Eliminating the nondeterminism from two-way finite automata

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Università degli Studi di Milano

Frankfurt – December 11, 2007

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Outline of the talk

- Finite state automata and their descriptional complexity
- Eliminating nondeterminism using two–way motion: the problem of Sakoda and Sipser
- Unary automata, unary languages and their properties

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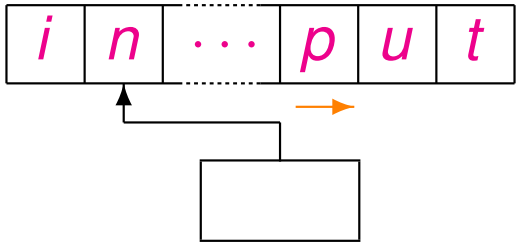

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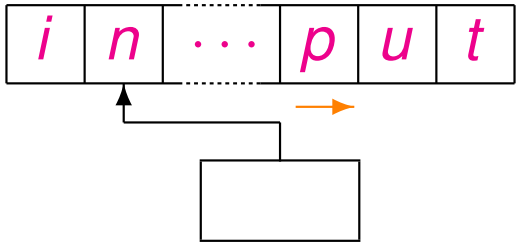
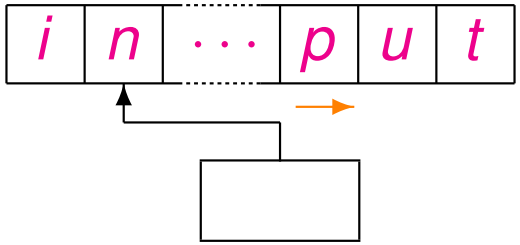


Base version:

One-way determistic finite automata (1dfa)

- one-way input tape
- deterministic

Finite state automata



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- non determinism
- two-way input head motion
- alternation
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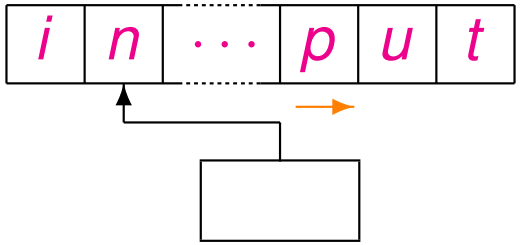

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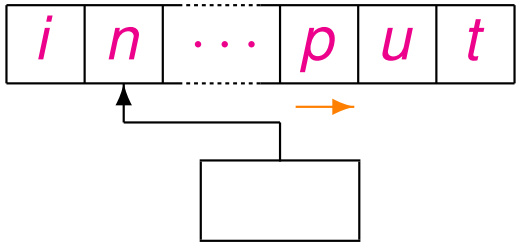

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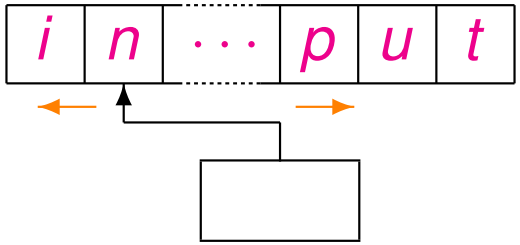

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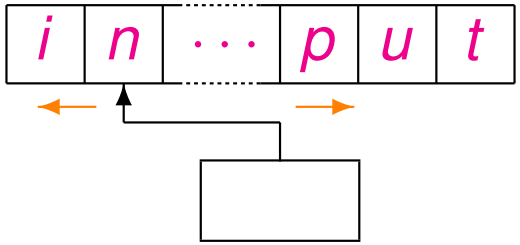

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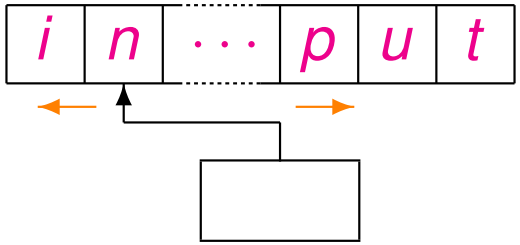

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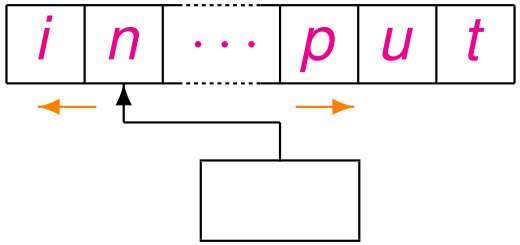

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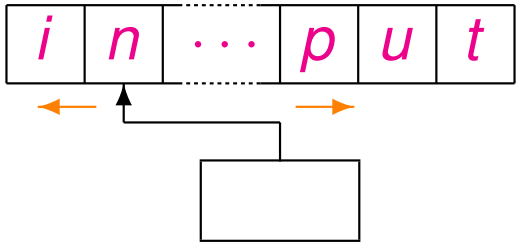

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- Each n-state 1nfa can be simulated by a 1dfa with 2^n states (subset construction) [Rabin and Scott '59], and:
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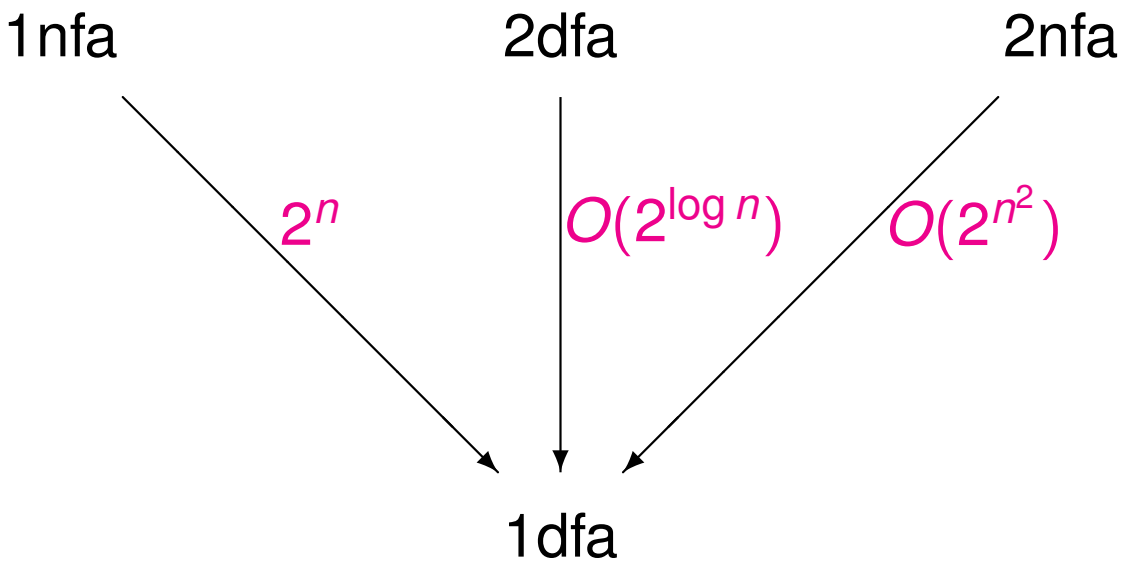

