Limited Automata and Regular Languages

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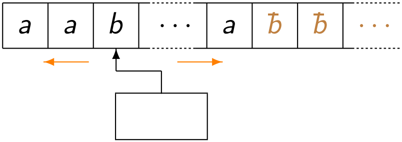
UNIVERSITÀ DEGLI STUDI
DI MILANO

One-Tape Turing Machine



Very simple but powerful model!

Recursive enumerable languages

- No rewritings: *two-way finite automata*
  Regular languages
- Linear space:
  Context-sensitive languages [Kuroda'64]
- Linear time:
  Regular languages [Hennie'65]

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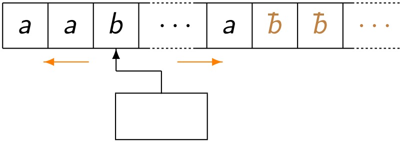

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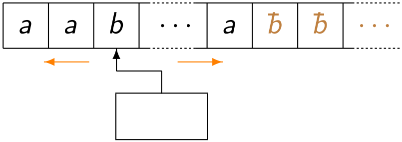

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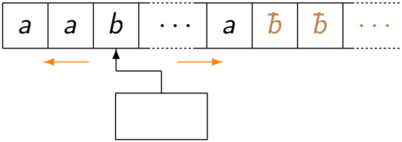

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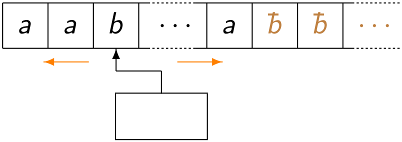

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Limited Automata [Hibbard'67]

One-tape Turing machines with restricted rewritings

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Fixed an integer $d \geq 1$, a *d-limited automaton* is

- a one-tape Turing machine
- which is allowed to rewrite the content of each tape cell *only in the first d visits*

- End-marked tape
- The space is bounded by the input length
  (this restriction can be removed without changing the computational power and the state upper bounds)

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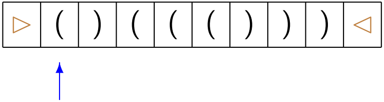

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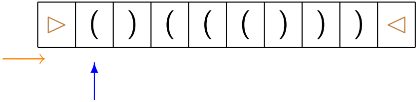

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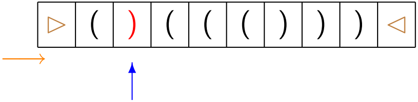

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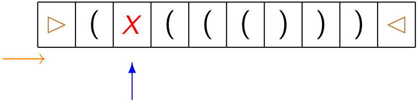

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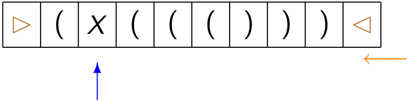

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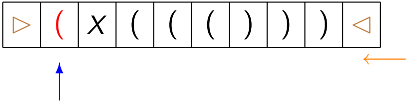

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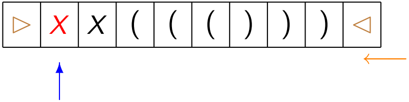

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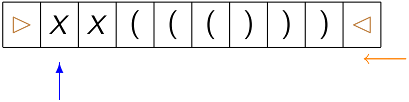

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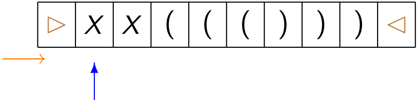

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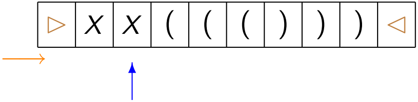

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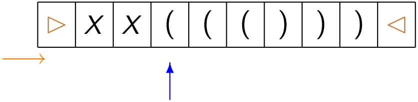

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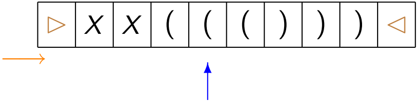

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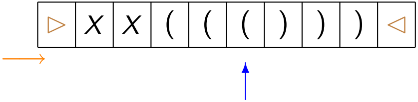

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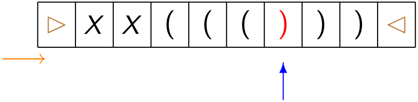

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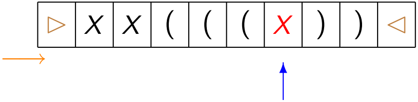

