Determinism versus Nondeterminism in Two-Way Finite Automata

Recent Results around the Sakoda and Sipser Question

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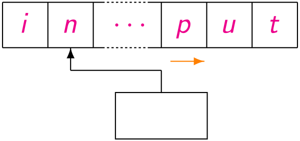
UNIVERSITÀ DEGLI STUDI
DI MILANO

Outline

Finite State Automata

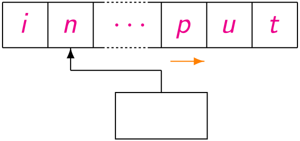


Base version:

one-way deterministic finite automata (1DFA)

- one-way input tape
- deterministic transitions

Finite State Automata



Possibile variants allowing:

- nondeterministic transitions
  - one-way nondeterministic finite automata (1NFA)
- input head moving forth and back
  - two-way deterministic finite automata (2DFA)
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- alternation
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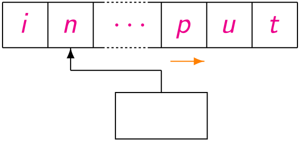

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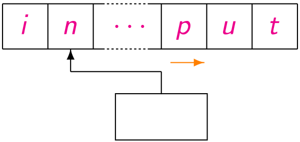

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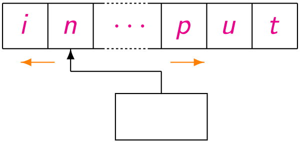

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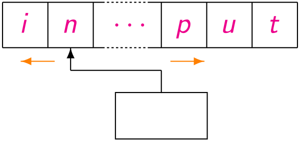

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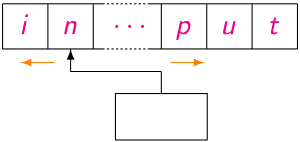

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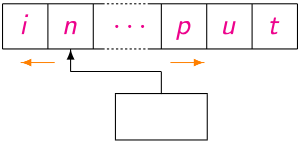

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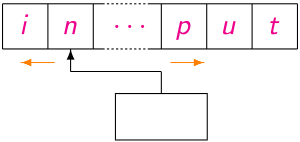

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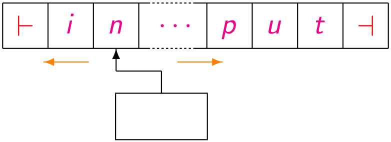
Two-Way Automata: Technical Details



- Input surrounded by the endmarkers ⊢ and ⊣
- $w \in \Sigma^*$ is accepted iff there is a computation
  - with input tape $\vdash w \dashv$
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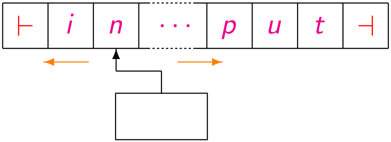

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1DFA, 1NFA, 2DFA, 2NFA

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They share the same computational power, namely they characterize the class of *regular languages*,

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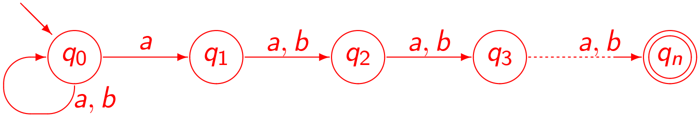
They share the same computational power, namely they characterize the class of *regular languages*, however...

...some of them are more succinct

Example: $I_n = (a+b)^* a (a+b)^{n-1}$

- I_n is accepted by a 1NFA with $n+1$ states



- The minimum 1DFA accepting I_n requires 2^n states
- We can get a *deterministic* automaton for I_n with $n+2$ states, which reverses the input head direction just one time
- Hence I_n is accepted by
  - a 1NFA and a 2DFA with approx. the same number of states
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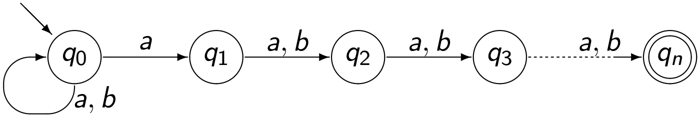

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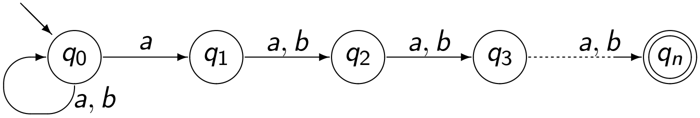

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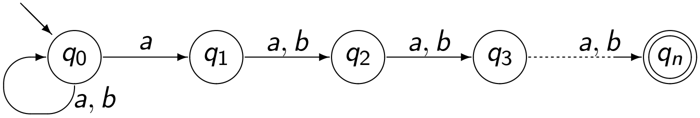

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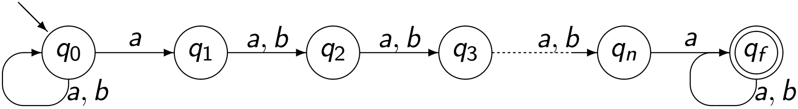