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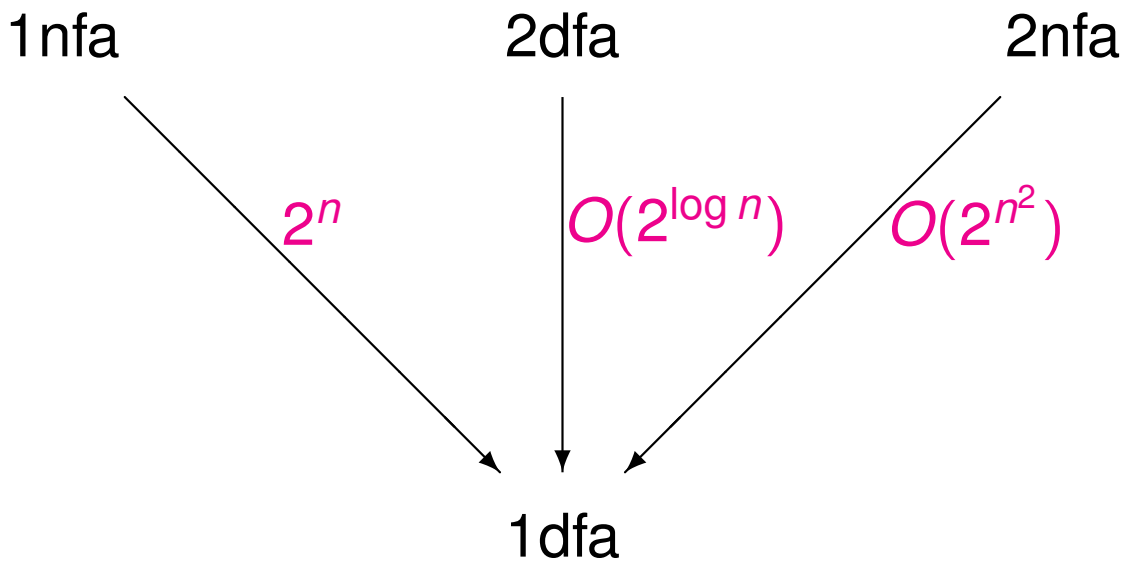

How much two-way motion is useful in the elimination of the nondeterminism?

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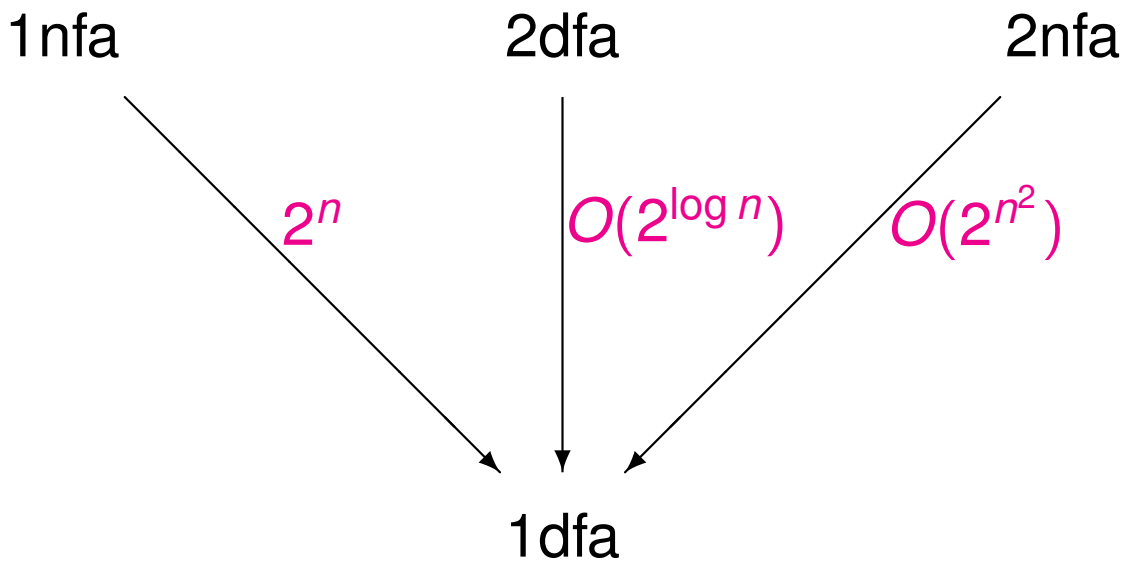

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Find the costs, in terms of states, of the optimal simulations of

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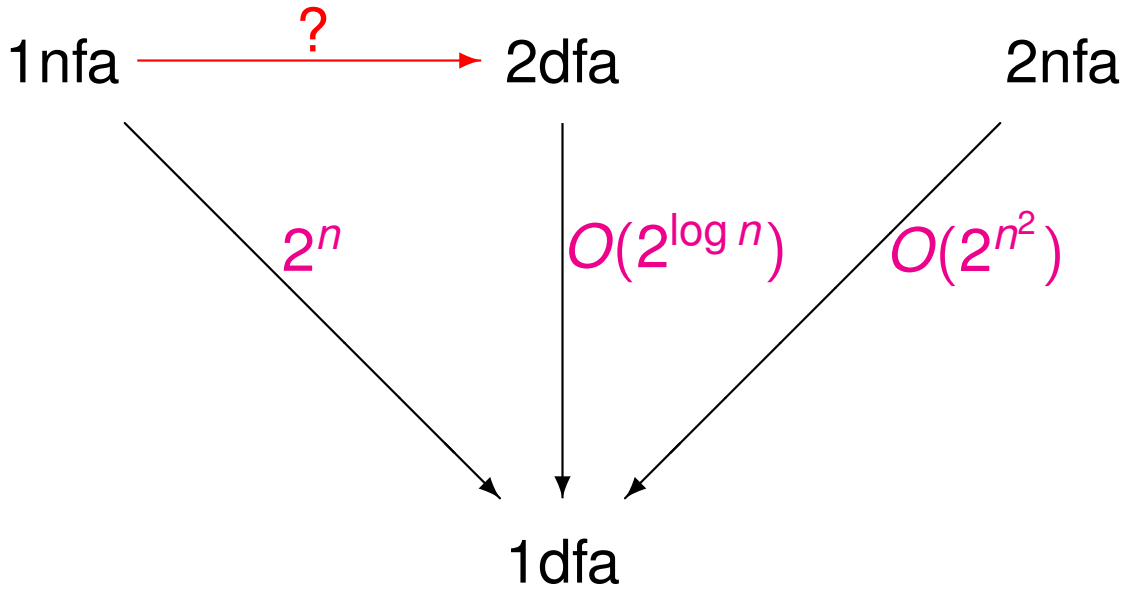

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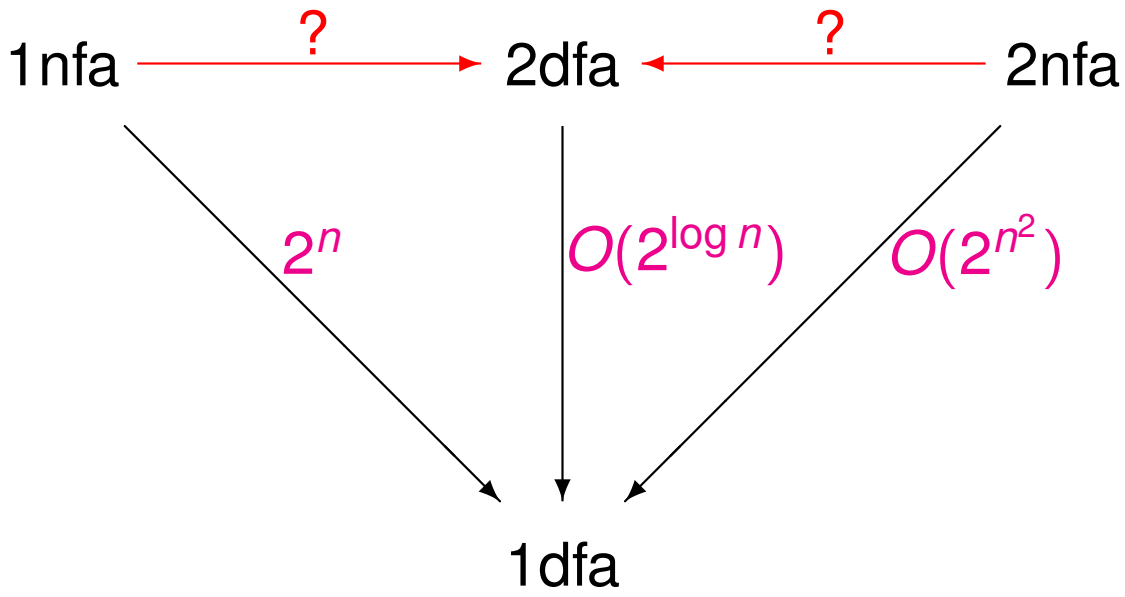