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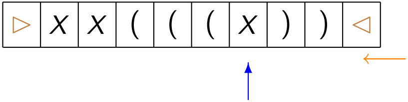

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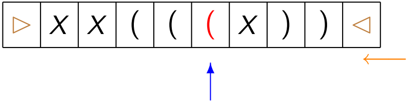

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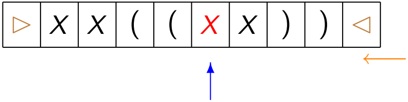

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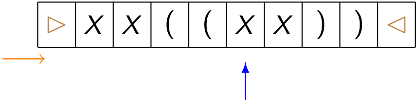

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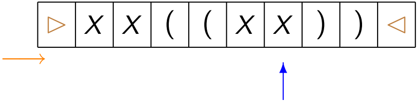

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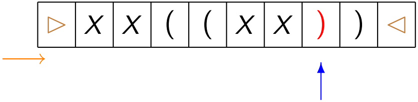

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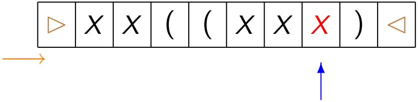

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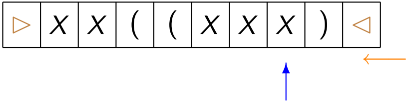

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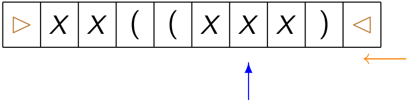

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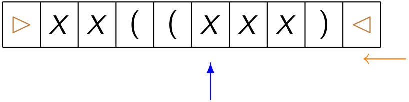

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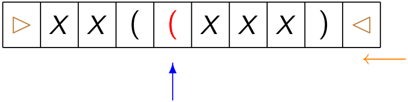

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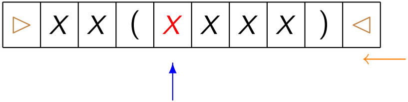

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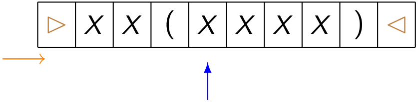

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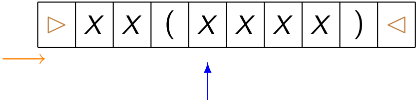

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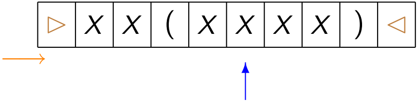

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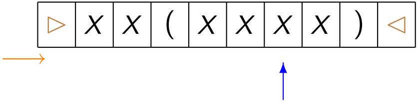

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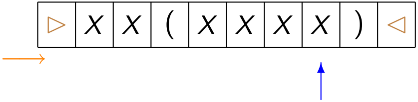

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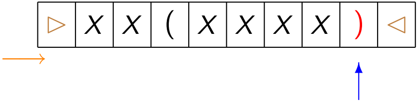

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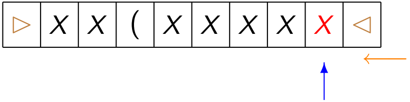

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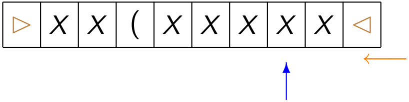

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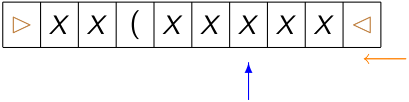

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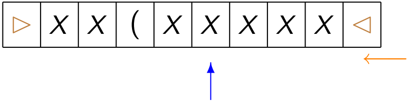

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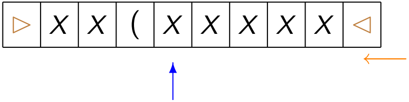

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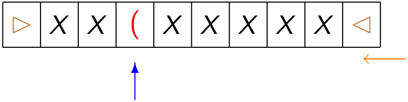

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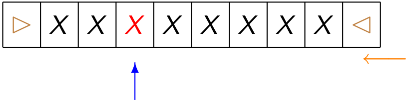

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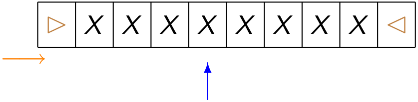

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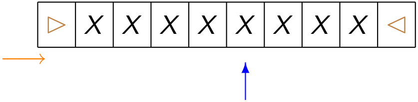