1NFA: $n+2$ states

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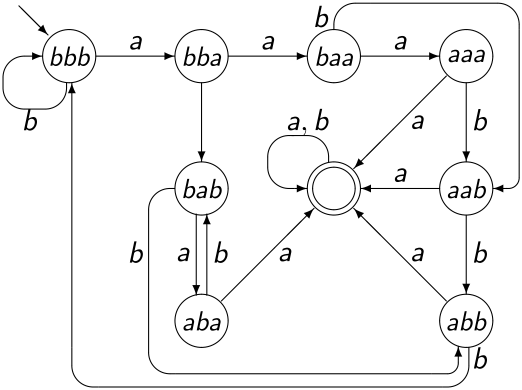

$n = 3$

Minimum 1DFA: $2^n + 1$ states

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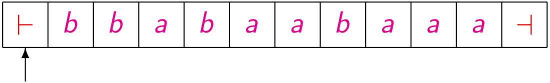

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while input symbol $\neq a$ **do** move to the right

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	$\uparrow$										

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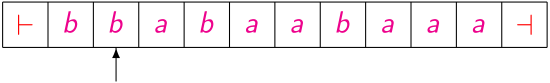

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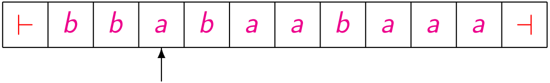

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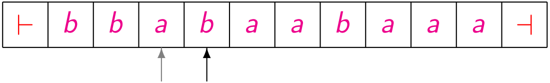

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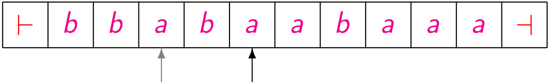

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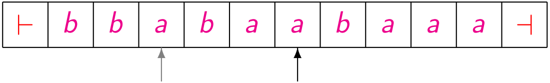

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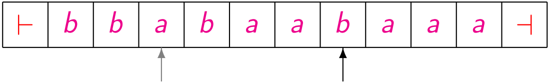

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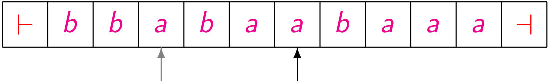

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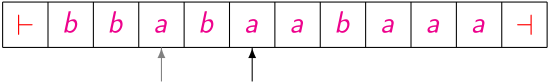

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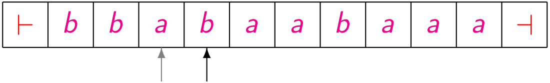

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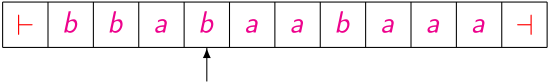

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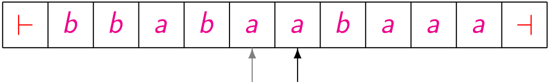

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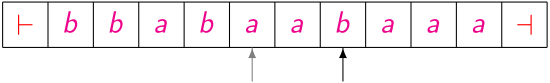

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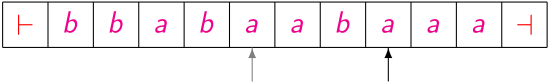

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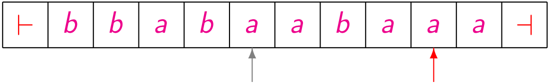

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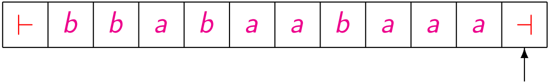

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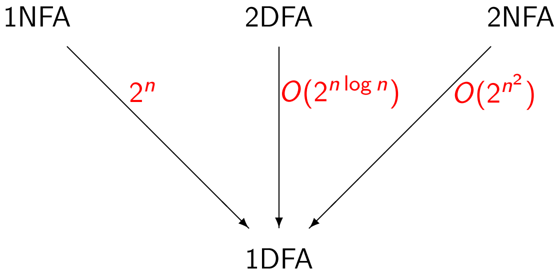
2DFA: $O(n)$ states

Costs of the Optimal Simulations Between Automata



[Rabin&Scott '59, Shepardson '59, Meyer&Fischer '71, . . .]

Question

How much the possibility of moving the input head forth and back is useful to eliminate the nondeterminism?

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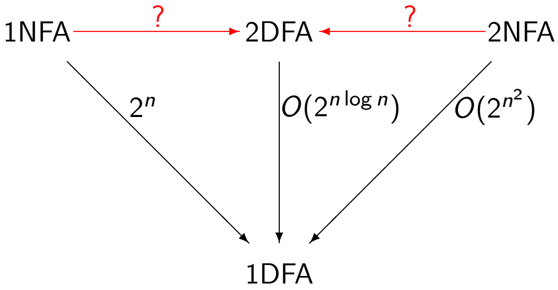

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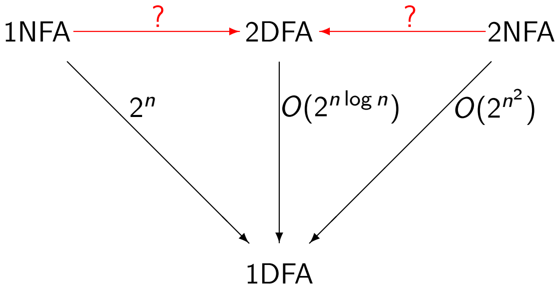

Problem ([Sakoda&Sipser '78])