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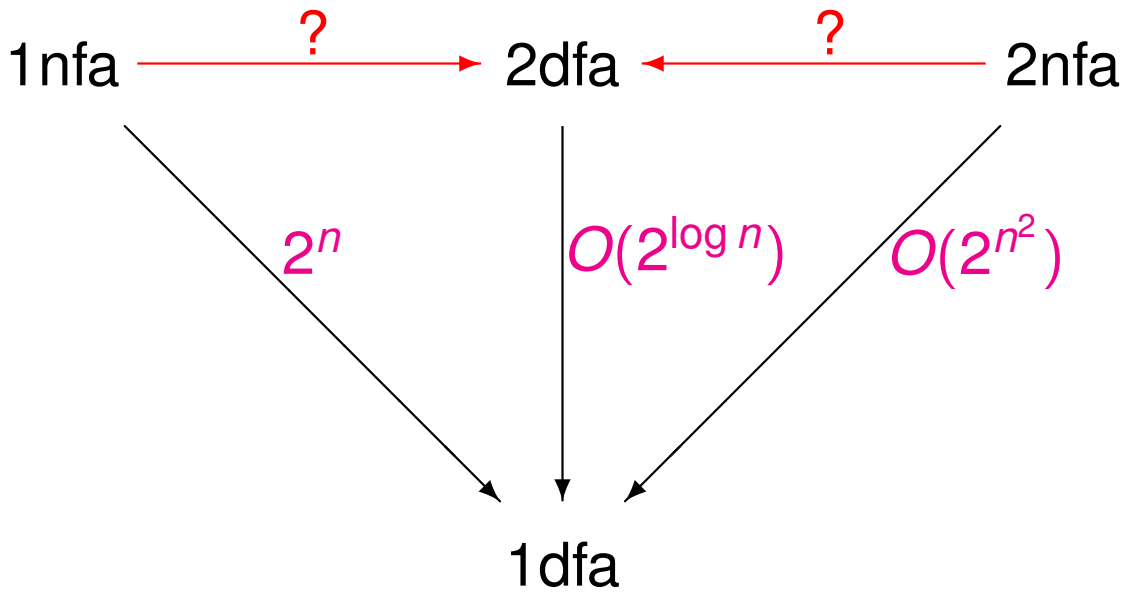

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Sakoda and Sipser question: complete languages

Theorem (Complete languages for 1nfa vs 2dfa)

There exists a sequence of languages $< B_1, B_2, \ldots, B_n, \ldots >$ *s.t. for each integer* $n \geq 1$*:*

- B_n *is accepted by a 1nfa with n states, and*
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Remark: the second condition implies that
the simulation of 1nfa by 2dfa is polynomial iff each B_n is accepted by a 2dfa with a polynomial (in n) number of states.

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Polynomial lower bounds have been proved for the cost $c(n)$ of simulation of 1nfa by 2dfa. In particular:

- $c(n) \in \Omega(\frac{n^2}{\log n})$ [Berman and Lingas 1977]
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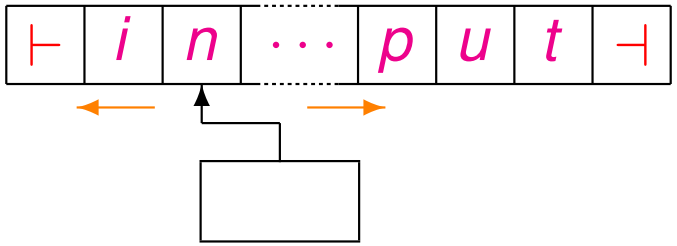

- Input string surrounded by two *endmarkers* $\vdash$ and $\dashv$
- Transition function

$$\delta : Q \times (\Sigma \cup \{\vdash, \dashv\}) \to 2^{Q \times \{-1,0,+1\}}$$

where $(p, d) \in \delta(q, a)$ means that the automaton

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- moving the input head in the direction specified by d: if $d = -1$ left; if $d = 0$ stationary; if $d = +1$ right; if $a = \vdash$ ($a = \dashv$, resp.) then $d \neq -1$ ($d \neq +1$, resp.)

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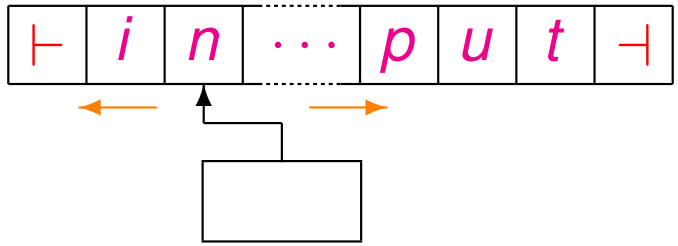

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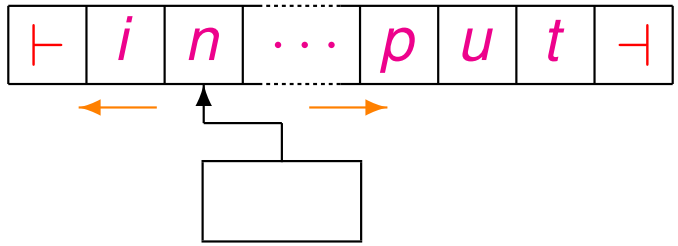

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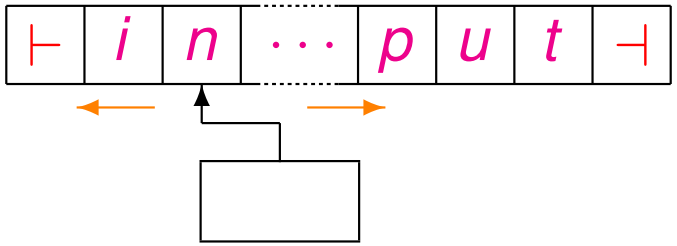

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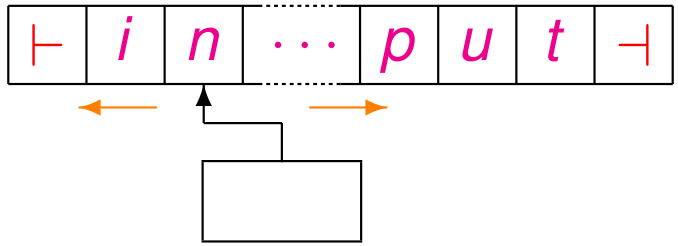

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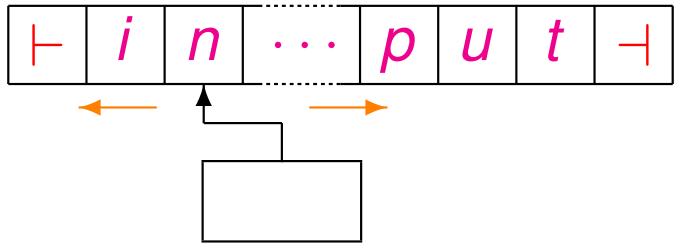

- Input string surrounded by two *endmarkers* $\vdash$ and $\dashv$
- Transition function

$$\delta : Q \times (\Sigma \cup \{\vdash, \dashv\}) \to 2^{Q \times \{-1,0,+1\}}$$

where $(p, d) \in \delta(q, a)$ means that the automaton

- in the state q, with the input head scanning the symbol $a \in \Sigma \cup \{\vdash, \dashv\}$
- can make a transition to the state p
- moving the input head in the direction specified by d:

  $d = -1$: left; $d = 0$: stationary; $d = +1$: right.

  If $a = \vdash$ ($a = \dashv$, resp.) then $d \neq -1$ ($d \neq +1$, resp.)