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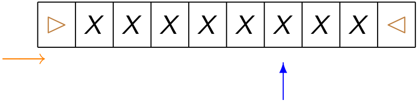

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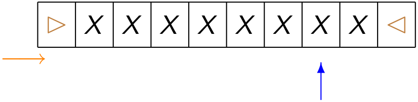

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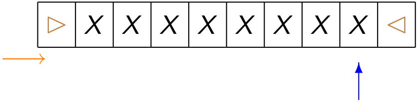

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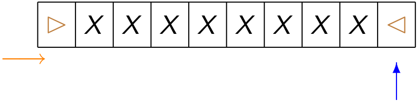

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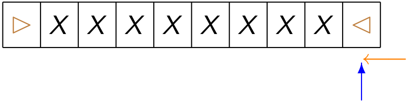

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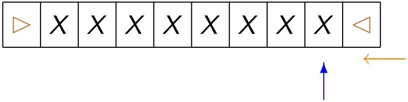

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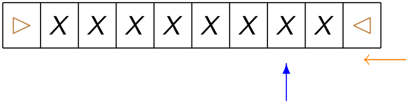

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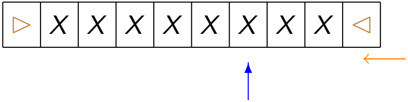

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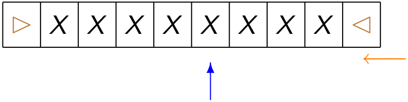

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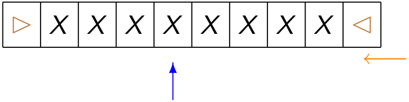

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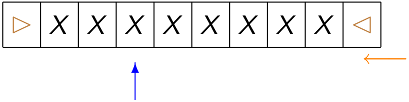

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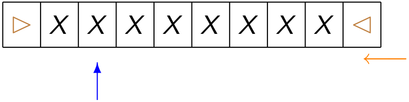

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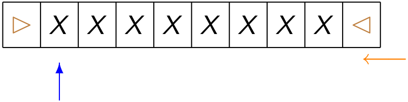

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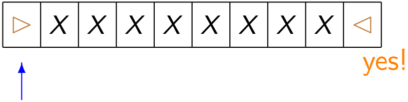

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Cells can be rewritten only in the first 2 visits!

d-Limited Automata: Computational Power

$d = 1$: regular languages [Wagner&Wechsung'86]

$d \geq 2$: context-free languages [Hibbard'67]

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New transformation

context-free languages $\rightarrow$ 2-limited automata

based on the Chomsky-Schützenberger Theorem

Simulation of 1-Limited Automata by Finite Automata

- Main idea:
  transformation of *two-way* NFAs into *one-way* DFAs:
  [Shepherdson'59]
  - First visit to a cell: direct simulation
  - Further visits: *transition tables*
  - Finite control of the simulating DFA:
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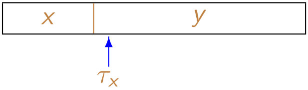

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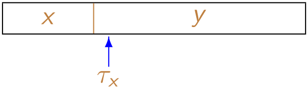

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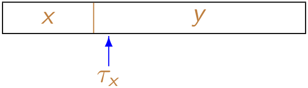

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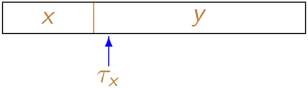

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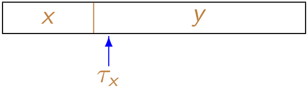

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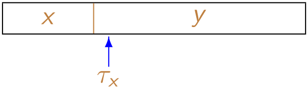

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1-Limited Automata $\longrightarrow$ Finite Automata: Upper Bounds

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Let M be a 1-LA *with n states.*

- *There exists an equivalent* DFA *with $2^{n \cdot 2^{n^2}}$ states.*
- *There exists an equivalent* NFA *with $n \cdot 2^{n^2}$ states.*

If M is deterministic then there exists an equivalent DFA *with no more than $n \cdot (n+1)^n$ states.*

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These upper bounds do not depend on the alphabet size of M!

