Descriptional complexity of automata and languages

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Outline of the talk

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1. Introduction
2. Regular languages and finite automata
3. The problem of Sakoda and Sipser
4. Unary automata
5. Regular languages vs context-free grammars and pushdown automata
6. State complexity of language operations
7. State complexity of the complementation

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Descriptional complexity

Given

- $\mathcal{C}$, a class of languages
- $\mathcal{S}$, a formal system (e.g., class of devices, class of grammars,..) able to represent all the languages in $\mathcal{C}$

What is the *size* of the representations of the languages in $\mathcal{C}$ by the system $\mathcal{S}$?

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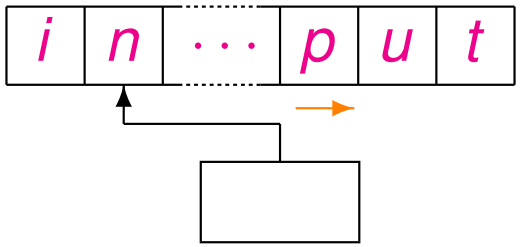

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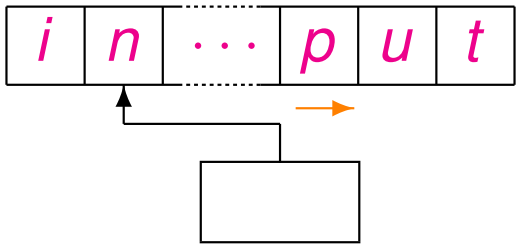

Base version:

One-way determistic finite automata (1dfa)

- one-way input tape
- deterministic

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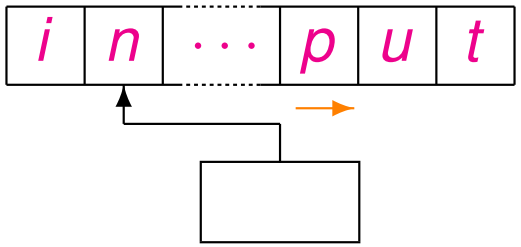

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- two-way input head motion
- alternation
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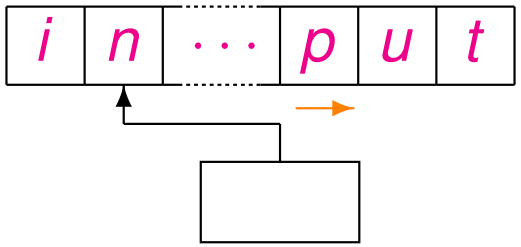

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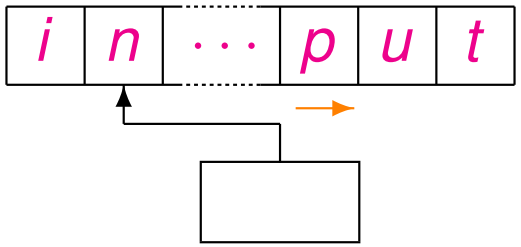

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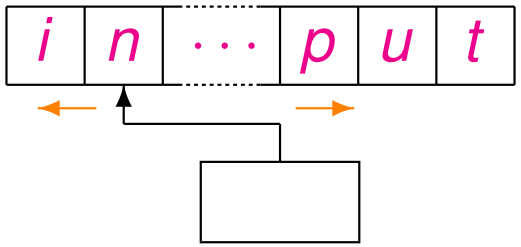

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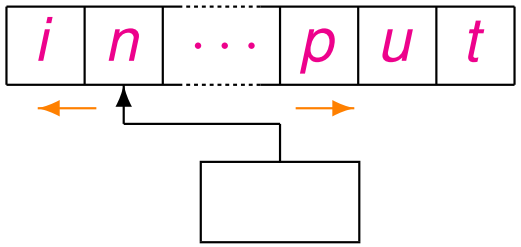

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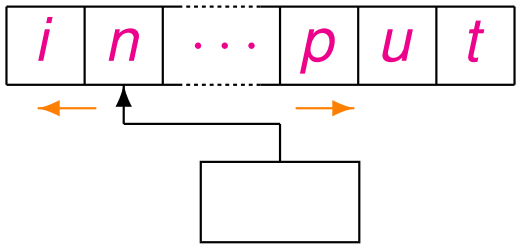

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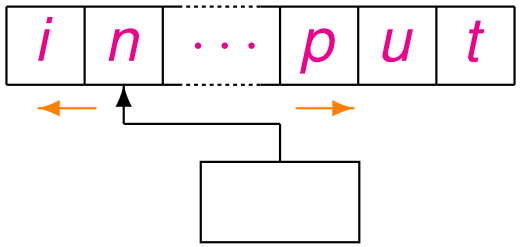

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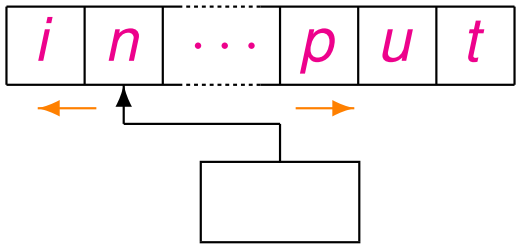

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Finite state automata: different variants

1dfa, 1nfa, 2dfa, 2nfa, ...

- Formal language point of view:

  What about the *power* of these models?

  All of them characterize the class of *regular languages*...

- Descriptional complexity point of view:

  What about the *size* of their descriptions?

  ...however some of them are more succinct.

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- Each n-state 1nfa can be simulated by a 1dfa with 2^n states (subset construction) [Rabin and Scott '59], and:
- For each integer $n \geq 1$ there is a language which is accepted by a n-state 1nfa which requires 2^n states to be accepted by a 1dfa [Meyer and Fischer '71].

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- Deterministic automata (dfa):
  number of states.
- Nondeterministic automata (nfa):
  number of states, or
  number of transitions (more precise).

Remarks:

- Each nfa with n states has $O(n^2)$ transitions.
- Many results have been obtained for the measure “number of the states”.
- The measure “number of transitions” was recently investigated by some authors,
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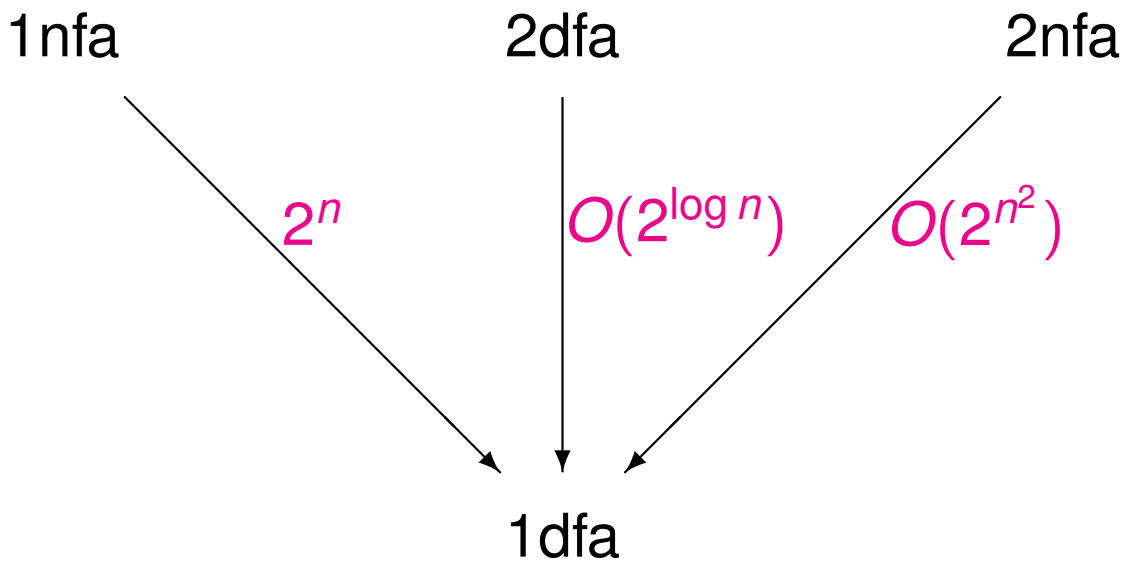
Costs of the optimal simulations by 1dfa (in terms of states):



[Rabin and Scott '59, Shepardson '59,Meyer and Fischer '71,...]

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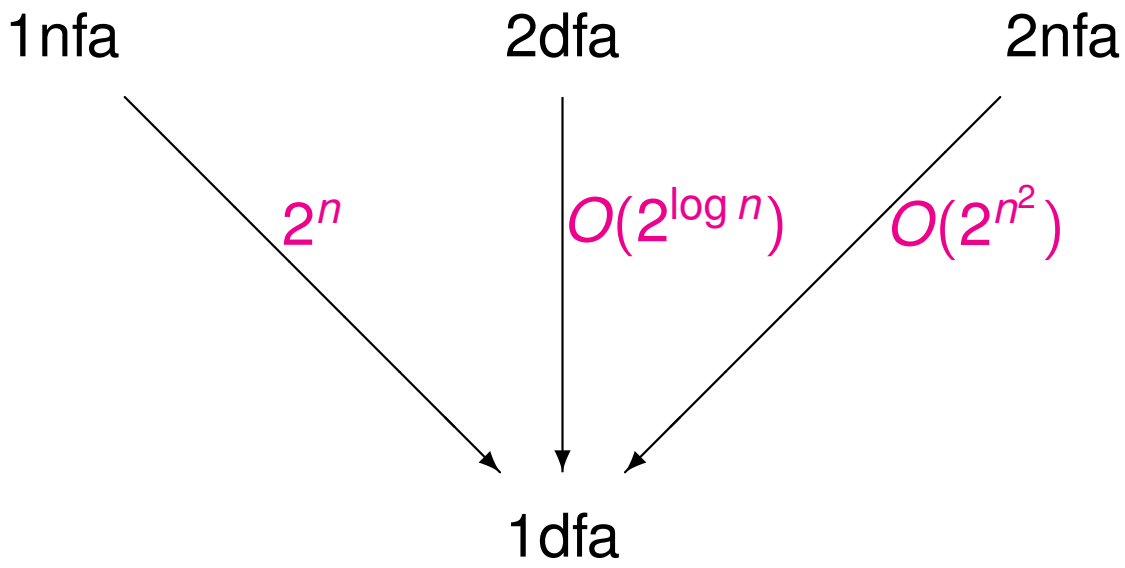

How much two-way motion can be useful in order to eliminate the nondeterminism?