Do there exist polynomial simulations of

- *1NFAs by 2DFAs*
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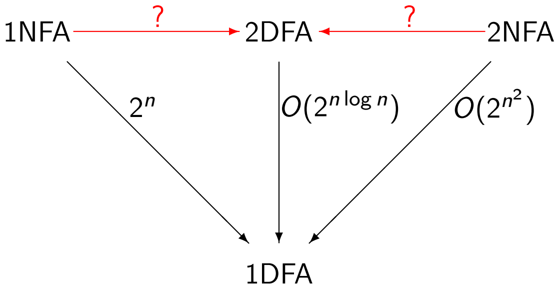

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Do there exist polynomial simulations of

- *1NFAs by 2DFAs*
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Conjecture

These simulations are not polynomial

Sakoda&Sipser Question: Upper and Lower Bounds

- Exponential upper bounds
  deriving from the simulations of 1NFAs and 2NFAs by 1DFAs

- Polynomial lower bounds
  for the cost $c(n)$ of simulation of 1NFAs by 2DFAs:
  - $c(n) \in \Omega(\frac{n^2}{\log n})$ [Berman&Lingas '77]
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- Complete languages
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Sakoda and Sipser Question

- Very difficult in its general form
- Not very encouraging obtained results:

  Lower and upper bounds too far
  (Polynomial vs exponential)

- Hence:

  Try to attack restricted versions of the problem!

2NFAs vs 2DFAs: Restricted Versions

(i) Restrictions on the resulting machines (2DFAs)

- sweeping automata [Sipser '80]
- oblivious automata [Hromkovič&Schnitger '03]
- "few reversal" automata [Kapoutsis '11]

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- unary regular languages [Geffert Mereghetti&P '03]

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A two-way automaton A is said to be sweeping if and only if

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- the input head of A can change direction only at the endmarkers

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  - $L_k = (\{uv \mid u, v \in \{a, b\}^k \text{ and } u \neq v\}\#)^*$
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Theorem ([Kapoutsis '11])

- *Few reversal DFAs can be exponentially larger than few reversal NFAs and, hence, than 2NFAs*
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Sakoda&Sipser Question

Problem ([Sakoda&Sipser '78])

Do there exist polynomial simulations of

- *1NFAs by 2DFAs*
- *2NFAs by 2DFAs ?*

Another possible restriction:

The unary case $\#\Sigma = 1$

Optimal Simulation Between Unary Automata

The costs of the optimal simulations between automata are different in the unary and in the general case

1DFA 1NFA

2DFA 2NFA

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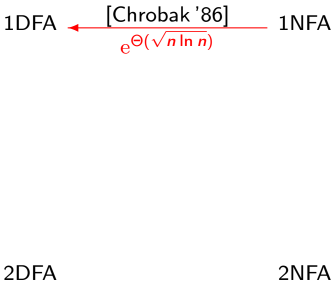

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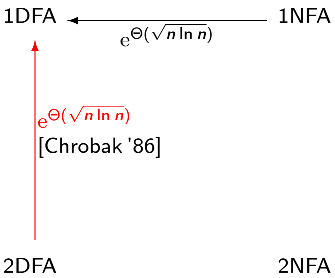

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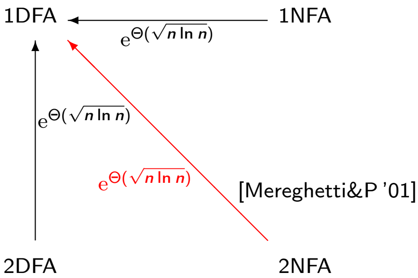

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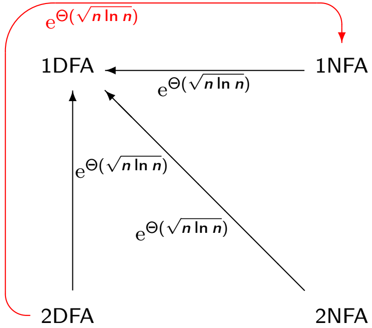

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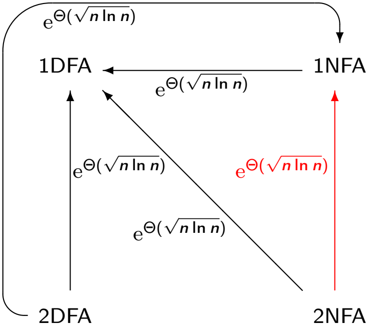

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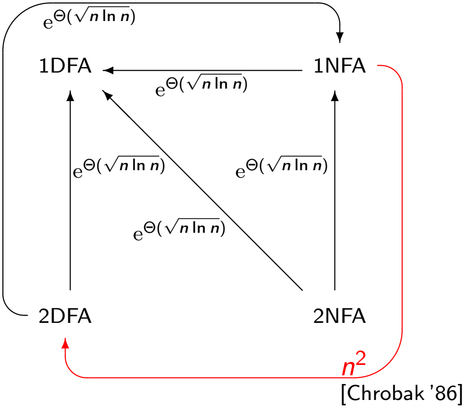

1NFA → 2DFA
In the unary case
this question is solved!
(polynomial conversion)

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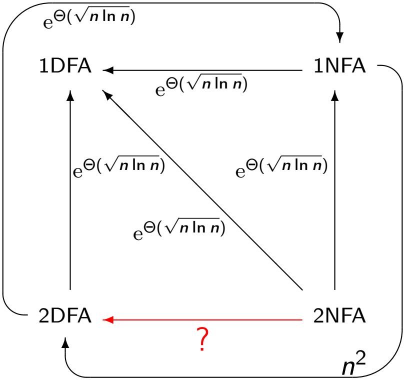

2NFA → 2DFA
Even in the unary case
this question is open!