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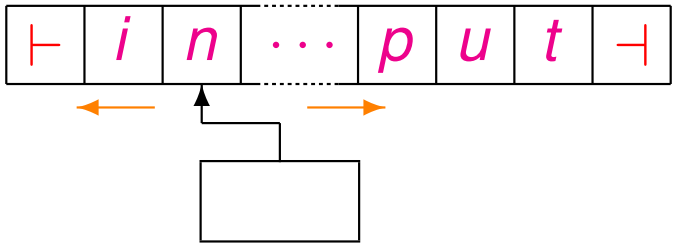
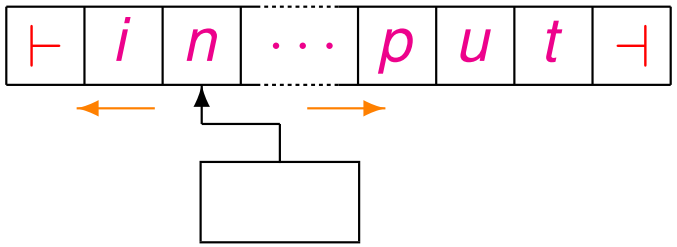


- The automaton is *deterministic* if and only if $\#\delta(q, a) \leq 1$ for each $q \in Q$, $a \in \Sigma \cup \{\vdash, \dashv\}$

Sweeping automata



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A two-way automaton A is said to be *sweeping* if and only if

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Note: the computation of a sweeping automaton is a sequence of left-to-right and right-to-left traversals of the input

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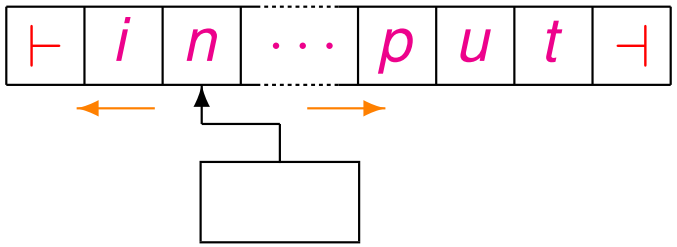

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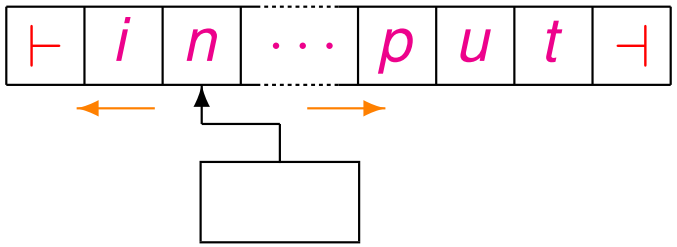

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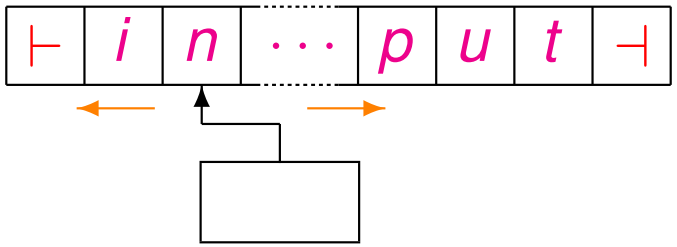

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Theorem ([Sipser 1980])

There exists a family of languages $< B_1, B_2, \ldots, B_n, \ldots >$ s.t. for each integer $n \geq 1$:

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However, 2dfa can be exponentially more succinct than sweeping automata:

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Problem ([Sakoda and Sipser 1978])

Find the costs, in terms of states, of the optimal simulations of

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- Very difficult in its general form
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Input alphabet $\Sigma = \{a\}$

Theorem

$L \subseteq \{a\}^*$ is regular iff $\exists \mu \geq 0, \lambda \geq 1$ s.t.

$$\forall n \geq \mu : a^n \in L \text{ iff } a^{n+\lambda} \in L.$$

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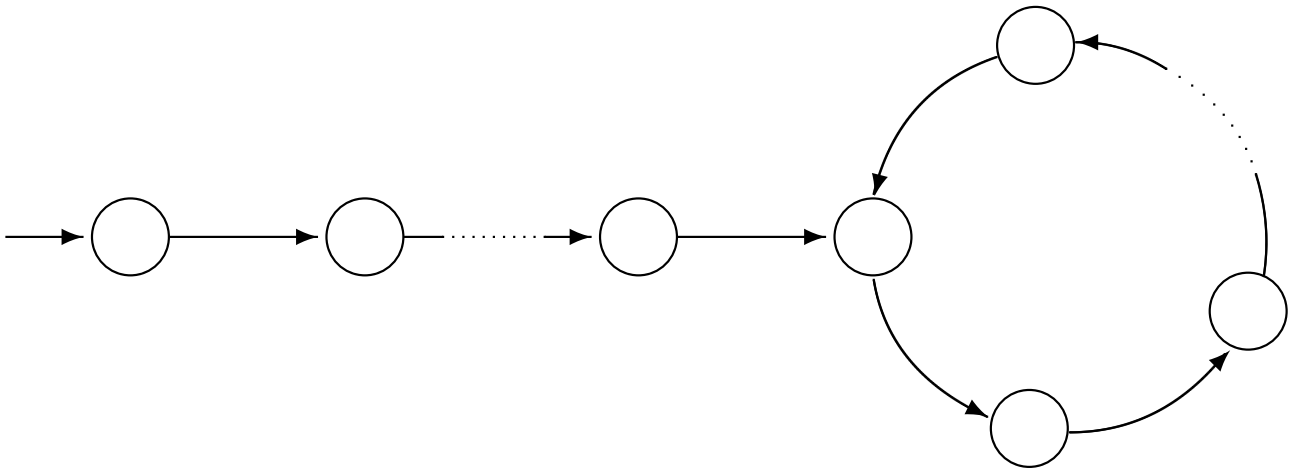

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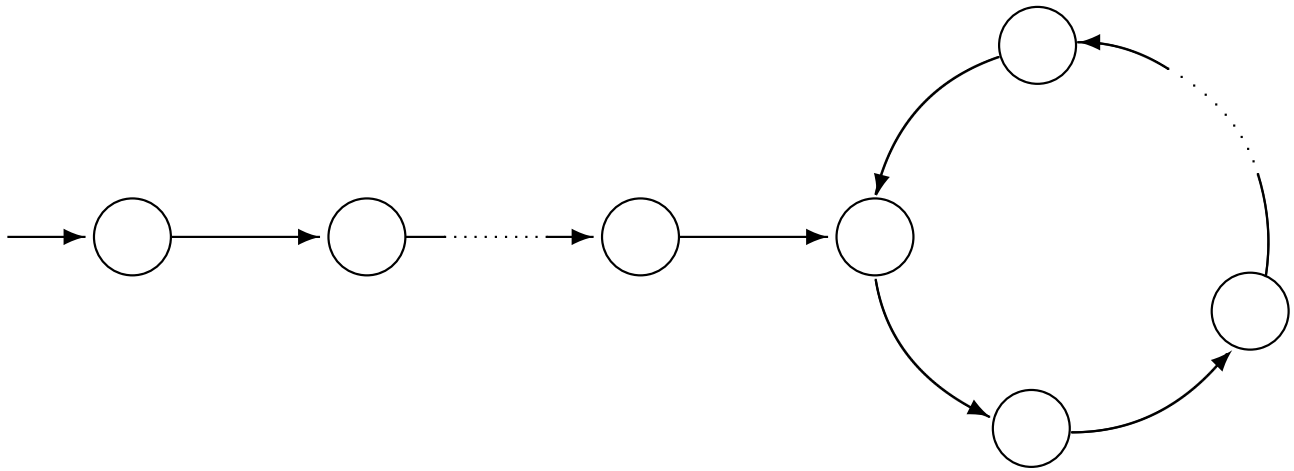