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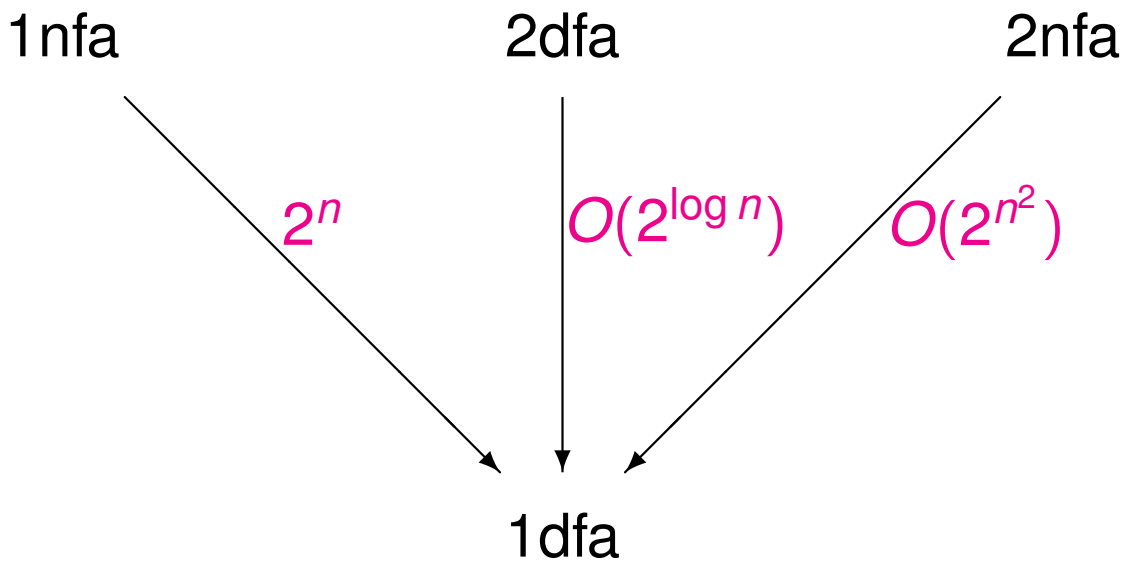

Problem ([Sakoda and Sipser 1978])

Find the costs, in terms of states, of the optimal simulations of

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Conjecture: these costs are exponential

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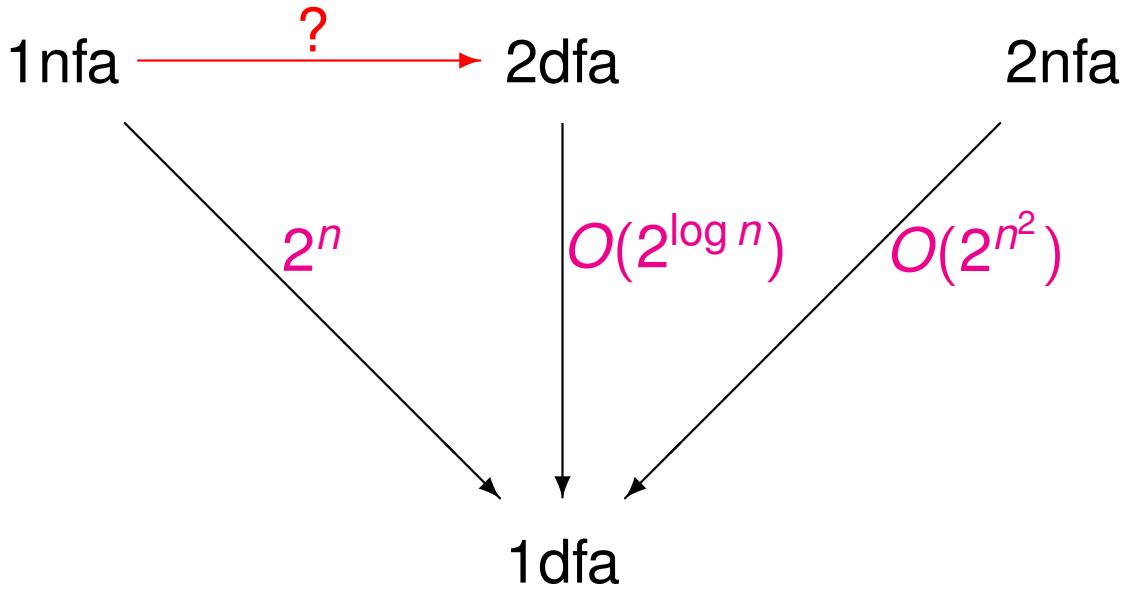

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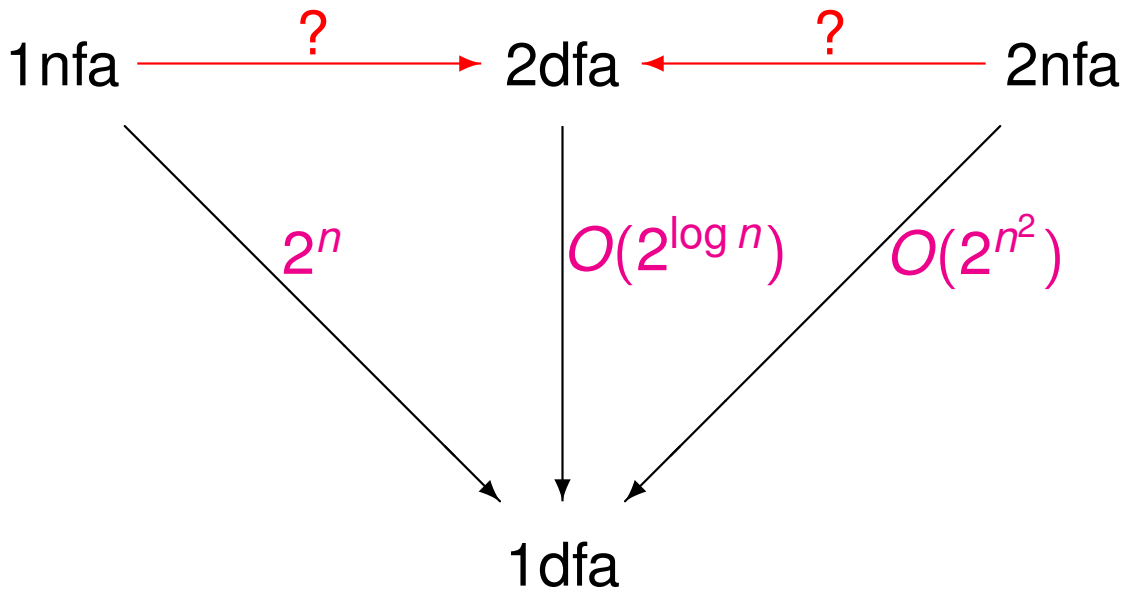

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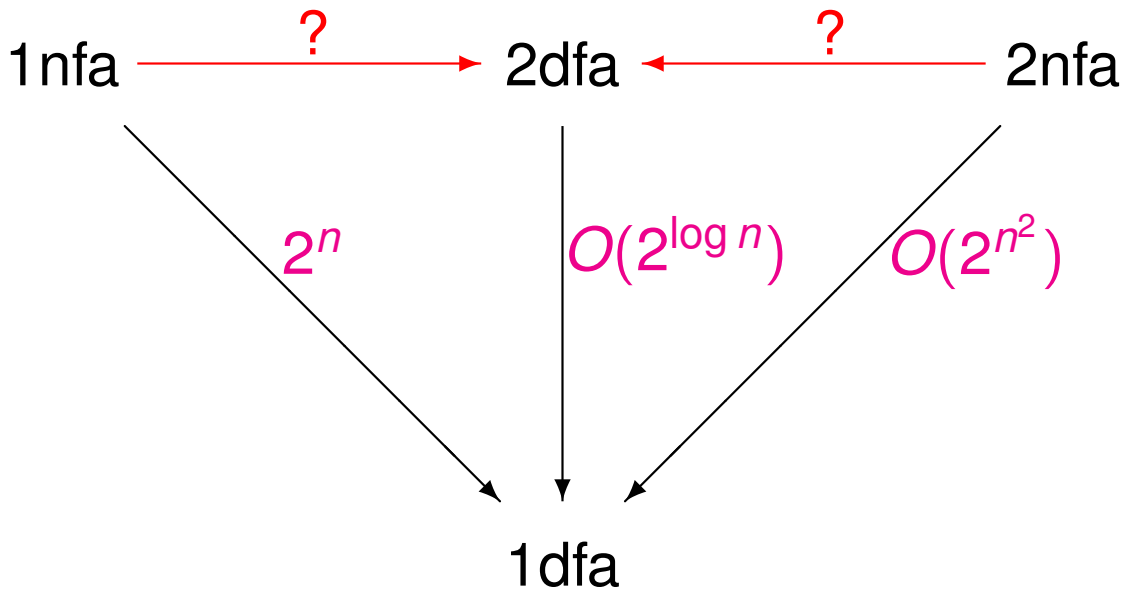

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Theorem (Complete languages for 1nfa vs 2dfa)

There exists a sequence of languages $< B_1, B_2, \ldots, B_n, \ldots >$ *s.t. for each integer* $n \geq 1$*:*

- B_n *is accepted by a 1nfa with* n *states, and*
- *among all languages accepted by* n*-state 1nfa,* B_n *requires the largest 2dfa.*

Remark: the second condition implies that
the simulation of 1nfa by 2dfa is polynomial iff each B_n is accepted by a 2dfa with a polynomial (in n) number of states.

In a similar way:

Theorem (Complete languages for 2nfa vs 2dfa)

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- *oblivious automata* [Hromkovič and Schnitger 2003]

Sweeping automata

Theorem ([Sipser 1980])

There exists a family of languages $< B_1, B_2, \ldots, B_n, \ldots >$ *s.t. for each integer* $n \geq 1$*:*

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Sakoda&Sipser question

Current knowledge

- Upper bounds

	1nfa $\longrightarrow$ 2dfa	2nfa $\longrightarrow$ 2dfa
general case	exponential	exponential
unary case	$O(n^2)$ [1] optimal	$n^{O(\log n)}$ [2]

[1: Chrobak 1986]

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  For all the cases, the best known lower bound is $\Omega(n^2)$ [1]

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Input alphabet $\Sigma = \{a\}$

Theorem

$L \subseteq \{a\}^*$ is regular iff $\exists \mu \geq 0, \lambda \geq 1$ s.t.

$$\forall n \geq \mu : a^n \in L \text{ iff } a^{n+\lambda} \in L.$$

Special case $\mu = 0$: the language is *periodic* or *cyclic*.