- $e^{\Theta(\sqrt{n \ln n})}$ upper bound (from 2NFA → 1DFA)
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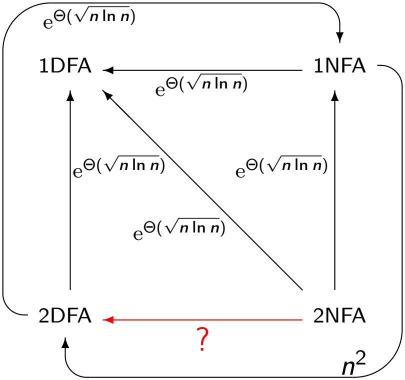

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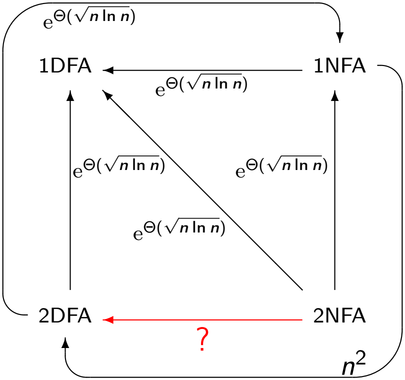

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A better upper bound $e^{O(\ln^2 n)}$ has been proved!

Sakoda&Sipser Question: Current Knowledge

- Upper bounds

	1NFA→ 2DFA	2NFA→ 2DFA
unary case	$O(n^2)$ optimal	$e^{O(\ln^2 n)}$
general case	exponential	exponential

Unary case [Chrobak '86, Geffert Mereghetti&P '03]

- Lower Bounds

In all the cases, the best known lower bound is $\Omega(n^2)$ [Chrobak '86]

Unary Case: Quasi Sweeping Automata

[Geffert Mereghetti&P '03]

In the study of unary 2NFA, sweeping automata with some *restricted nondeterministic capabilities* turn out to be very useful:

Definition

A 2NFA is quasi sweeping (qsNFA) iff both

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Theorem (Quasi Sweeping Simulation)

Each n-state unary 2NFA A can be transformed into a 2NFA M s.t.

- *M is quasi sweeping*
- *M has at most $N \leq 2n + 2$ states*
- *M and A are "almost equivalent"*
  (differences are possible only for inputs of length $\leq 5n^2$)

Quasi Sweeping Simulation: Consequences

Several results using quasi sweeping simulation of unary 2NFAs have been found:

(i) Subexponential simulation of unary 2NFAs by 2DFAs
Each unary n-state 2NFA can be simulated by a 2DFA with $e^{O(\ln^2 n)}$ states [Geffert Mereghetti&P '03]

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Inductive counting argument for qsNFAs
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Logspace Classes and Graph Accessibility Problem

L: class of languages accepted in logarithmic space by *deterministic* machines

NL: class of languages accepted in logarithmic space by *nondeterministic* machines

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More in general, GAP $\in \mathcal{C}$ implies $\mathcal{C} \supseteq$ NL for each class $\mathcal{C}$ closed under logspace reductions

- Let A be an n-state unary 2NFA
- Reduction from $L(A)$ to GAP
  i.e, from each string a^m we compute a graph $G(m)$ s.t.

$$a^m \in L(A) \iff G(m) \in \text{GAP}$$

- *Under the hypothesis* L = NL
  this reduction is used to build a 2DFA equivalent to A,
  with a number of states polynomial in n
- Actually we do not work directly with A:
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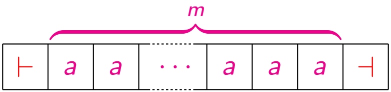

Given the qsNFA M with N states and an input a^m the graph $G(m)$ is defined as:

- the vertices are the states of M
- (p, q) is an edge iff M can traverse the input
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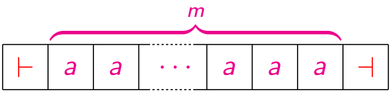

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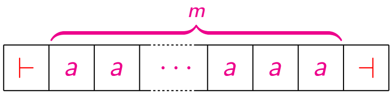

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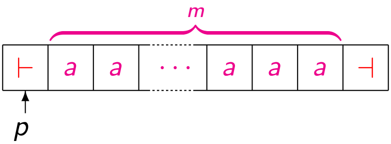

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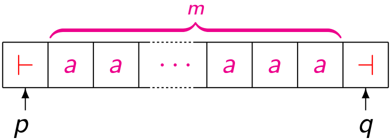

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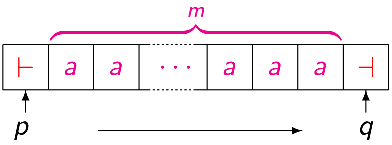

Given the qsNFA M with N states and an input a^m the graph $G(m)$ is defined as:

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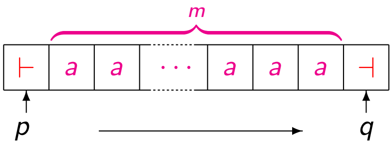

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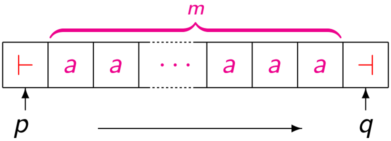