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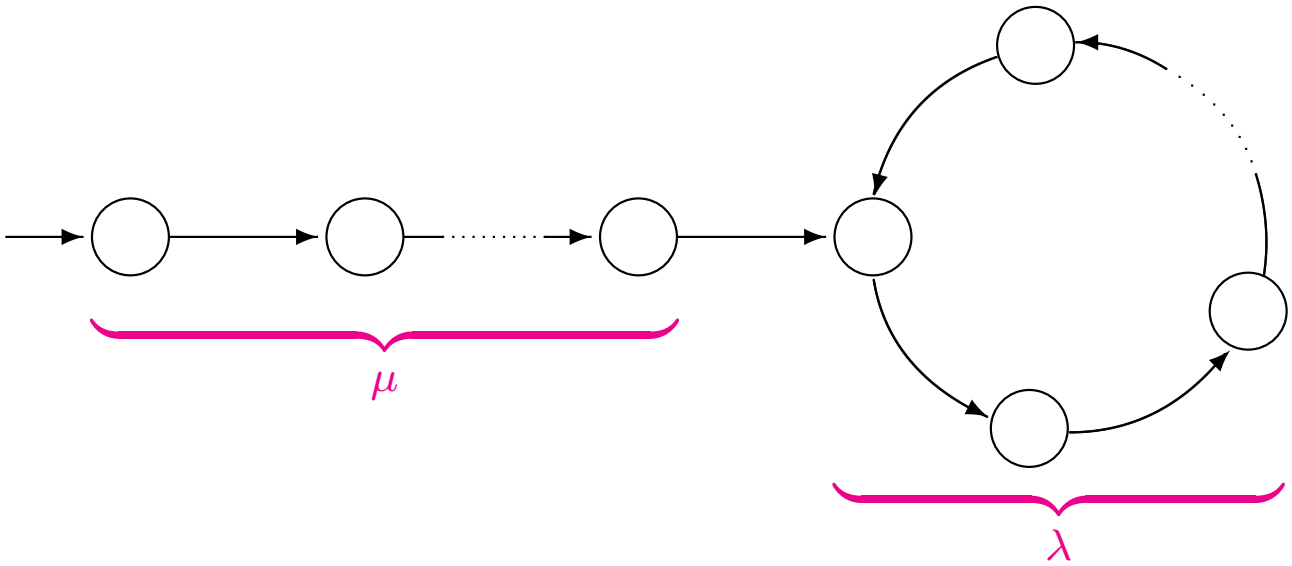

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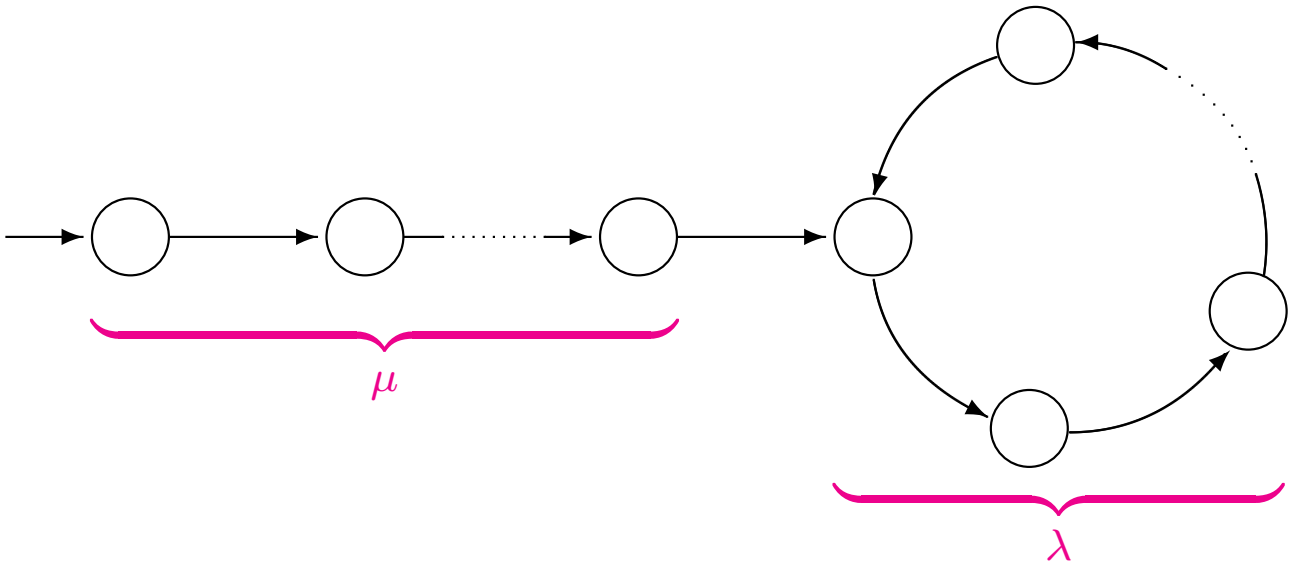

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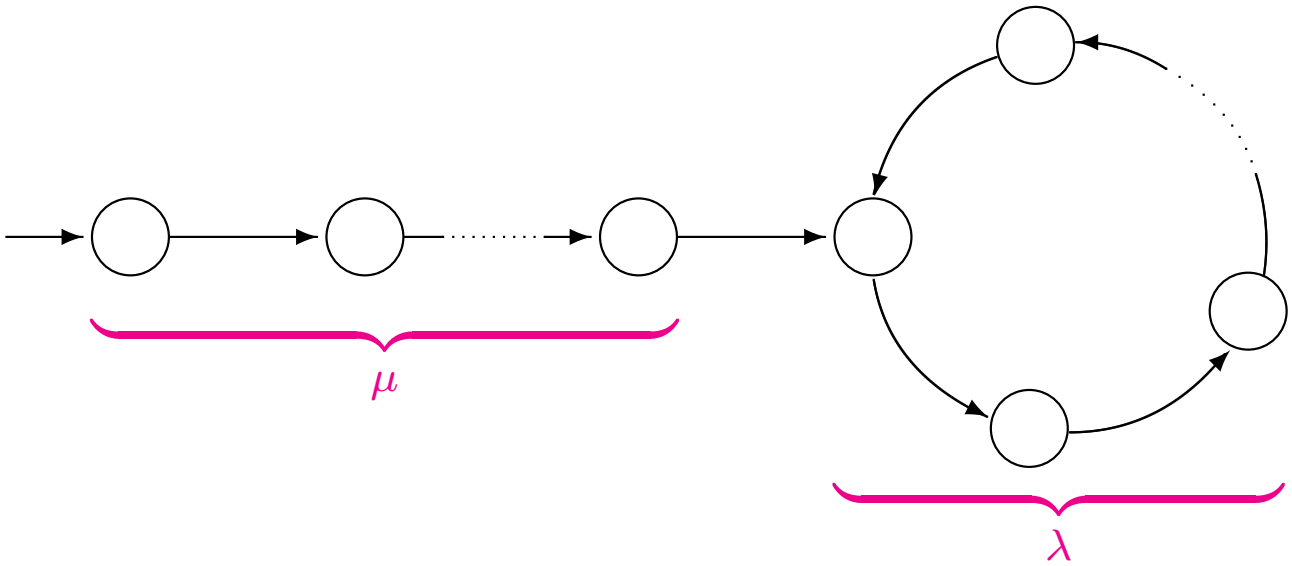