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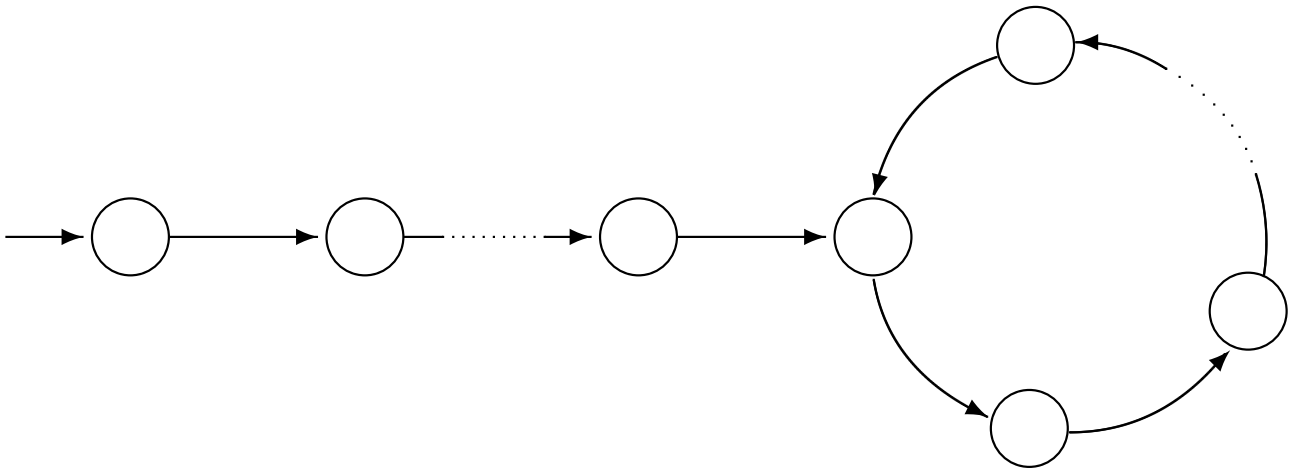

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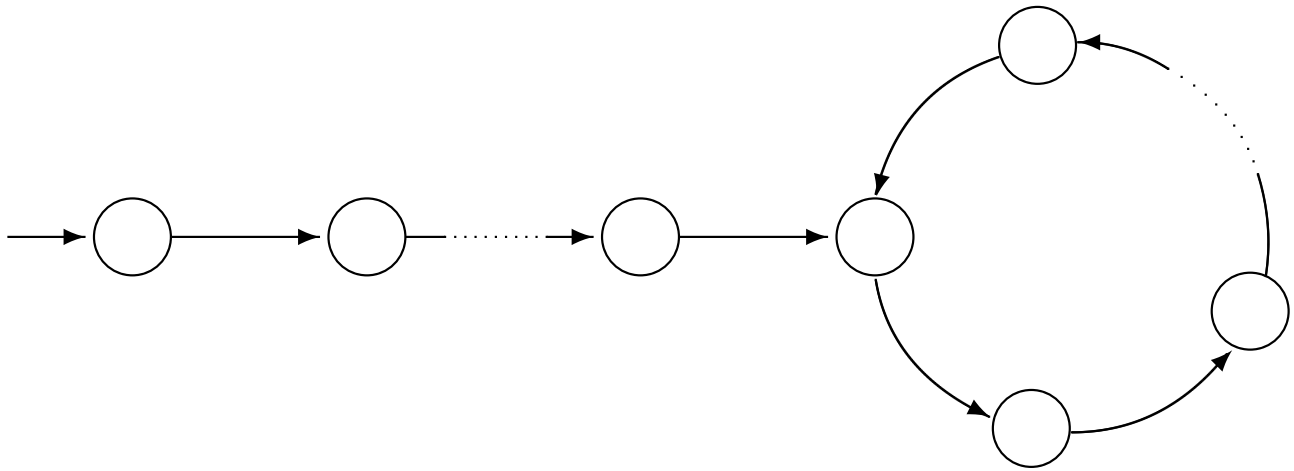

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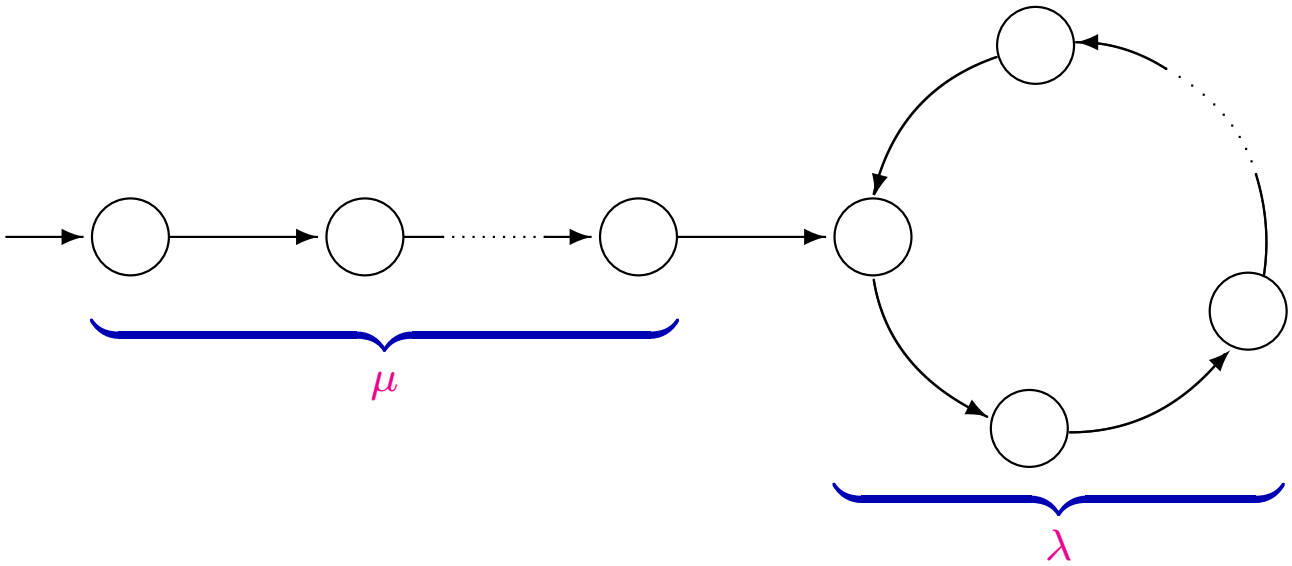

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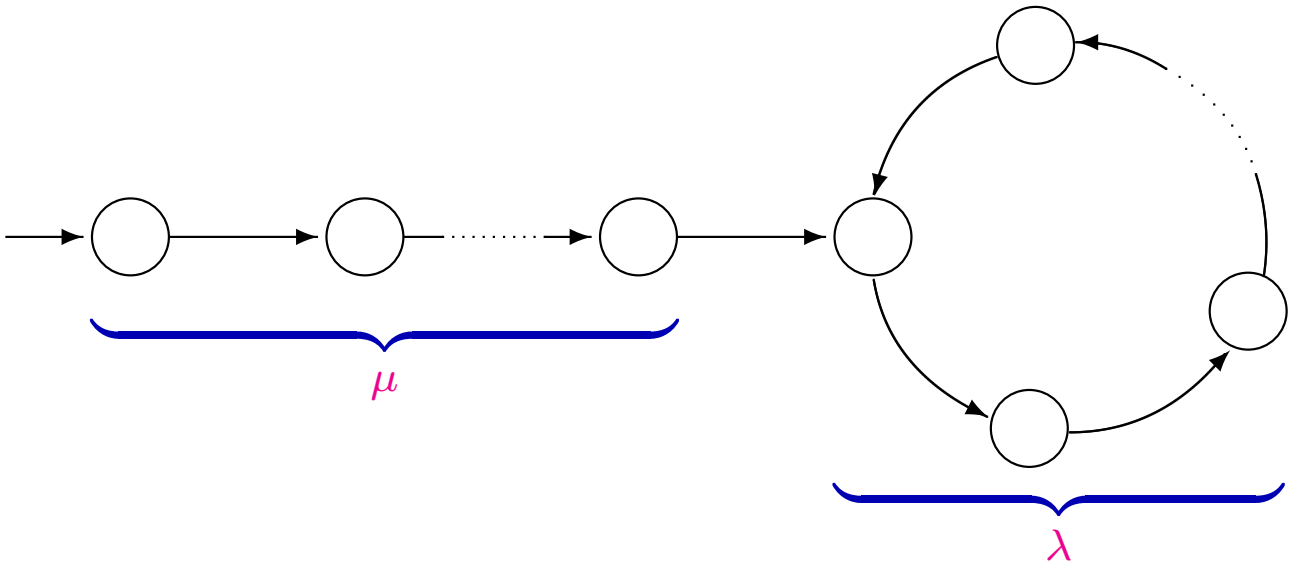