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The existence of the edge (p, q) can be verified by a subroutine, implemented by a finite automaton $A_{p,q}$ with N states

Deterministic Simulation

- Suppose L = NL
- Let D_{GAP} be a logspace bounded *deterministic* machine solving GAP
- On input a^m, compute $G(m)$ and give the resulting graph as input to D_{GAP}
- This decides whether or not $a^m \in L(M)$

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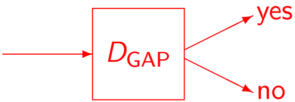

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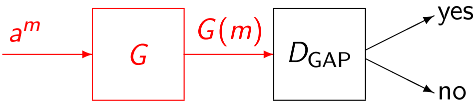

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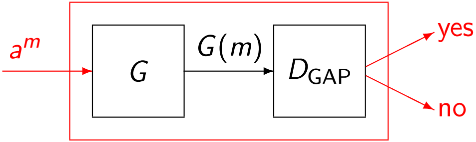

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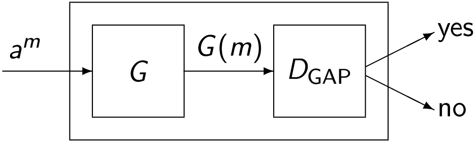

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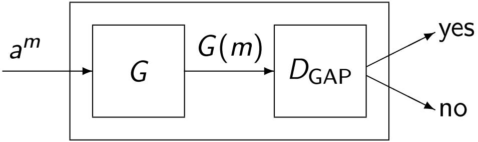

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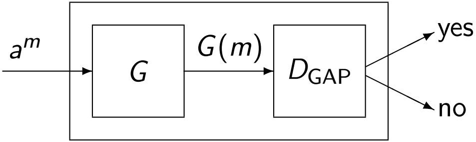

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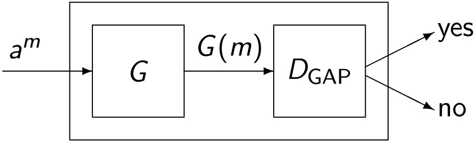

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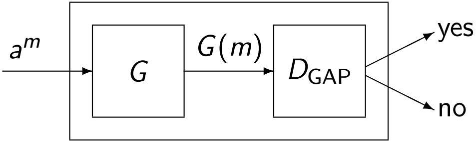

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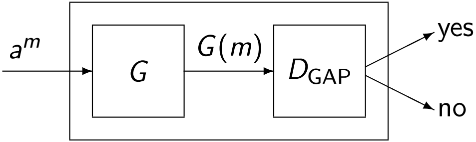

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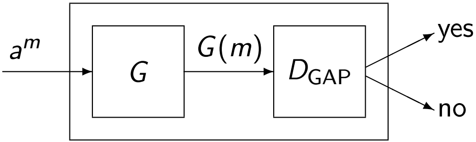

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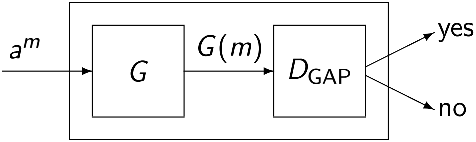

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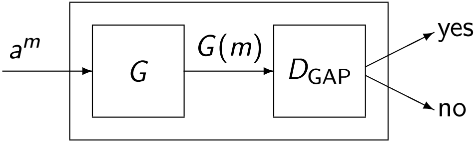

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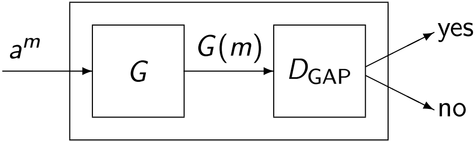

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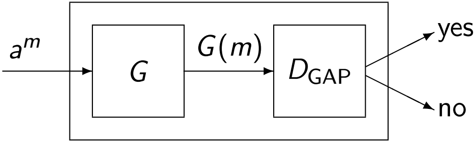

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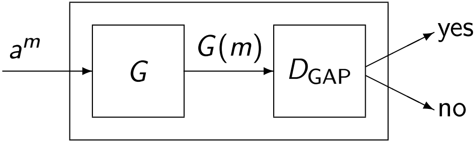

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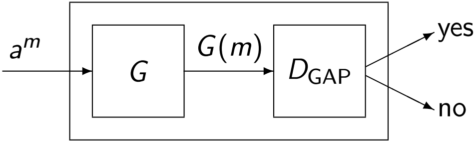

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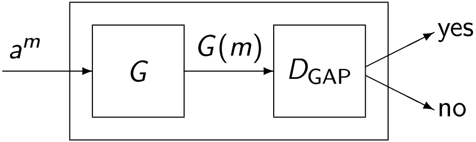

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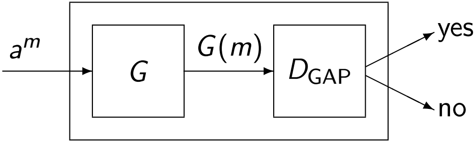

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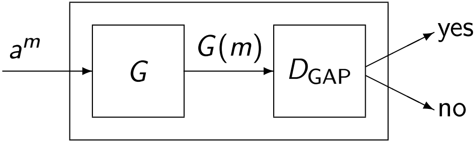