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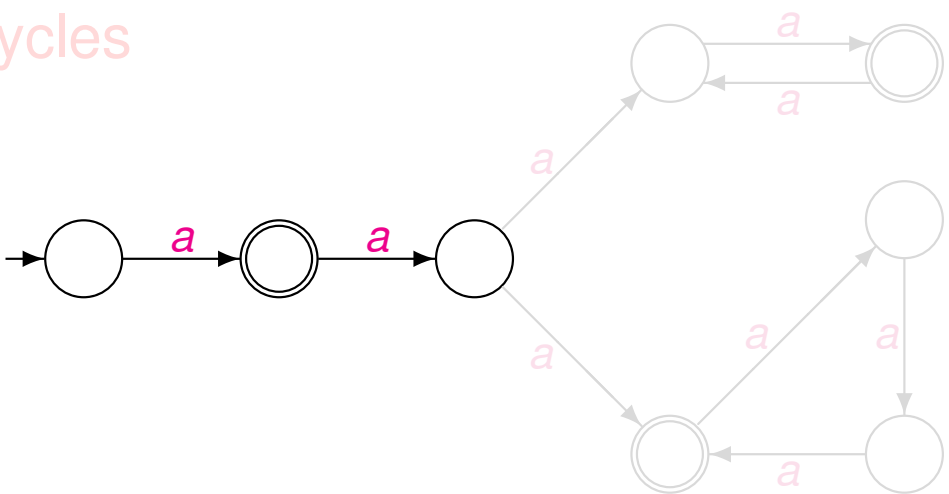

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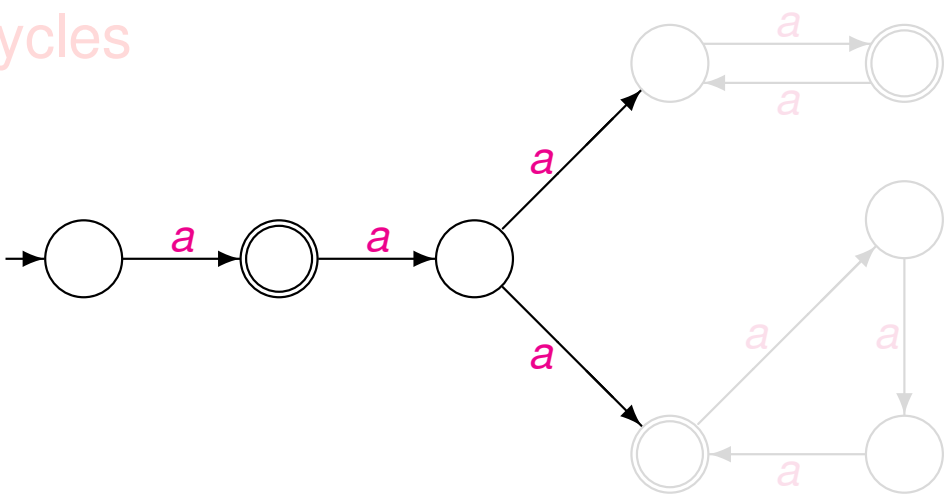

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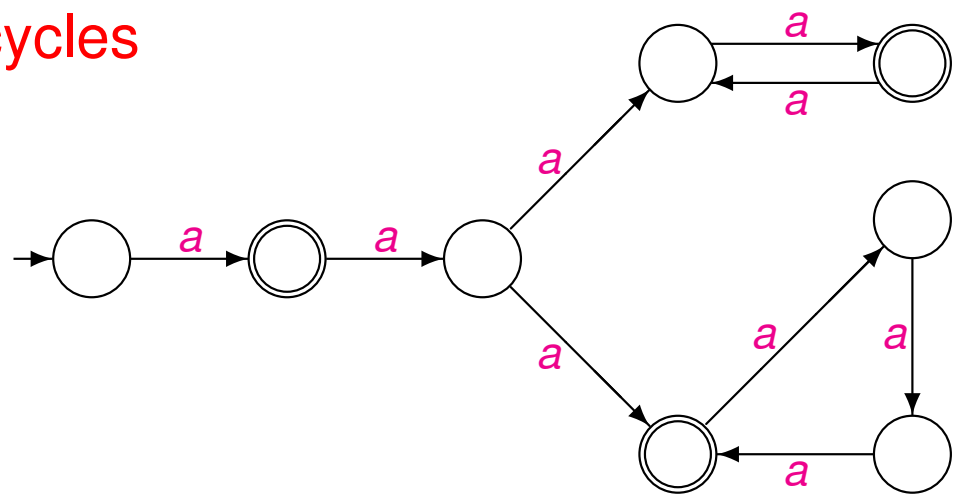

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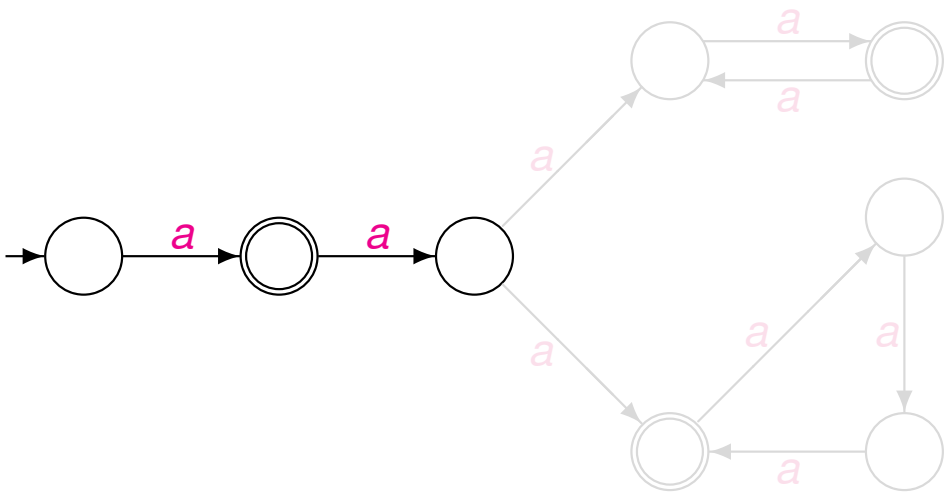

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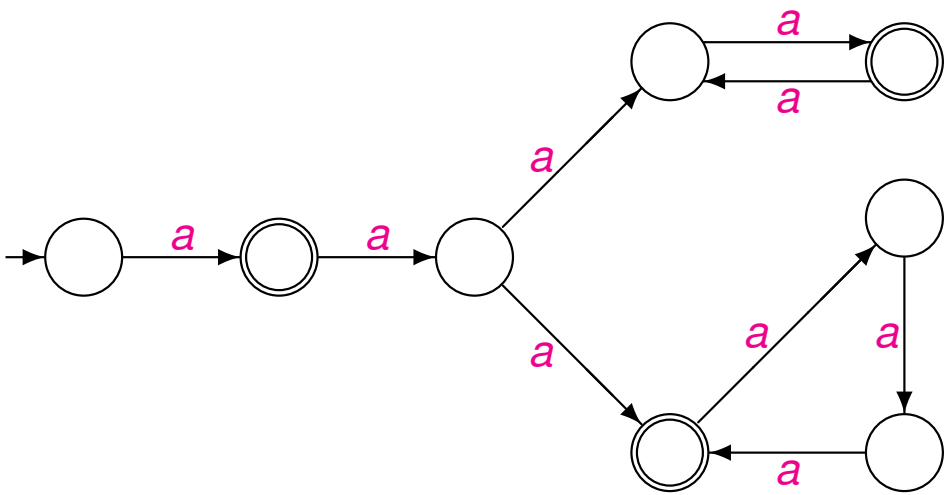

Unary nondeterministic automata

How to eliminate the nondeterminism from a unary 1nfa?