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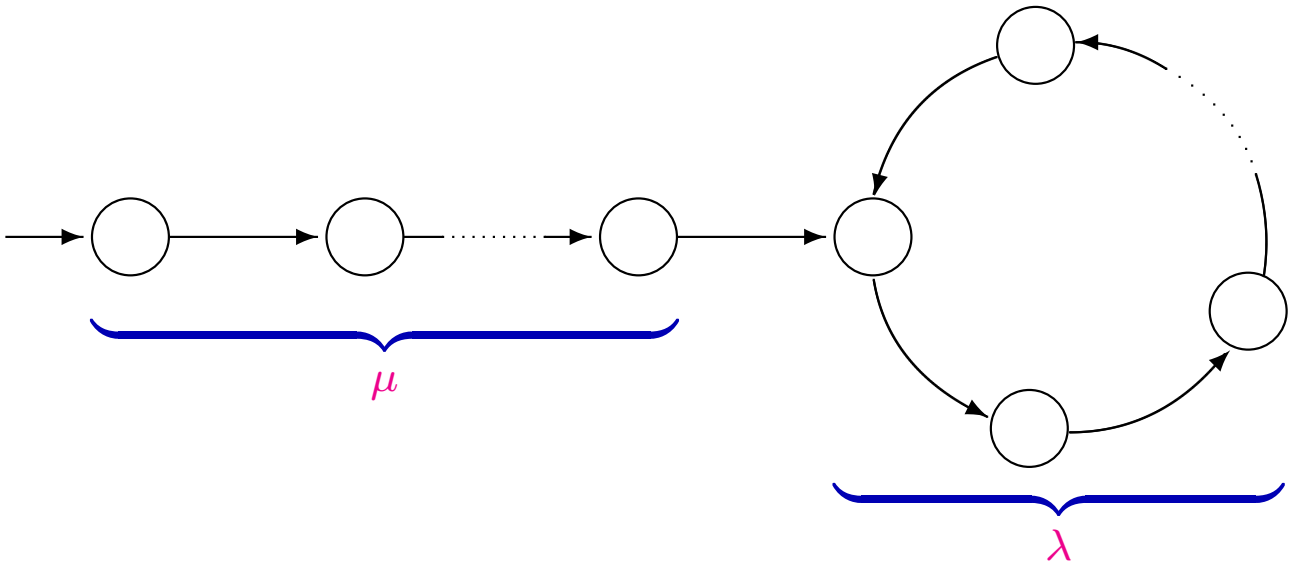

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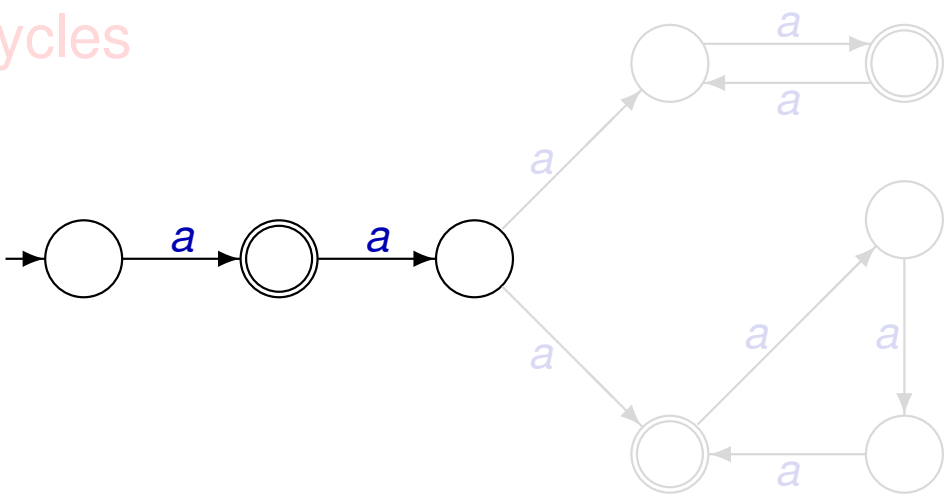

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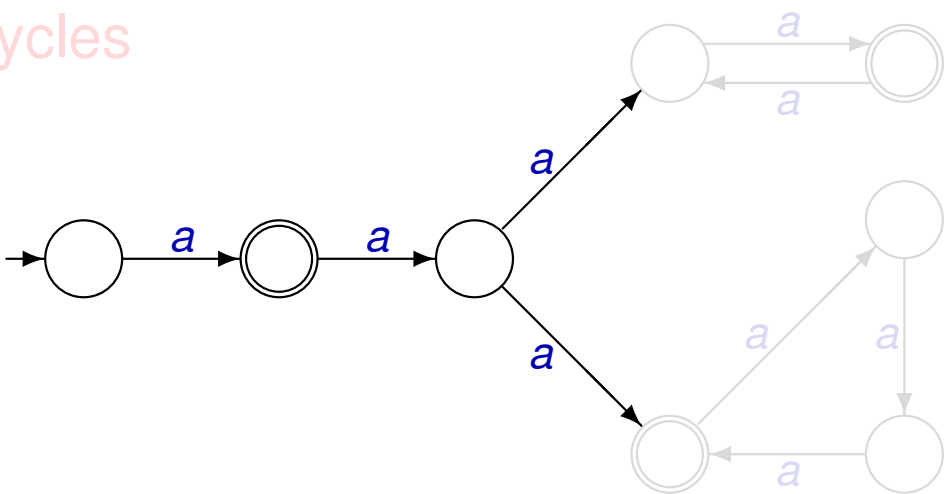

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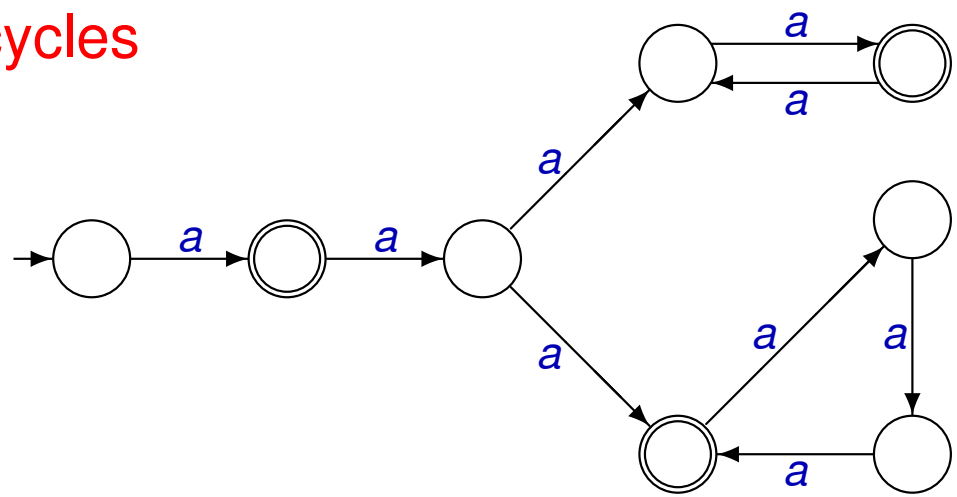

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For any unary n-state 1nfa there exists an equivalent 1nfa in Chrobak normal form s.t.

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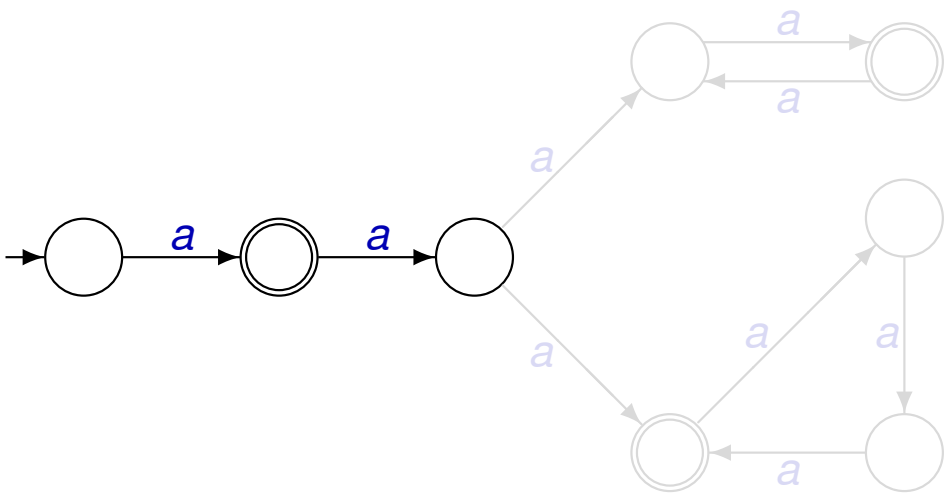

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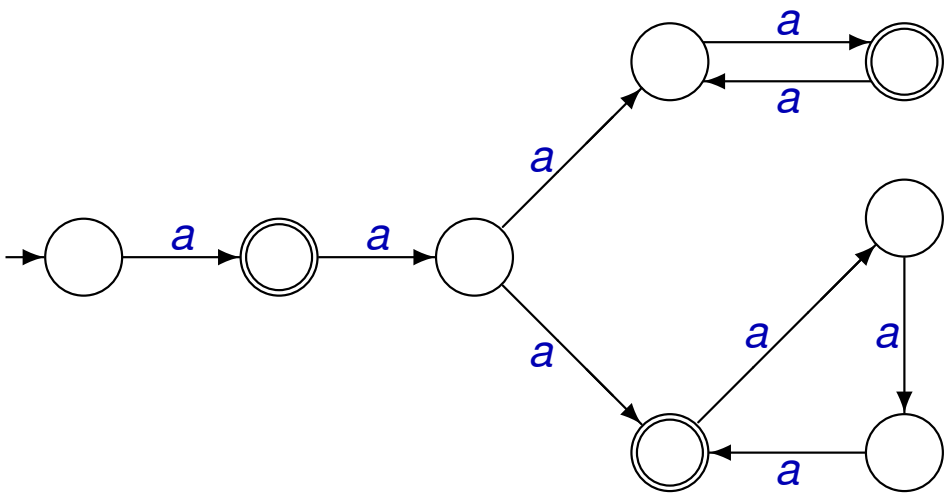

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How to eliminate the nondeterminism from a unary 1nfa?

