } \geq \exp(n) \\ \text{states} \end{array} \quad \text{det-1-LA} \arrow[urr, "\text{exp}"'] & &
\end{tikzcd}
$$

Corollary

Removing nondeterminism from 1-LAs requires exponentially many states.

Cfr. Sakoda and Sipser question [Sakoda&Sipser'78]:

How much it costs in states to remove nondeterminism from two-way finite automata?

More Than One Rewriting

For each $d \geq 2$, d-limited automata characterize CFLs [Hibbard'67]

We present a construction of 2-LAs from CFLs based on:

Theorem ([Chomsky&Schützenberger'63])

Every context-free language $L \subseteq \Sigma^*$ can be expressed as

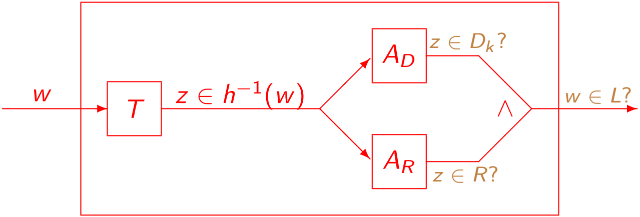
$$L = h(D_k \cap R)$$

where, for $\Omega_k = \{(_1,)_1, (_2,)_2, \ldots, (_k,)_k\}$:

- $D_k \subseteq \Omega_k^*$ *is a Dyck language*
- $R \subseteq \Omega_k^*$ *is a regular language*
- $h : \Omega_k \to \Sigma^*$ *is an homomorphism*

Furthermore, it is possible to restrict to *non-erasing* homomorphisms [Okhotin'12]

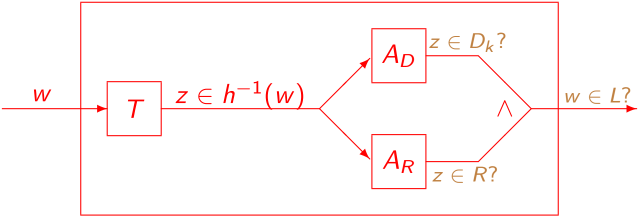
From CFLs to 2-LAs



L context-free language, with $L = h(D_k \cap R)$

- T nondeterministic transducer computing h^{-1}
- A_D 2-LA accepting the Dyck language D_k
- A_R finite automaton accepting R

From CFLs to 2-LAs



$$\underbrace{\begin{array}{|c|c|c|c|}\hline u_1 & u_2 & \cdots & u_k \\ \hline\end{array}}_{\text{input of } T}$$

$z = \sigma_1\sigma_2 \cdots \sigma_k \in h^{-1}(w)$

$h(\sigma_i) = u_i$

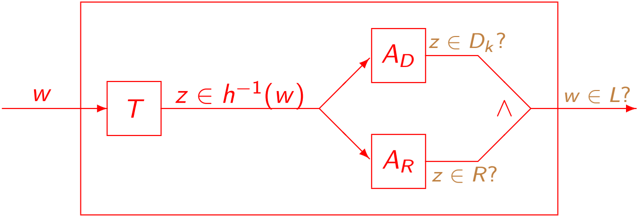
$$\underbrace{\begin{array}{|c|c|c|c|}\hline \#\#\#\#\sigma_1 & \#\#\sigma_2 & \cdots & \#\#\#\sigma_k \\ \hline\end{array}}_{\text{(padded) input of } A_D \text{ and } A_R}$$

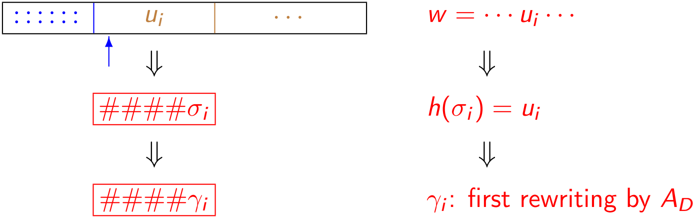
Non erasing homomorphism!

Not stored into the tape!

Each σ_i is produced "on the fly"

From CFLs to 2-LAs





- On the tape, u_i is replaced directly by $\#\#\#\#\gamma_i$
- One move of A_R on input σ_i is also simulated

Final Remarks: 1-Limited Automata

- Nondeterministic 1-LAs can be
  - double exponentially smaller than one-way deterministic automata
  - exponentially smaller than one-way nondeterministic and two-way deterministic/nondeterminstic automata
- Witness languages over a two letter alphabet

What about the unary case?

Theorem

For each prime p, the language $(a^{p^2})^*$ is accepted by a deterministic 1-LAs with $p+1$ states, while it needs p^2 states to be accepted by any 2NFA.

We expect state gaps smaller than in the general case

Final Remarks: d-Limited Automata, $d \geq 2$

- Descriptional complexity aspects
  - Case $d = 2$ [P&Pisoni NCMA2013]
  - Case $d > 2$ under investigation
- Determinism vs. nondeterminism
  - Deterministic 2-LAs characterize deterministic CFLs
    [P&Pisoni NCMA2013]
  - Infinite hierarchy
    For each $d \geq 2$ there is a language which is accepted by a deterministic d-limited automaton and that cannot be accepted by any deterministic $(d-1)$-limited automaton
    [Hibbard'67]

Thank you for your attention!