1. Convert the given 1nfa to the Chrobak normal form:

2. Copy the initial path
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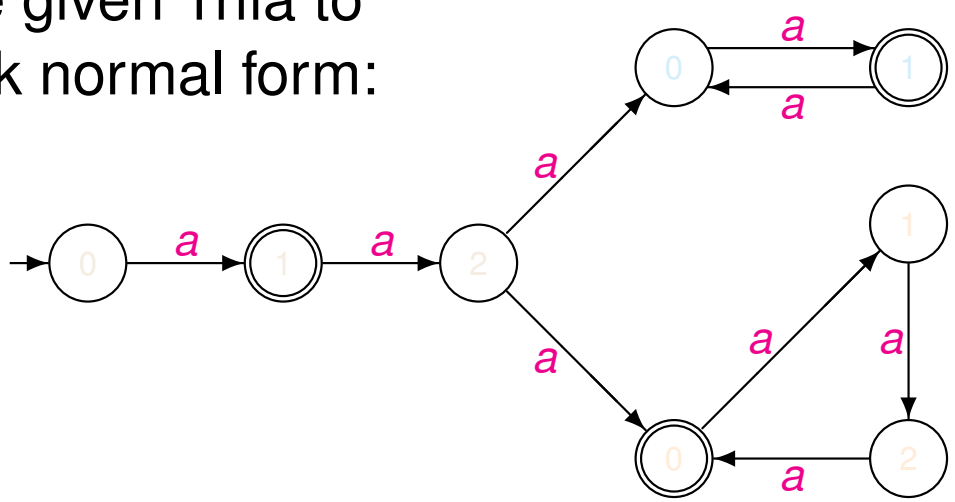

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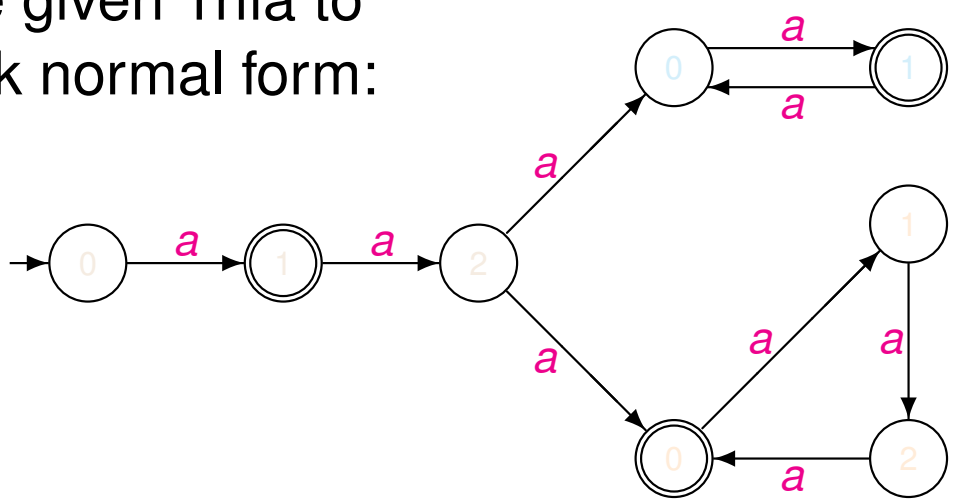

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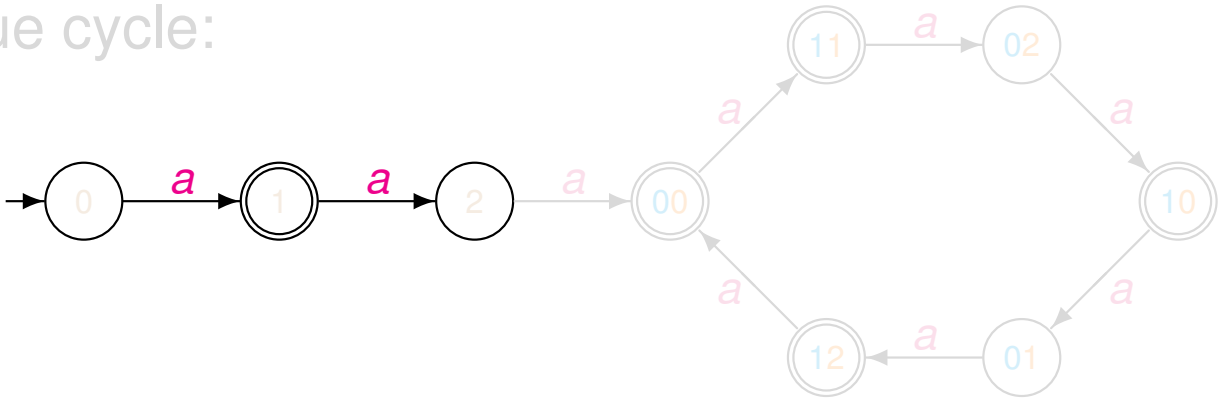

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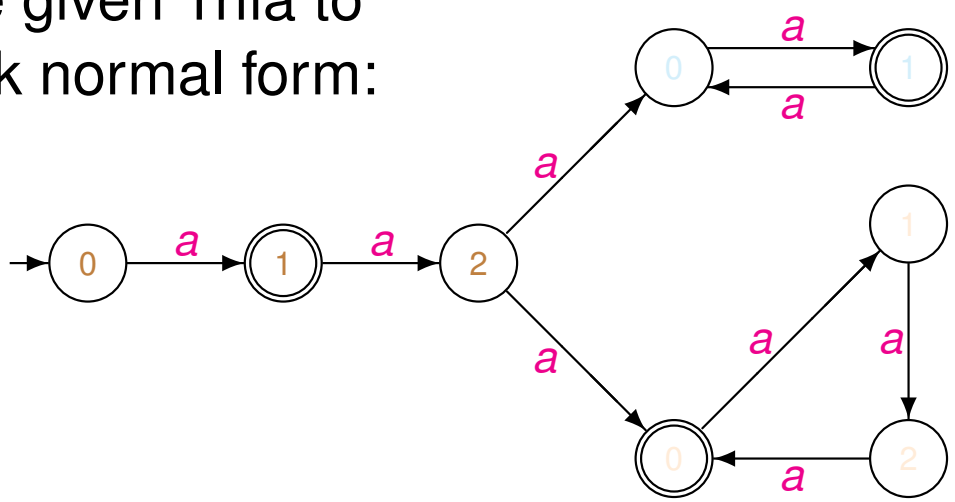

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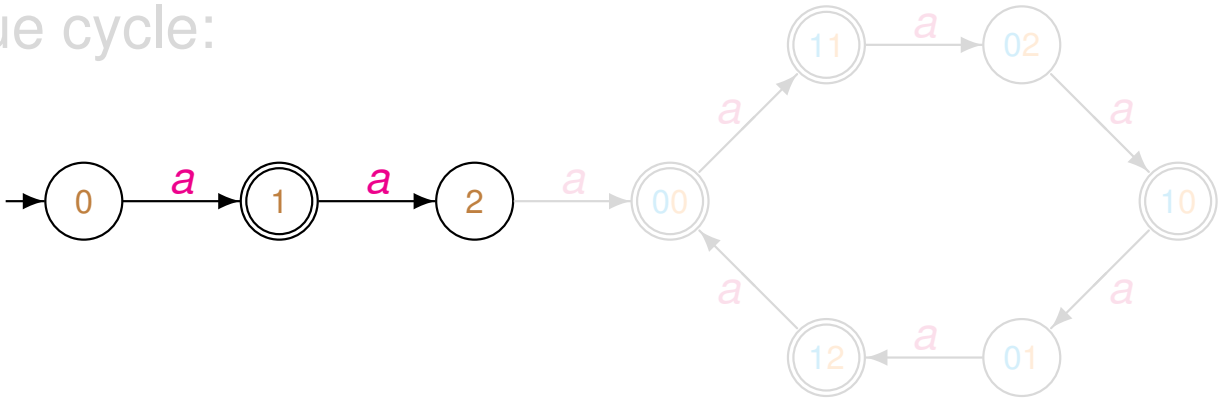