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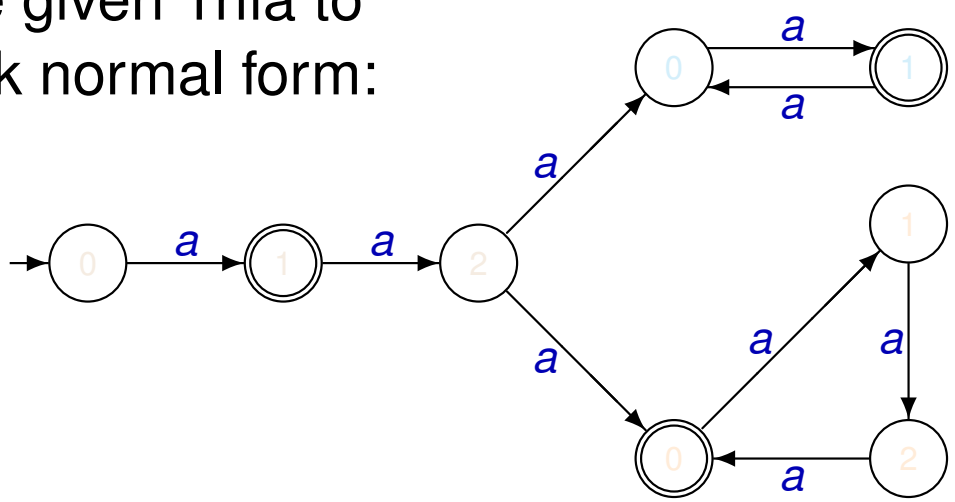

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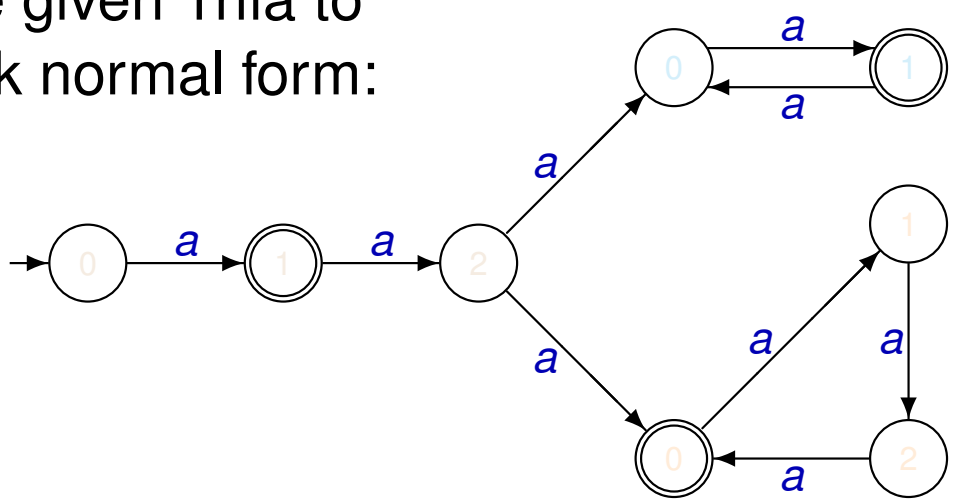

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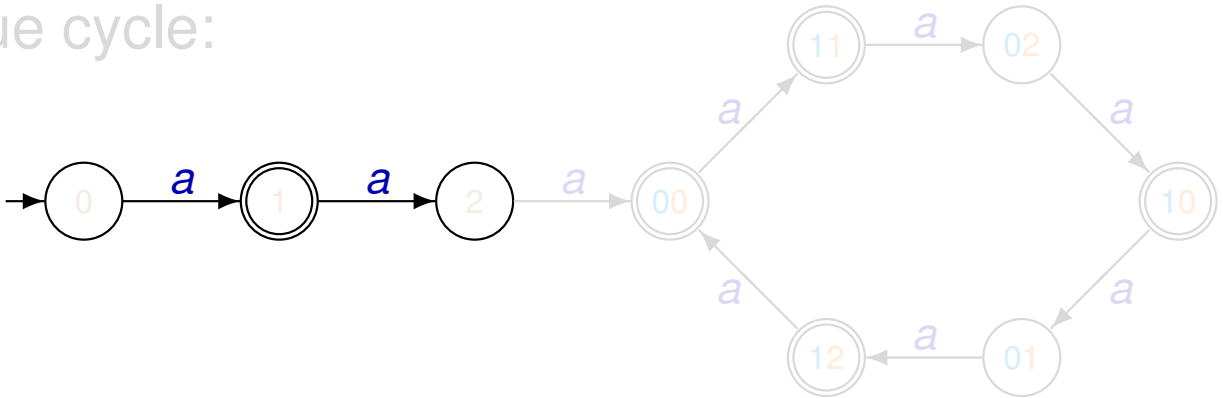

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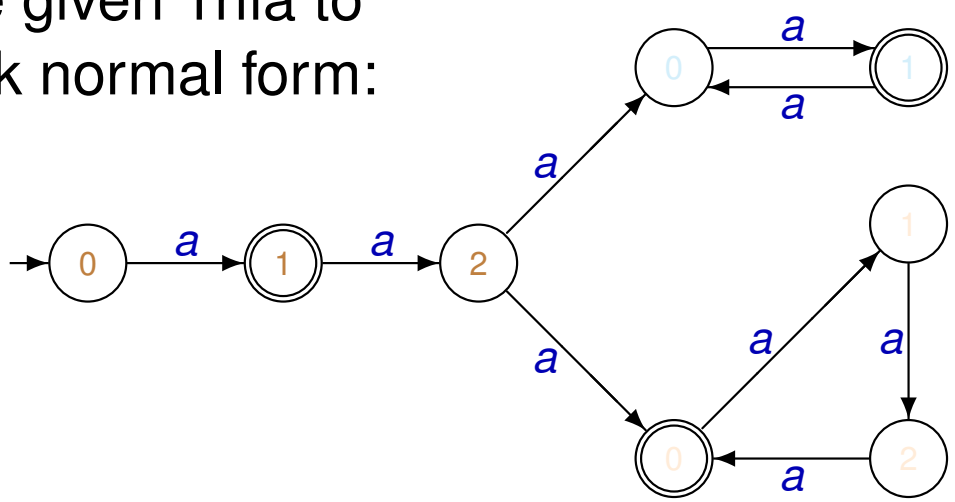

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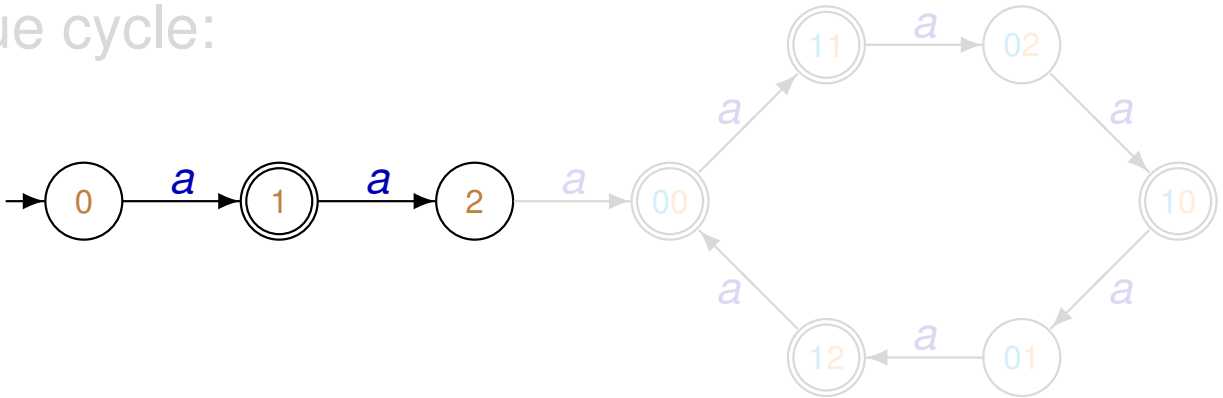

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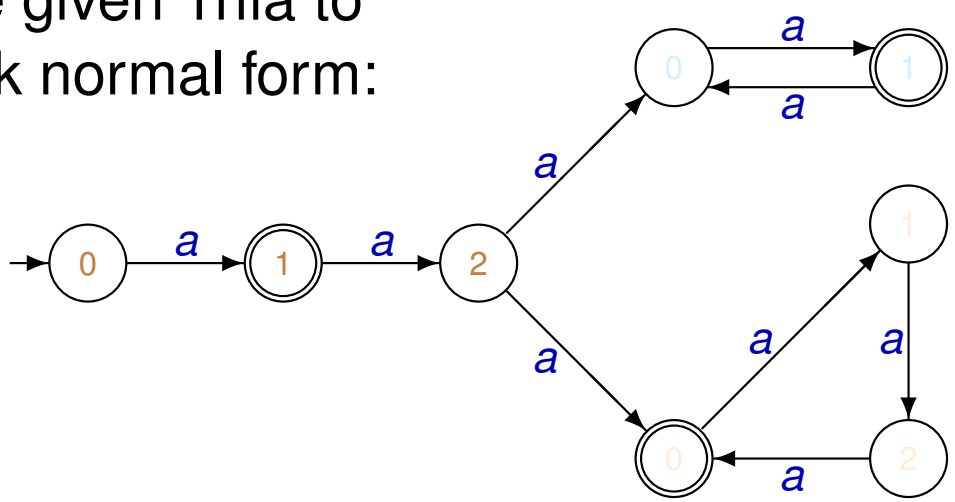

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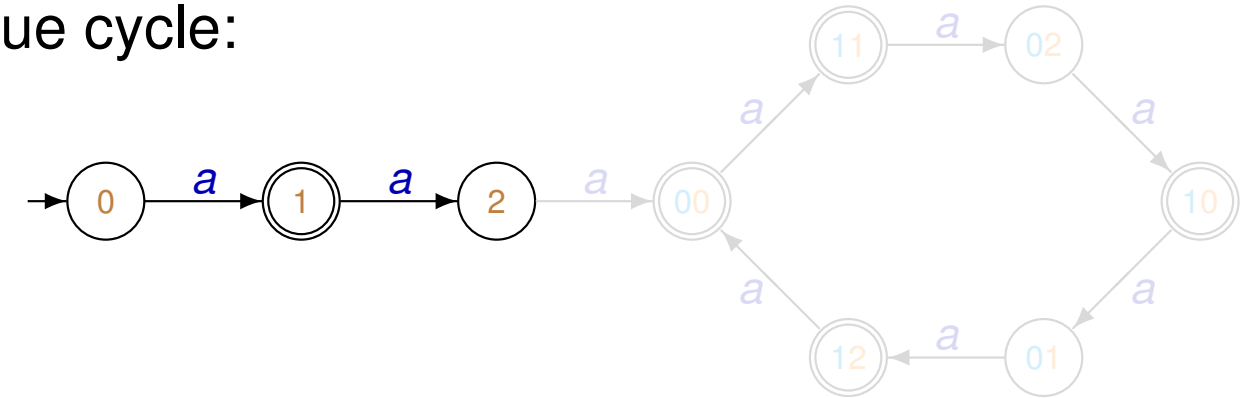