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Optimality: the Witness Languages

Given $n \geq 1$:

$$a_1 \quad a_2 \quad \cdots \quad a_n \quad a_{n+1}\, a_{n+2} \,\cdots\, a_{2n} \quad \cdots \quad a_{\dots} \quad a_{\dots} \quad \cdots \quad a_{kn}$$

$$L_n \quad =$$

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At least n of these blocks contain the same factor

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$$\begin{aligned} L_n \quad = \{x_1 x_2 \cdots x_k \mid \quad & k \geq 0,\ \ x_1, x_2, \ldots, x_k \in \{0,1\}^n, \\ & \exists i_1 < i_2 < \cdots < i_n \in \{1, \ldots, k\}, \\ & x_{i_1} = x_{i_2} = \cdots = x_{i_n} \} \end{aligned}$$

Example ($n = 3$): 0 0 1 1 1 0 0 1 1 1 1 0 1 1 0 1 1 1 0 1 1

Optimality: the Witness Languages

Given $n \geq 1$:

$$\underbrace{a_1 \quad a_2 \quad \cdots \quad a_n}_{x_1} \; \underbrace{a_{n+1}\, a_{n+2} \quad \cdots \quad a_{2n}}_{x_2} \quad \cdots \quad \underbrace{a_{\ldots} \quad a_{\ldots} \quad \cdots \quad a_{kn}}_{x_k}$$

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How to Recognize L_n: 1-Limited Automata

- Nondeterministic strategy:
  Guess the leftmost positions of n input blocks containing the same factor and *Verify*

- Implementation:
  1. Mark n tape cells
  2. Count the tape modulo n to check whether or not:
     - the input length is a multiple of n, and
     - the marked cells correspond to the leftmost symbols of some blocks of length n
  3. Compare, symbol by symbol, each two consecutive blocks of length n that start from the marked positions

- $O(n)$ states

How to Recognize L_n: 1-Limited Automata

0 0 1 1 1 0 0 1 1 1 1 0 1 1 0 1 1 1 0 1 1 $(n = 3)$

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How to Recognize L_n: 1-Limited Automata

$$0\,0\,1\,\hat{1}\,1\,0\,0\,1\,1\,\hat{1}\,1\,0\,\hat{1}\,1\,0\,1\,1\,1\,0\,1\,1 \qquad (n = 3)$$

$\longrightarrow$

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$$0\ 0\ 1 | \hat{1}\ 1\ 0 | 0\ 1\ 1 | \hat{1}\ 1\ 0 | \hat{1}\ 1\ 0 | 1\ 1\ 1 | 0\ 1\ 1 \qquad (n = 3)$$

$$\longleftarrow$$

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How to Recognize L_n: 1-Limited Automata

$0\,0\,1|\hat{1}\,1\,0|0\,1\,1|\hat{1}\,1\,0|\hat{1}\,1\,0|1\,1\,1|0\,1\,1$ $\qquad (n = 3)$

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How to Recognize L_n: 1-Limited Automata

$$0\,0\,1 | \textcolor{red}{\hat{1}\,1\,0} | 0\,1\,1 | \textcolor{red}{\hat{1}\,1\,0} | \textcolor{red}{\hat{1}\,1\,0} | 1\,1\,1 | 0\,1\,1 \qquad (n = 3)$$

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How to Recognize L_n: Deterministic Finite Automata

- Idea:
  - For each $x \in \{0,1\}^n$ count how many blocks coincide with x
  - Accept if and only if one of the counters reaches the value n
- State upper bound:
  - Finite control:
    a counter (up to n) for each possible block of length n
  - There are 2^n possible different blocks of length n
  - Number of states double exponential in n
    more precisely $(2^n - 1) \cdot n^{2^n} + n$
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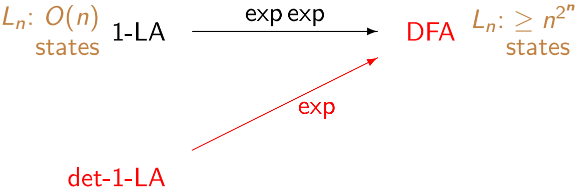
The state gap between 1-LAs and DFAs is double exponential!