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Summing Up... (under L = NL)

We described the following simulation:

M	2NFA in normal form	N states
$\Downarrow$		Deterministic Simulation
M'	2DFA equivalent to M	$poly(N)$ states

Summing Up... (under L = NL)

- M is *almost equivalent* to the original 2NFA A
- Hence, M' is *almost equivalent* to A
- Possible differences for input length $\leq 5n^2$
- They can be fixed in a preliminary scan ($5n^2 + 2$ more states)
- The resulting automaton has polynomially many states

A	given unary 2NFA	n states
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Theorem ([Geffert&P '10])

If $L = NL$ *then each* n*-state unary 2NFA can be simulated by an equivalent 2DFA with* $poly(n)$ *many states*

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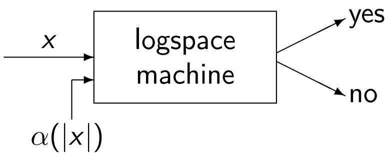
Uniformity?

Nonuniform Deterministic Logspace

- L/poly
  class of languages accepted by deterministic logspace machines with a *polynomial advice*



Problem

L/poly $\supseteq$ NL ?

Polynomial Deterministic Simulation (under L = NL)

We did not used the uniformity of L !

- L can be replaced by L/poly:

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- We can prove the converse using GAP:

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Solving GAP with Two-Way Automata

Binary Encoding: Languages BGAP

- Let n be a fixed integer
- GAP_n denotes GAP restricted to graphs with vertex set $V_n = \{0, \ldots, n-1\}$
- The *binary encoding* of a graph $G = (V_n, E)$ is the standard encoding of its adjacency matrix, i.e., a string

$$\langle G \rangle_2 = x_1 x_2 \cdots x_{n^2} \in \{0,1\}^{n^2}$$
with $x_{i \cdot n + j + 1} = 1$ if and only if $(i, j) \in E$

- $\mathrm{BGAP}_n := \{\langle G \rangle_2 \mid G \text{ has a path from } 0 \text{ to } n-1\}$
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- GAP_n denotes GAP restricted to graphs with vertex set $V_n = \{0, \ldots, n-1\}$
- The *binary encoding* of a graph $G = (V_n, E)$ is the standard encoding of its adjacency matrix, i.e., a string
  $$\langle G \rangle_2 = x_1 x_2 \cdots x_{n^2} \in \{0,1\}^{n^2}$$
  with $x_{i \cdot n + j + 1} = 1$ if and only if $(i, j) \in E$
- $\mathrm{BGAP}_n := \{\langle G \rangle_2 \mid G \text{ has a path from 0 to } n-1\}$
  $= \{\langle G \rangle_2 \mid G \in \mathrm{GAP}_n\}$

Standard nondeterministic algorithm solving graph accessibility

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from vertex i with the input head on the left endmarker

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Standard nondeterministic algorithm solving graph accessibility

$i \leftarrow 0$ // input head on the left endmarker
while $i \neq n - 1$ **do**
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  move to the input cell $i \cdot n + j + 1$
  if the input symbol is 0 **then reject** // $(i, j) \notin E$
  move the input head to the left endmarker
  $i \leftarrow j$
endwhile

Standard nondeterministic algorithm solving graph accessibility

```
i ← 0                                        // input head on the left endmarker
while i ≠ n − 1 do
    guess j ≠ i                              // try the edge (i, j)
    move to the input cell i · n + j + 1
    if the input symbol is 0 then reject     // (i, j) ∉ E
    move the input head to the left endmarker
    i ← j
endwhile
accept
```

Standard nondeterministic algorithm solving graph accessibility

$i \leftarrow 0$ // input head on the left endmarker
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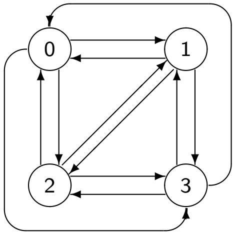
accept

- Implementation using $O(n^3)$ states

- K_n := complete directed graph with vertex set $V_n = \{0, \ldots, n-1\}$
- With each edge (i, j) we associate a different prime number $p_{(i,j)}$
- A subgraph $G = (V_n, E)$ of K_n is encoded by the string a^{m_G}, where

$$m_G = \prod_{(i,j)\in E} p_{(i,j)}$$



Solving GAP with Two-Way Automata

Unary Encoding: Languages UGAP

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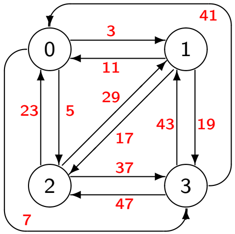

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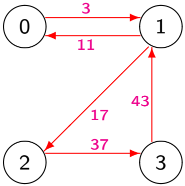

$$m_G = 3 \cdot 11 \cdot 17 \cdot 37 \cdot 43 = 892551$$

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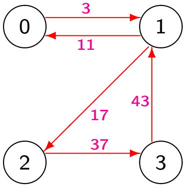

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- Graph $K_n(m)$: $\exists$ edge (i,j) iff $p_{(i,j)}$ divides m
- UGAP$_n$:= $\{a^m \mid K_n(m)$ has a path from 0 to $n-1\}$

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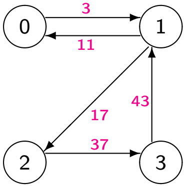