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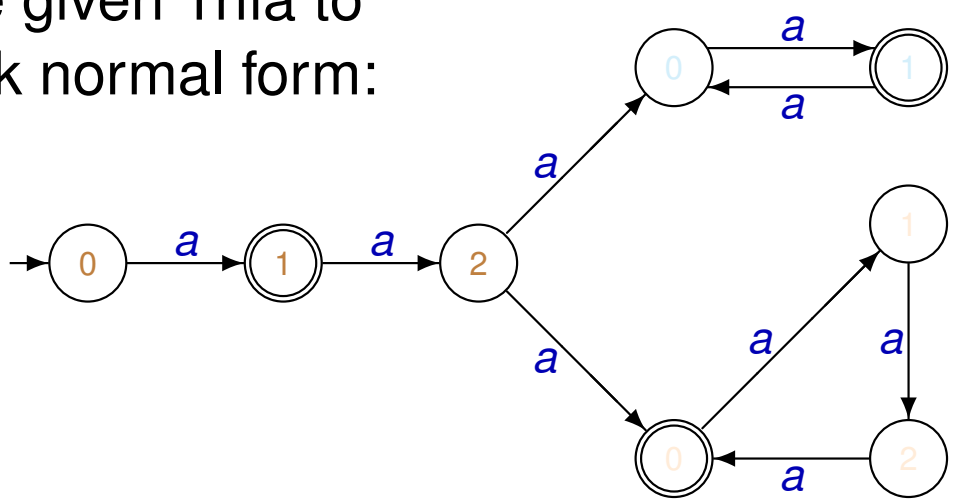

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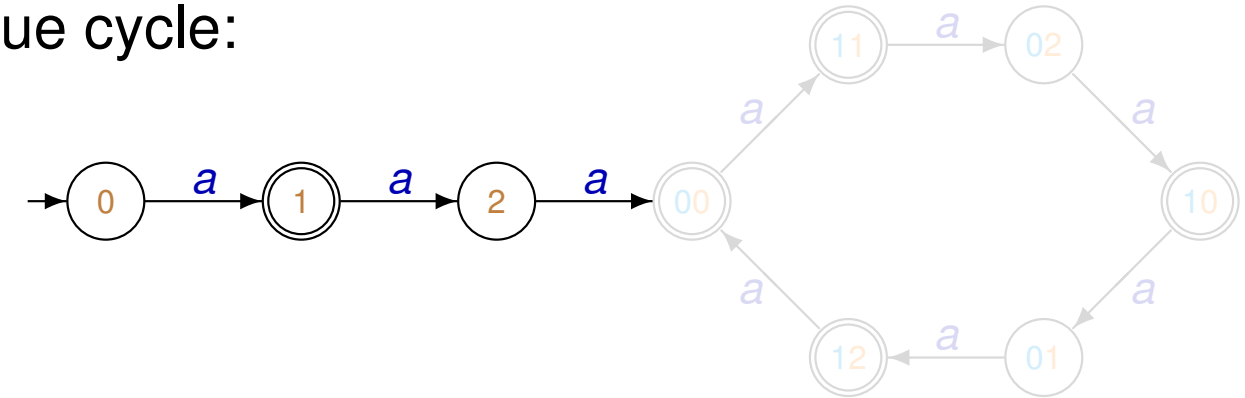

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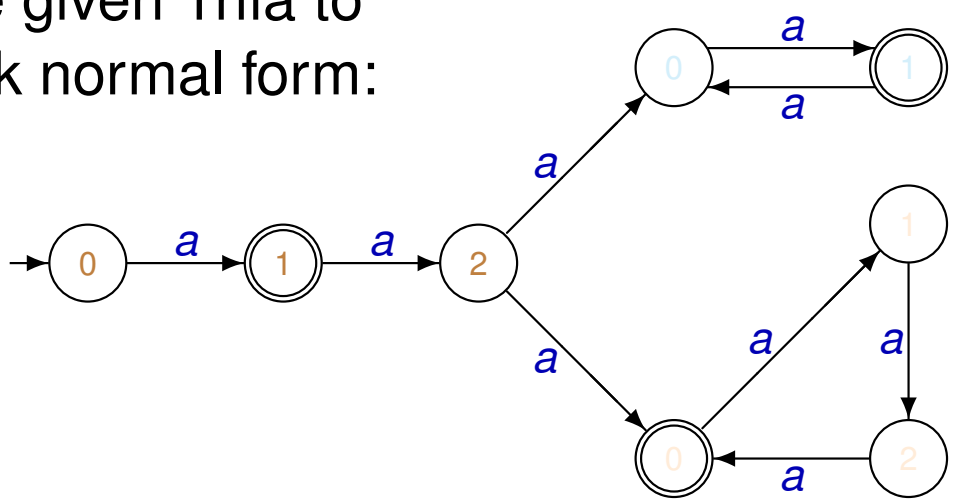

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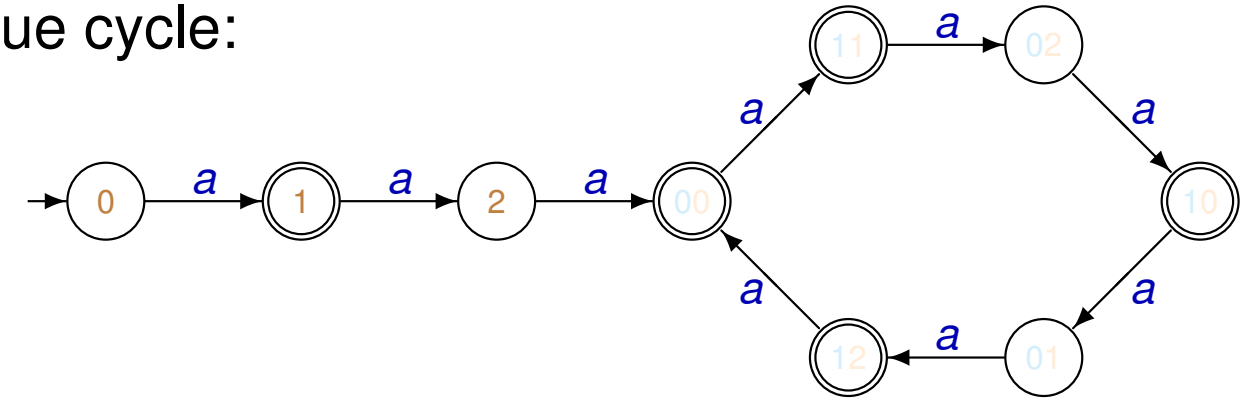

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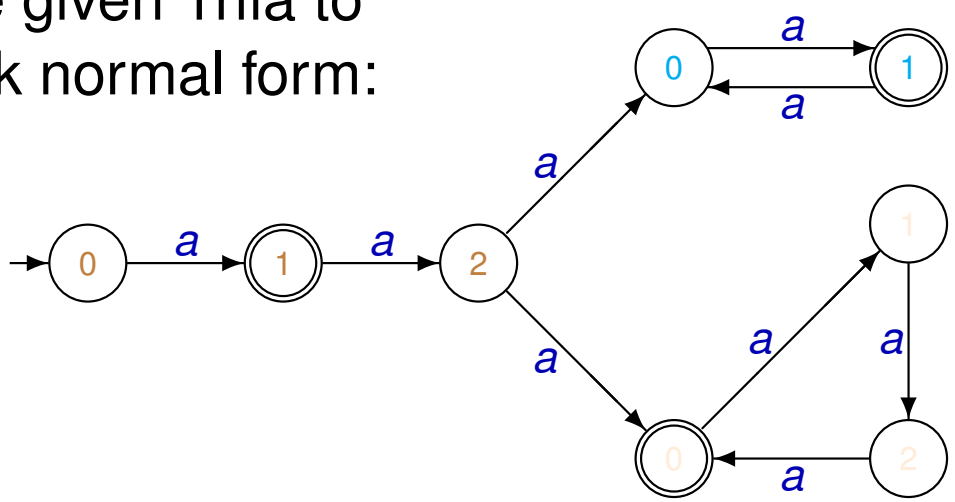

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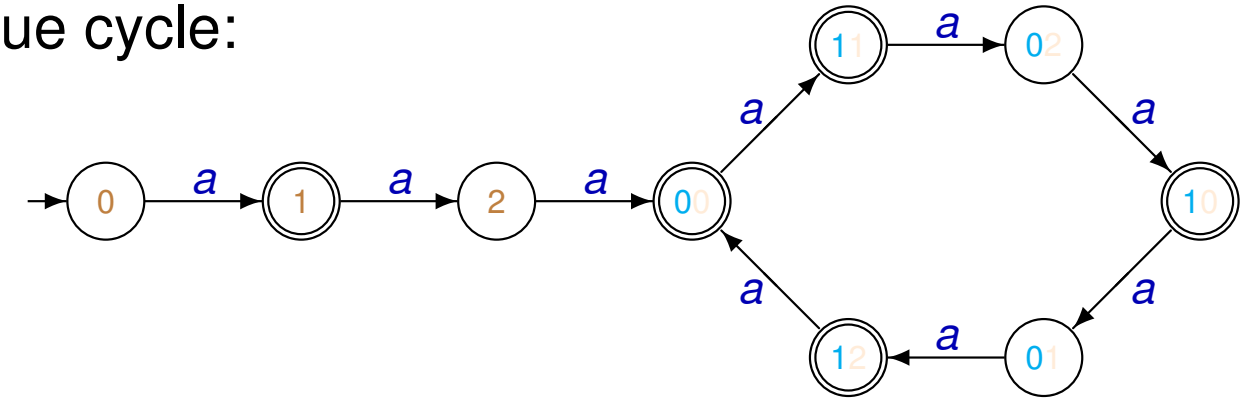