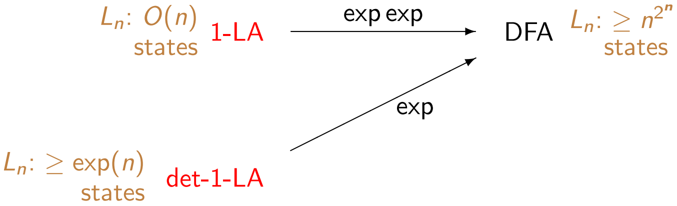
Nondetermism vs. Determinism in 1-LAs

L_n: $O(n)$ states **1-LA** $\xrightarrow{\text{exp exp}}$ **DFA** L_n: $\geq n^{2^n}$ states

Nondetermism vs. Determinism in 1-LAs



Nondetermism vs. Determinism in 1-LAs



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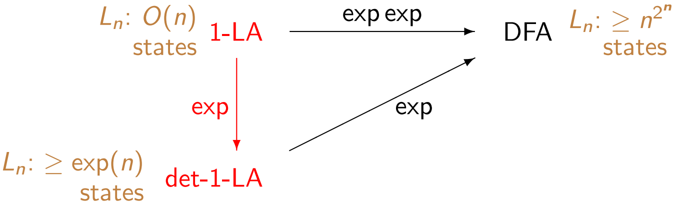
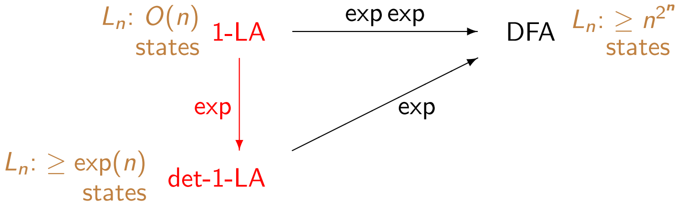


Corollary

Removing nondeterminism from 1-LAs requires exponentially many states.

Nondetermism vs. Determinism in 1-LAs



Corollary

Removing nondeterminism from 1-LAs *requires exponentially many states.*

Cfr. Sakoda and Sipser question [Sakoda&Sipser'78]:

How much it costs in states to remove nondeterminism from two-way finite automata?

More Than One Rewriting

For each $d \geq 2$, d-limited automata characterize CFLs [Hibbard'67]

We present a construction of 2-LAs from CFLs based on:

Theorem ([Chomsky&Schützenberger'63])

Every context-free language $L \subseteq \Sigma^*$ can be expressed as

$$L = h(D_k \cap R)$$

where, for $\Omega_k = \{(_1,)_1, (_2,)_2, \ldots, (_k,)_k\}$:

- $D_k \subseteq \Omega_k^*$ *is a Dyck language*
- $R \subseteq \Omega_k^*$ *is a regular language*
- $h : \Omega_k \to \Sigma^*$ *is an homomorphism*

Furthermore, it is possible to restrict to *non-erasing* homomorphisms [Okhotin'12]

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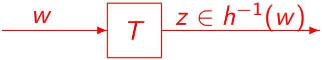
From CFLs to 2-LAs

L context-free language, with $L = h(D_k \cap R)$

- T nondeterministic transducer computing h^{-1}
- A_D 2-LA accepting the Dyck language D_k
- A_R finite automaton accepting R

From CFLs to 2-LAs



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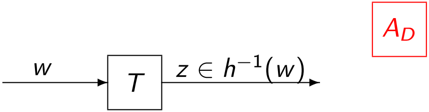

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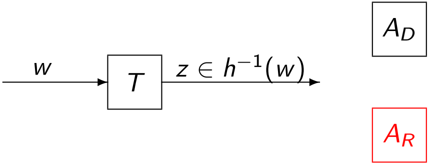

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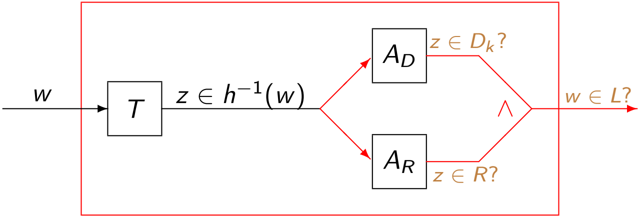