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- Implementation using $O(n^4 \log n)$ states

- Suppose the conversion of unary 2NFAs into 2DFAs is polynomial
- Let B_n be a 2DFA with $poly(n)$ states recognizing UGAP_n
- Given a graph $G = (V_n, E)$, compute its unary encoding a^{m_G} and give it as input to B_n
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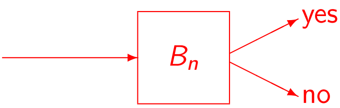
Solving GAP with Two-Way Automata

Outline of the Construction



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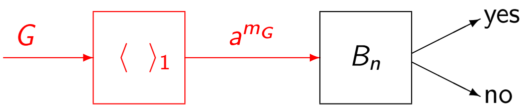

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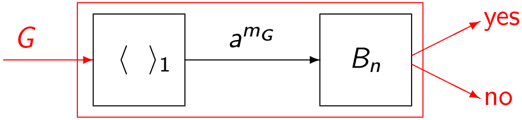

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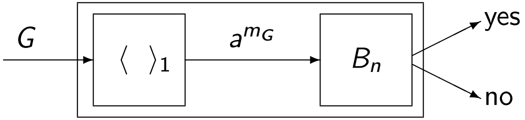

- Our goal:
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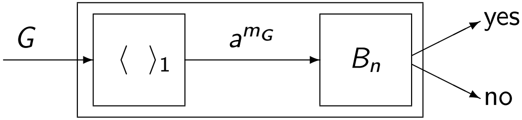
  - working in logarithmic space
  - using a polynomial advice
- The input is the graph G (size n^2)
- B_n *is the advice:* polynomial size in n
- Representing a^{m_G} would require too much space!

Solving GAP with Two-Way Automata
Outline of the Construction



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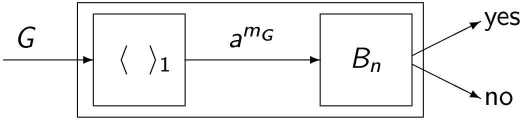

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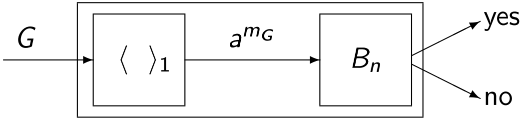
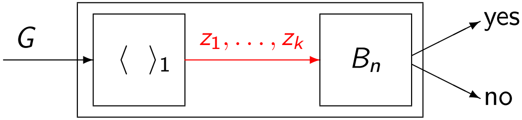


Prime encoding:

A list of prime powers $z_1, \ldots, z_k$ factorizing m_G



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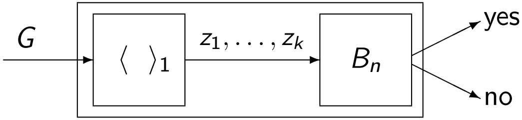
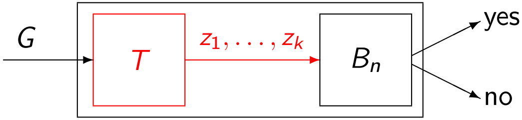
a^{m_G} is replaced by the prime encoding



- $m_G = \prod_{(i,j)\in E} p_{(i,j)}$
- *Prime encoding of a^{m_G}:*
  list of all $p_{(i,j)}$ associated with the edges of G
- It can be computed by in logarithmic space
  by *a deterministic transducer T*
- We replace B_n by an "equivalent" 2DFA B'_n:

  B'_n inputs represent prime encodings of B_n inputs



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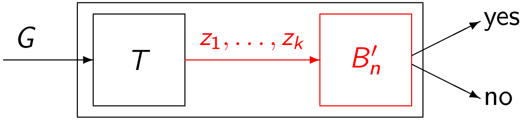
Solving GAP with Two-Way Automata

Replacing Unary Encodings by Prime Encodings



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How to Obtain B_n'?

- $s :=$ number of states of B_n
- $B_n \to M_n$

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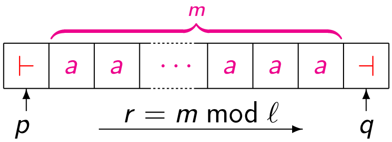
  - M_n and B_n almost equivalent (differences for length $O(s)$)
- $M_n \to B'_n$
  - $poly(s)$ many states
  - B'_n reads the prime encoding of an integer m
  - If m is "small" then B'_n gives the output according to a finite table
  - otherwise, B'_n on its input simulates M_n on a^m

How to Obtain B'_n?