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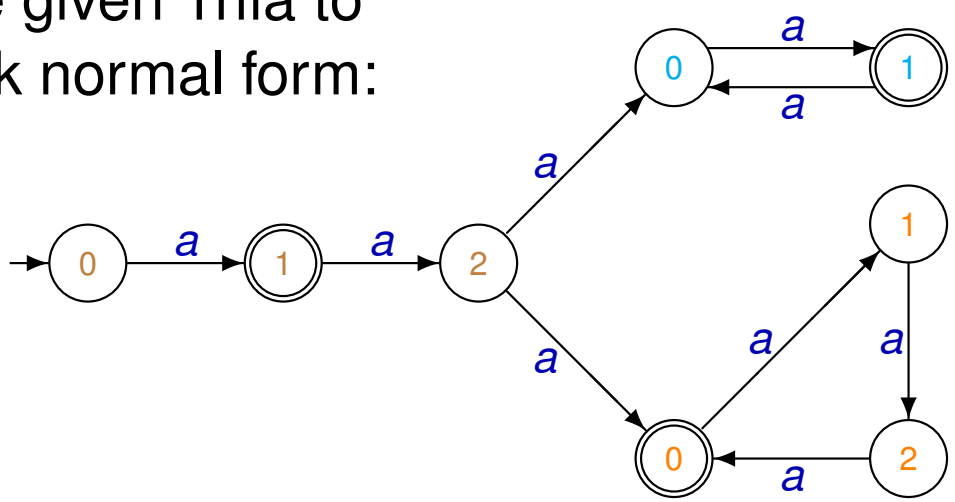

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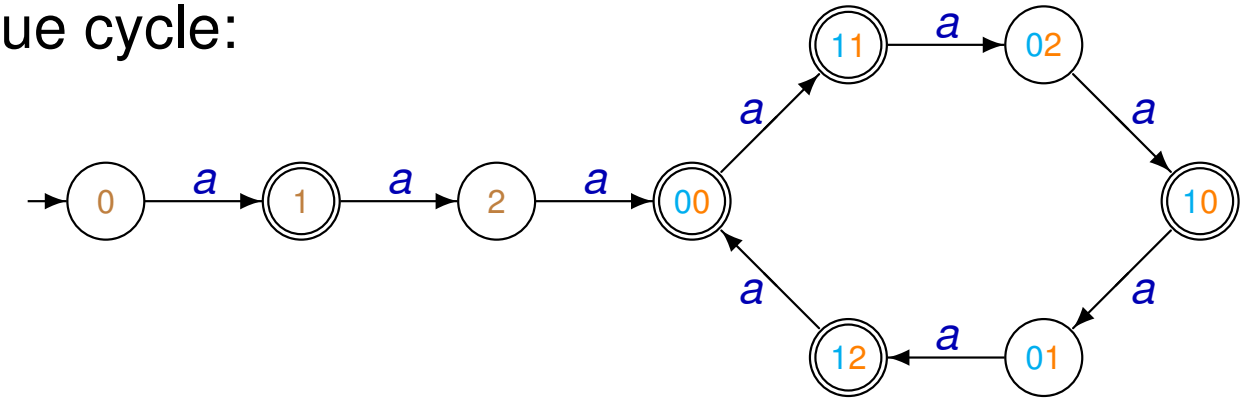

Simulation of unary 1nfa by 1dfa

Given an n-state 1nfa:

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Hence:

- The number of states in the resulting cycle is bouned by $F(n) = \max\{\mathrm{lcm}(x_1, \ldots, x_k) \mid x_1 + \ldots + x_k = n\}$

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- $F(n) = e^{O(\sqrt{n \log n})}$ [Landau 1903]

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1dfa 1nfa

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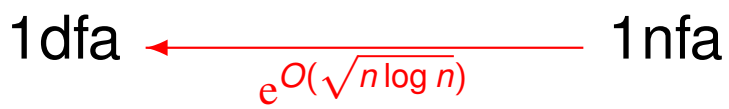

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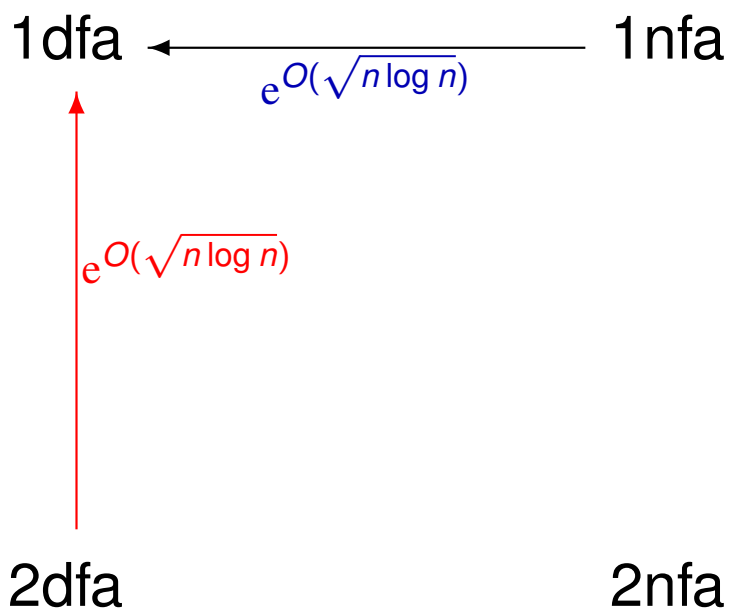

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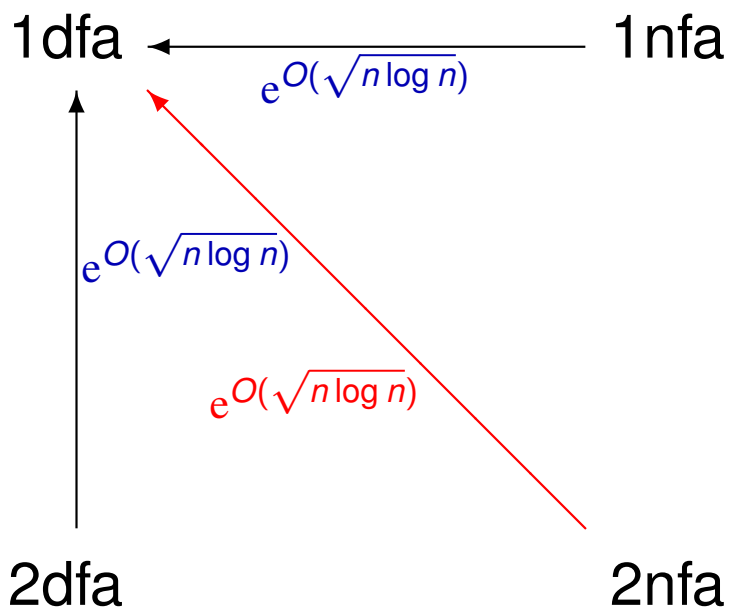

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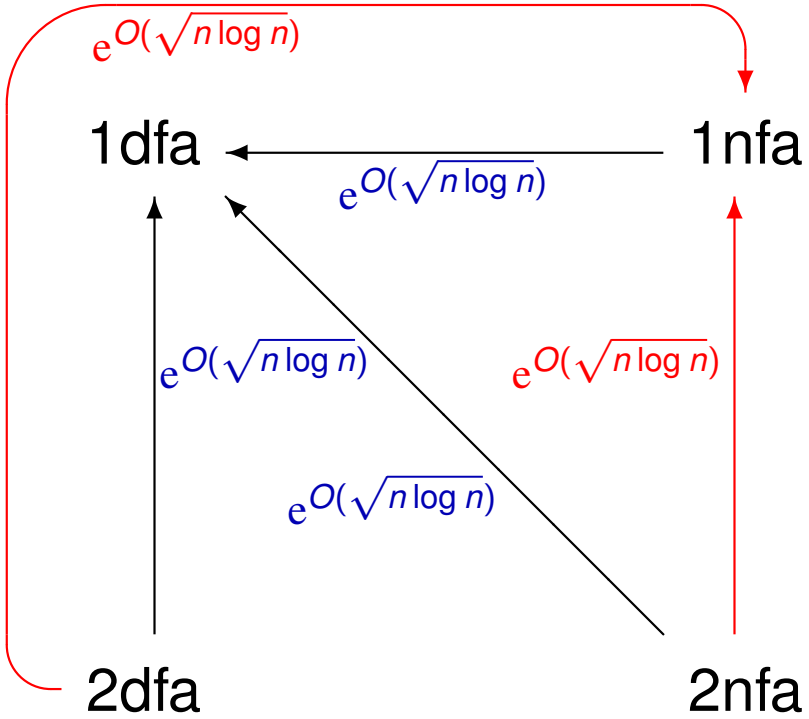

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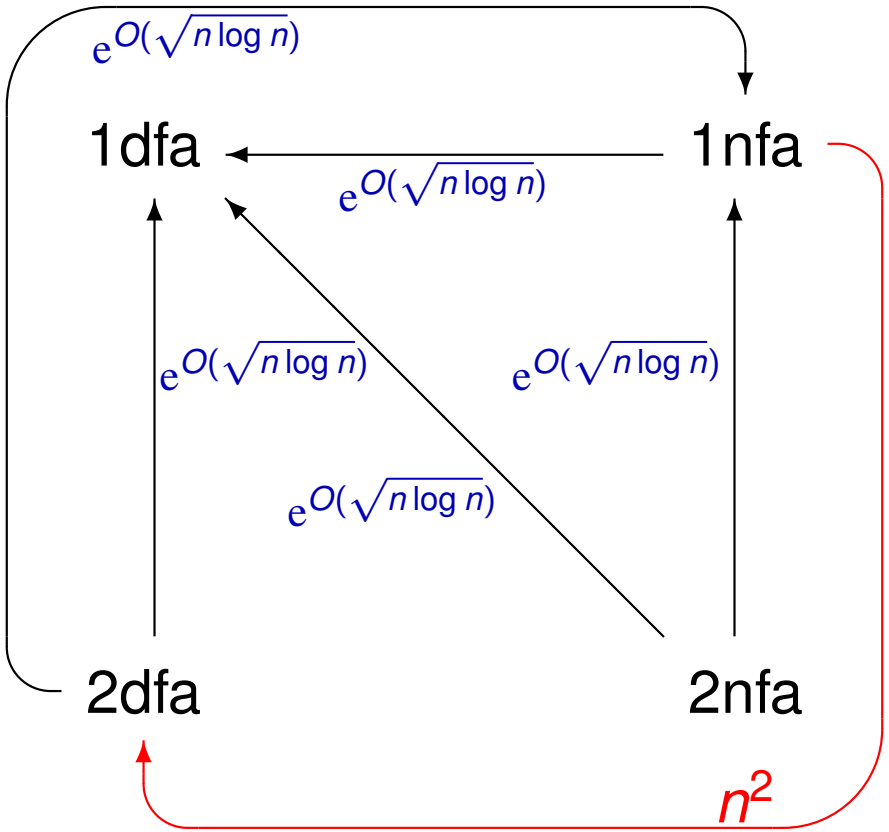