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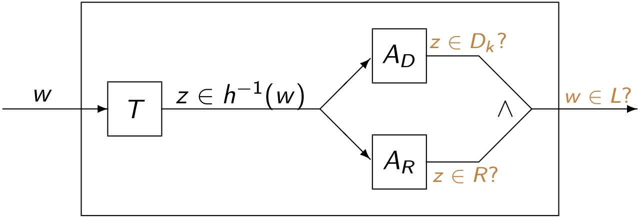
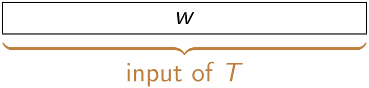
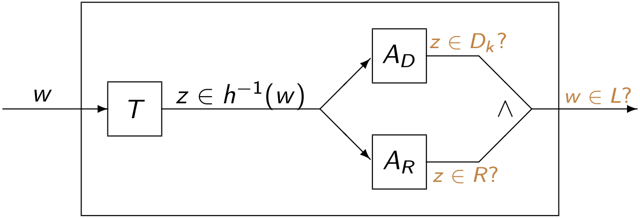




$$z = \sigma_1 \sigma_2 \cdots \sigma_k \in h^{-1}(w)$$

From CFLs to 2-LAs



u_1	u_2	$\cdots$	u_k

input of T

$$z = \sigma_1\sigma_2\cdots\sigma_k \in h^{-1}(w)$$

$$h(\sigma_i) = u_i$$

$\#\#\#\#\sigma_1$	$\#\#\sigma_2$	$\cdots$	$\#\#\#\sigma_k$

Non erasing homomorphism!

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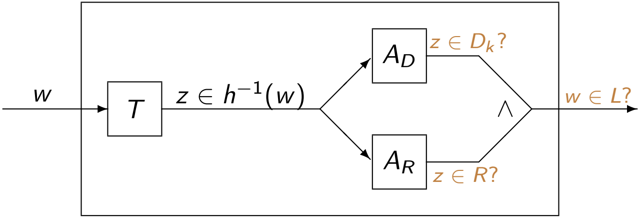
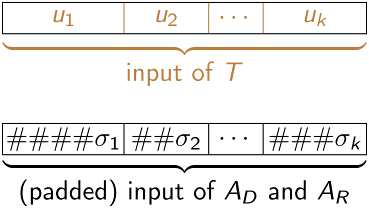


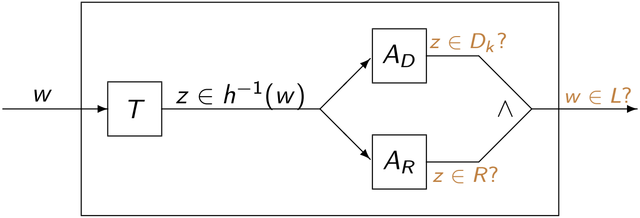


$z = \sigma_1\sigma_2\cdots\sigma_k \in h^{-1}(w)$

$h(\sigma_i) = u_i$

Non erasing homomorphism!

From CFLs to 2-LAs



u_1	u_2	$\cdots$	u_k

input of T

$$z = \sigma_1\sigma_2\cdots\sigma_k \in h^{-1}(w)$$

$$h(\sigma_i) = u_i$$

$\#\#\#\#\sigma_1$	$\#\#\sigma_2$	$\cdots$	$\#\#\#\sigma_k$

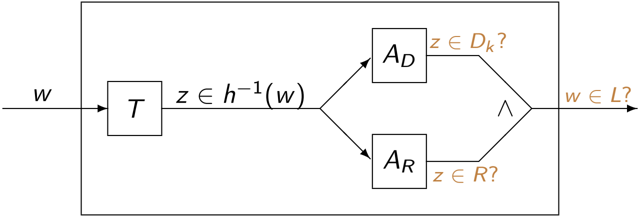
(padded) input of A_D and A_R

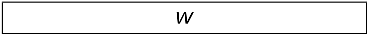
Non erasing homomorphism!

Not stored into the tape!

Each σ_i is produced "on the fly"

From CFLs to 2-LAs





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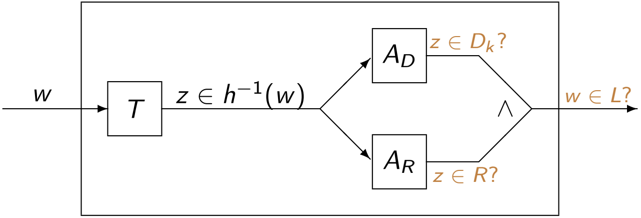
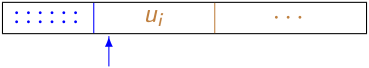
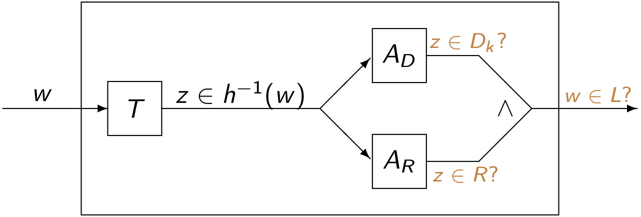
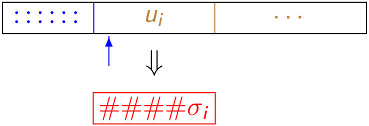