Simulation on Long Inputs



In a sweep:

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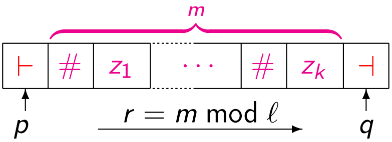

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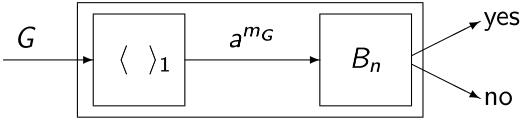
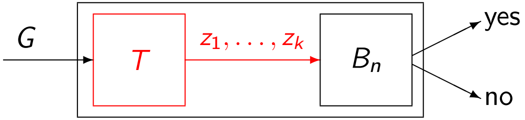
B'_n simulates the same sweep on input $z_1, z_2, \ldots, z_k$,
a prime encoding of m:

$$m \bmod \ell = ((\cdots((z_1 \bmod \ell) \cdot z_2) \bmod \ell \cdots) \cdot z_k) \bmod \ell$$



- We replace:
  - The machine which computes $m_G = \langle G \rangle_1$ by a logspace transducer T which outputs a prime encoding of m_G
  - The unary 2DFA B_n by an "equivalent" 2DFA B'_n working on prime encodings
- The resulting machine still decides whether $G \in \text{GAP}_n$
- The symbols of $z_1, \ldots, z_k$ are computed "on fly", by restarting T each time B'_n needs them



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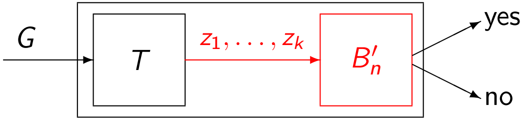
Solving GAP with Two-Way Automata

Combining All Together



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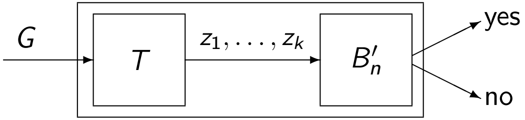

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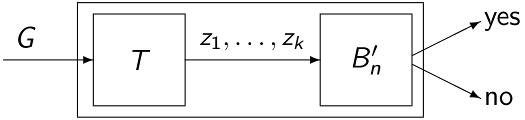

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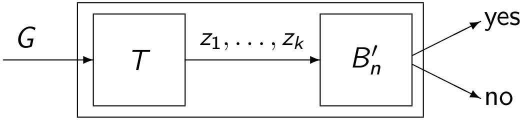

- B'_n has number of states polyomial in n
- T works in space $O(\log n)$
- Hence the resulting machine works in logarithmic space

We did not provided B'_n in a constructive way!

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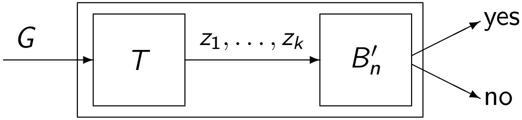

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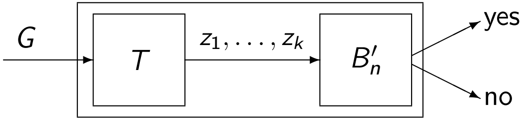

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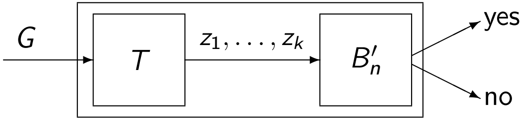

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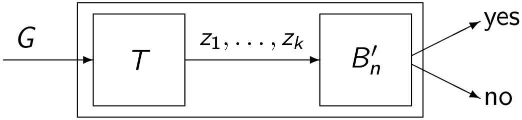
Solving GAP with Two-Way Automata

Combining All Together



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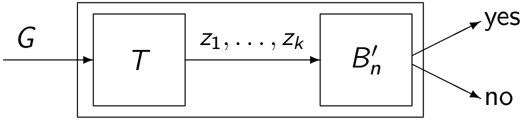

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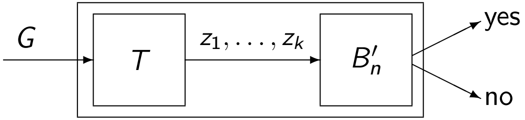

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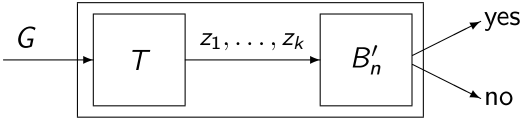
Theorem ([Kapoutsis&P '12])

If each n-state unary 2NFA can be simulated by a 2DFA with a polynomial number of states then L/poly $\supseteq$ NL



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Corollary

L/poly $\supseteq$ NL *if and only if*
the state cost of the simulation of unary 2NFAs by 2DFAs is poly

Outer Nondeterministic Automata (ONFAs)

Definition

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Hence:

- No restrictions on the *input alphabet*
- No restrictions on *head reversals*
- *Deterministic transitions* on "real" input symbols
- *Nondeterministic choices* only at the endmarkers

Outer Nondeterministic Automata (ONFAs)

All the results we obtained for the unary case
can be extended to ONFAs:

[Guillon Geffert&P '12, Kapoutsis&P '12]

(i) Subexponential simulation of 2ONFAs by 2DFAs

(ii) Polynomial complementation of unary 2ONFAs

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if and only if L/poly $\supseteq$ NL

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While in the unary case all the proofs rely on the conversion
of 2NFAs into quasi sweeping automata,
in the case of 2ONFAs we do not have a similar tool!

Final Remarks

- The question of Sakoda and Sipser is very challenging
- In the investigation of restricted versions many interesting and not artificial models have been considered
- The results obtained for restricted versions of the problem, even if not solving the full problem, are nontrivial and, in many cases, very deep
- Strong connections with open questions in structural complexity
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Two Further Directions

- The results obtained in the unary case have been extended to the general case for outer nondeterministic automata

Question

Does it is possible to extend the same results (or some of them) to some less restricted models of computation?

- Input head reversals are a critical resource that deserves further investigation

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