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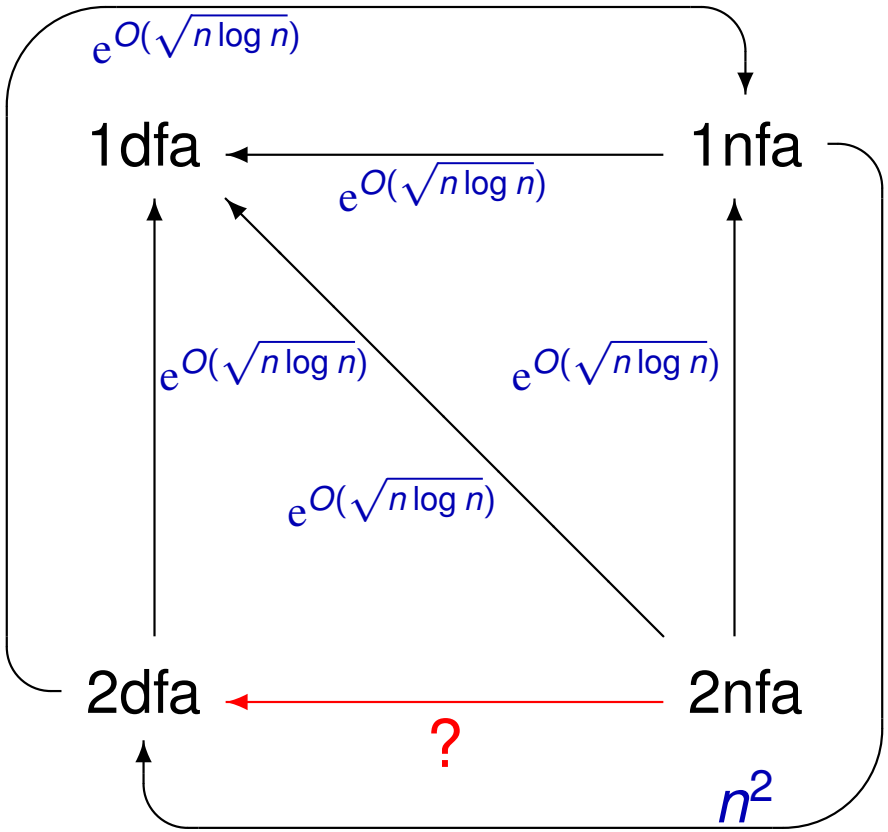

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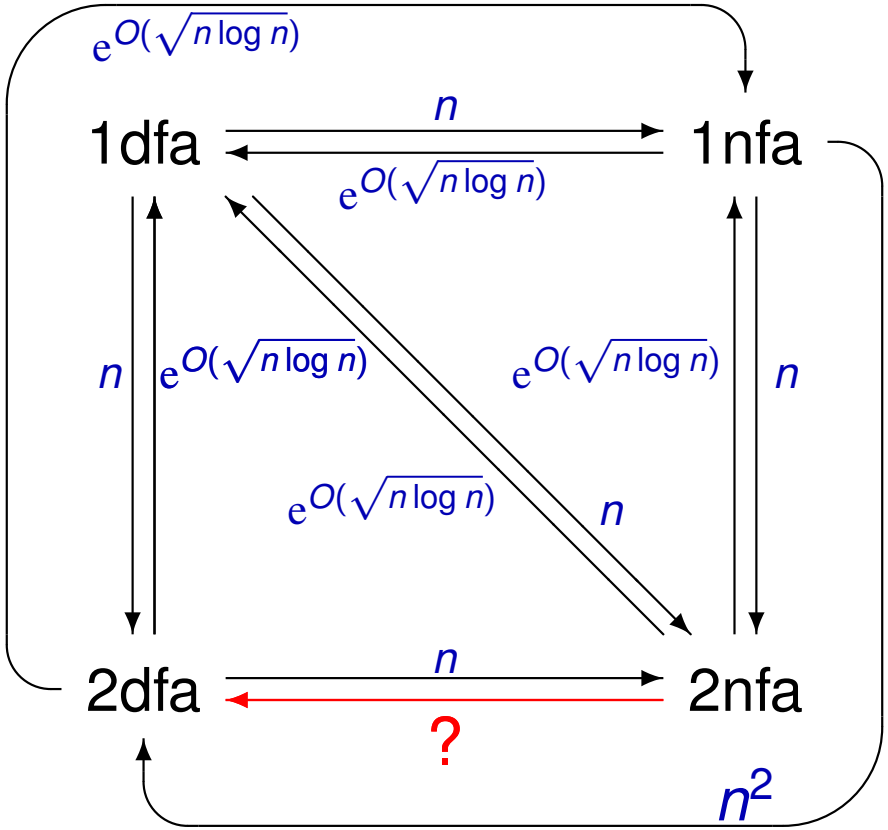

Descriptional complexity of regular languages

- Different variant of finite automata characterize regular languages.
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- Context-free grammars:
  number of variables?

  For $n \geq 1$, consider the language $L_n = (a^n)^*$:
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  - L_n is generated by the grammar with one variable S and the productions

$$S \to a^n \qquad S \to a^n S \qquad S \to \epsilon$$

Thus, the number of variables cannot be a descriptional complexity measure for context-free grammars.
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  W.l.o.g., we can consider only pda's s.t. push operations add exactly one symbol on the pushdown store.

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Given a context-free grammar (or a pushdown automaton) of size n, generating a regular language, how much is big an equivalent finite automaton, wrt n ?

Theorem ([Meyer and Fischer, 1971])

For any recursive function f and arbitrarily large integers n, there exists a cfg G of size n generating a regular language L, s.t. any dfa accepting L must have at least $f(n)$ states.

As a consequence, the trade-off between context-grammars and finite automata is not recursive.
However... The witness language L is defined over a unary alphabet.

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$\Sigma = \{a\}$

Theorem ([Ginsurg and Rice, 1962])

Every unary context-free language is regular.

Hence the classes of unary regular languages and unary *context-free* languages coincide!

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Study the equivalence between unary context-free and regular languages from the descriptional complexity point of view.

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Extension to bounded languages

Bounded languages:

Subsets of $w_1^* w_2^* \dots w_n^*$, for given words $w_1, \dots, w_n$ (*letter bounded* if $w_1, \dots, w_n \in \Sigma$).

- The class bounded regular languages is *properly* included in that of bounded cfl's, e.g., $\{a^n b^n \mid n \geq 0\}$.
- Bounded cfl's can be accepted by *finite-turn* pda's.

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Let M be a unary pda with n states and m stack symbols, s.t. each push adds exactly one symbol.

Using the simulation of unary cfg's by finite automata and the standard transformation of pda's into cfg's, it can be shown that M can be simulated by a 1dfa with with $2^{O(n^4 m^2)}$ states.

What about the deterministic case?

Theorem ([Pighizzini, 2008])

If M is deterministic then it can be simulated by a 1dfa with $2^{O(nm)}$ states.
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Unary dpda's can be exponentially more succinct than dfa's.
Does this is true for *each* unary regular language?

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For $m \geq 0$, let $L_m \subseteq a^*$ be a language accepted by a dfa with 2^m states.
Does there exists an equivalent dpda with $O(m)$ states?

The answer to this question is negative: [Pighizzini, 2008]

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For $m \geq 0$, let $L_m \subseteq a^*$ be a language accepted by a dfa with 2^m states.
Does there exists an equivalent dpda with $O(m)$ states?

The answer to this question is negative: [Pighizzini, 2008]

For each $m > 0$ there exists a language $L_m \subseteq a^*$ s.t.:

- L_m is accepted by a dfa with 2^m states.
- The size of any dpda accepting L_m is at least $d\frac{2^m}{m^2}$, for a constant d.

State complexity of language operations

- Let L_1 and L_2 two regular languages accepted by two automata with s_1 and s_2 states, and op a binary operation preserving the regularity.
- State a tight upper bound $f_{op}(s_1, s_2)$ for the number of the states of an automaton accepting the language $L_1 \, op \, L_2$.
- A same question can be formulated for other kinds of operations.

These problems have been extensively studied in the case of one-way deterministic and nondeterministic automata for "classical" regular operations.

There are interesting results and problems for the complement of languages in the case of nondeterministic machines.

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Complementation of two-way automata

Problem

Given a two-way automaton with n states accepting a language L, find the cost in term of states, of an automaton accepting the complement of L.

- Deterministic case:
  The cost is $4n$, for *any input alphabet*.
  [Geffert, Mereghetti, Pighizzini 2007]
  (note that 2dfa's can have infinite computations)
- Nondeterministic case:
  The cost is polynomial for a *unary alphabet*.
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Relationship with the Sakoda&Sipser question

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Hence:

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Theorem

If the complementation of 2nfa's requires an exponential number of states then the gap between 2nfa's and 2dfa's is exponential.

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Complementation of one-way automata

- Deterministic case: trivial
- Nondeterministic case:

  Worst case: [Jiraskova, 2005]

  For each integer $n \geq 1$ there exists a language L accepted by a 1nfa with n states such that each 1nfa accepting L^c needs 2^n states.

  Best case: [Mera, Pighizzini, 2005]

  For each integer $n \geq 1$ there exists a language L such that:
  – the smallest 1nfa accepting L has n states
  – the minimum 1dfa's accepting L and L^c have 2^n states
  – the smallest 1nfa accepting L^c has at most $n + 1$ states.

  Remarks:

  L is a witness of the gap between 1nfa's and 1dfa's.
  Both L and L^c have "small" 1nfa but "large" 1dfa's.

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Complementation of 1nfa: unary case

The last result does not hold in the case of unary languages:

For each unary language L such that:

- L is accepted by a 1nfa with n states
- L the minimum 1dfa accepting L has $e^{O(\sqrt{n \log n})}$ states

Then: [Mera, Pighizzini, 2005]

Each 1nfa accepting L^c should have at least n states.

Hence, if L has a "small" 1nfa and a "large" 1dfa, i.e., it is a witness of the gap between unary 1nfa's and 1dfa's, then the nondeterminism does not help in the recognition of L^c.

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