$w = \cdots u_i \cdots$

From CFLs to 2-LAs





$$w = \cdots u_i \cdots$$

$$\Downarrow$$

$$h(\sigma_i) = u_i$$

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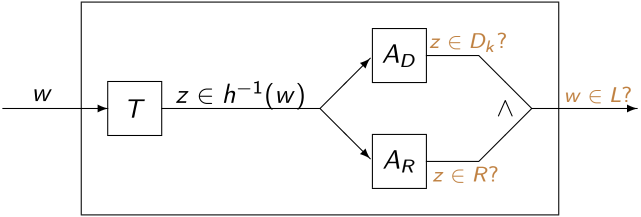

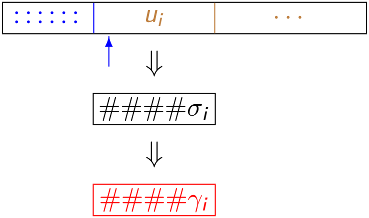

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$$\Downarrow$$

γ_i: first rewriting by A_D

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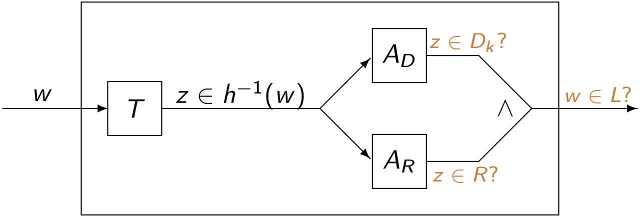

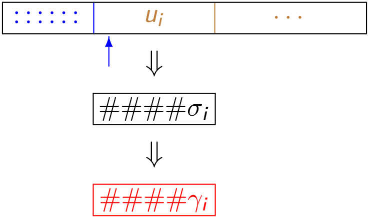

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$\Downarrow$

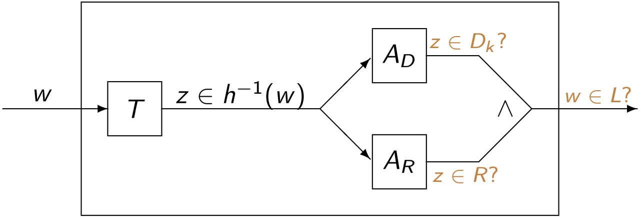
$h(\sigma_i) = u_i$

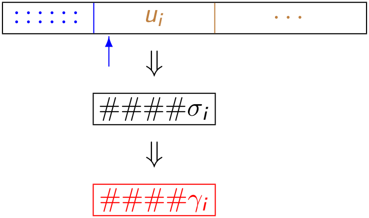
$\Downarrow$

γ_i: first rewriting by A_D

- On the tape, u_i is replaced directly by $\#\#\#\#\gamma_i$
- One move of A_R on input σ_i is also simulated

From CFLs to 2-LAs





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- Nondeterministic 1-LAs can be
  - double exponentially smaller than one-way deterministic automata
  - exponentially smaller than one-way nondeterministic and two-way deterministic/nondeterminstic automata
- Witness languages over a two letter alphabet

What about the unary case?

Theorem

For each prime p, the language $(a^{p^2})^*$ is accepted by a deterministic 1-LAs with $p+1$ states, while it needs p^2 states to be accepted by any 2NFA.

We expect state gaps smaller than in the general case

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- Descriptional complexity aspects
  - Case $d = 2$ [P&Pisoni NCMA2013]
  - Case $d > 2$ under investigation
- Determinism vs. nondeterminism
  - Deterministic 2-LAs characterize deterministic CFLs [P&Pisoni NCMA2013]
  - Infinite hierarchy
    For each $d \geq 2$ there is a language which is accepted by a deterministic d-limited automaton and that cannot be accepted by any deterministic $(d-1)$-limited automaton [Hibbard'67]

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Thank you for your attention!