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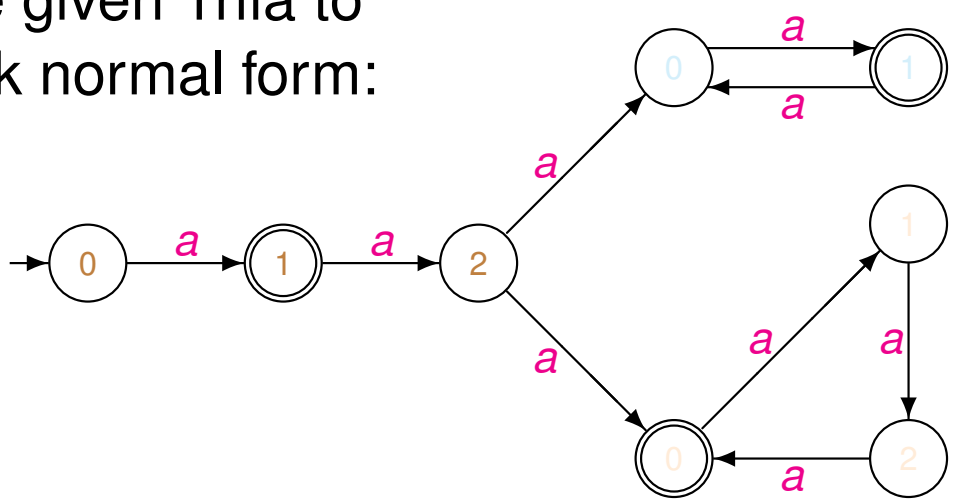

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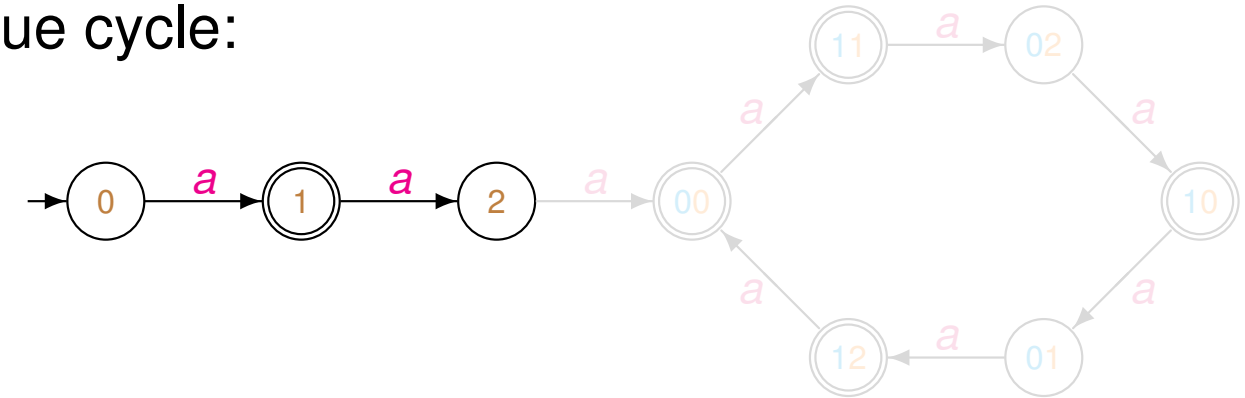

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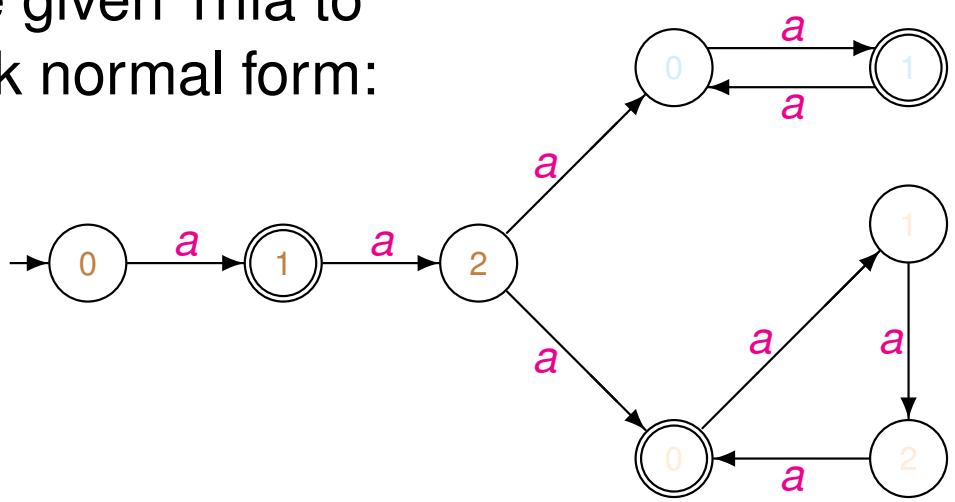

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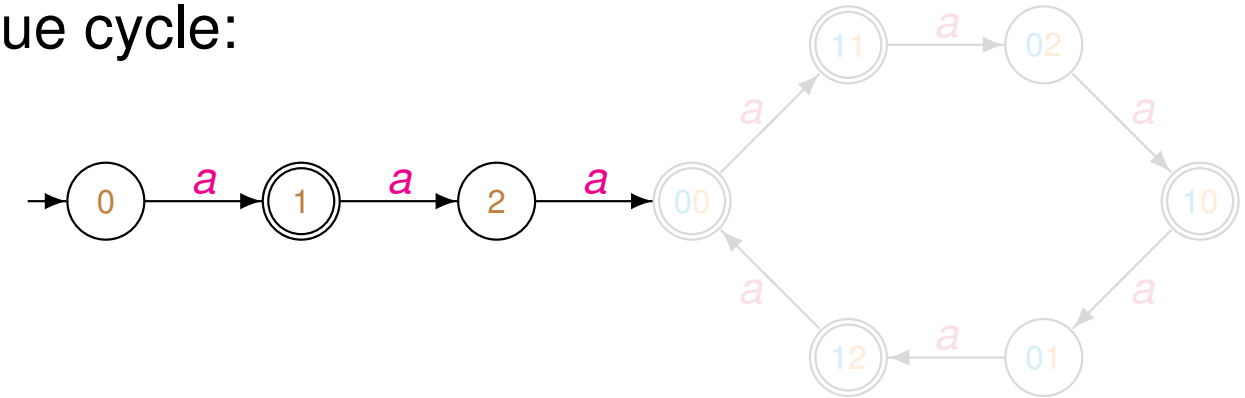

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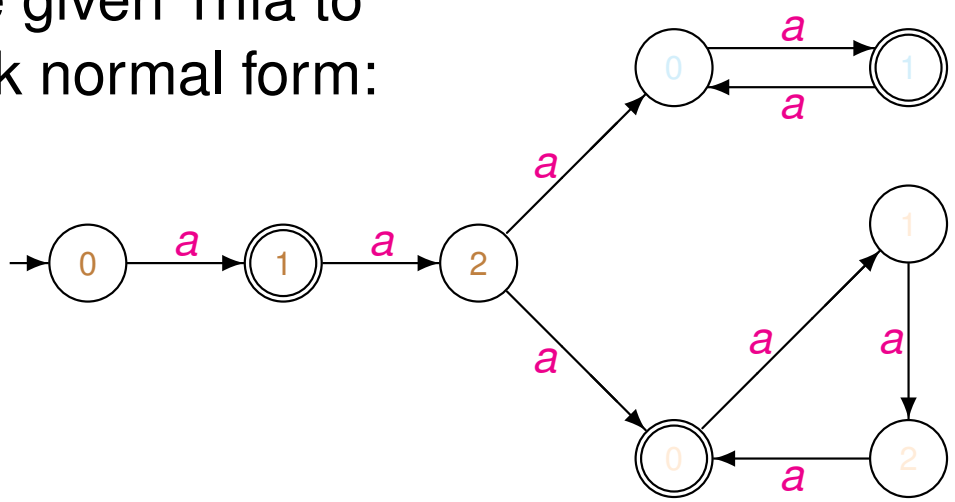

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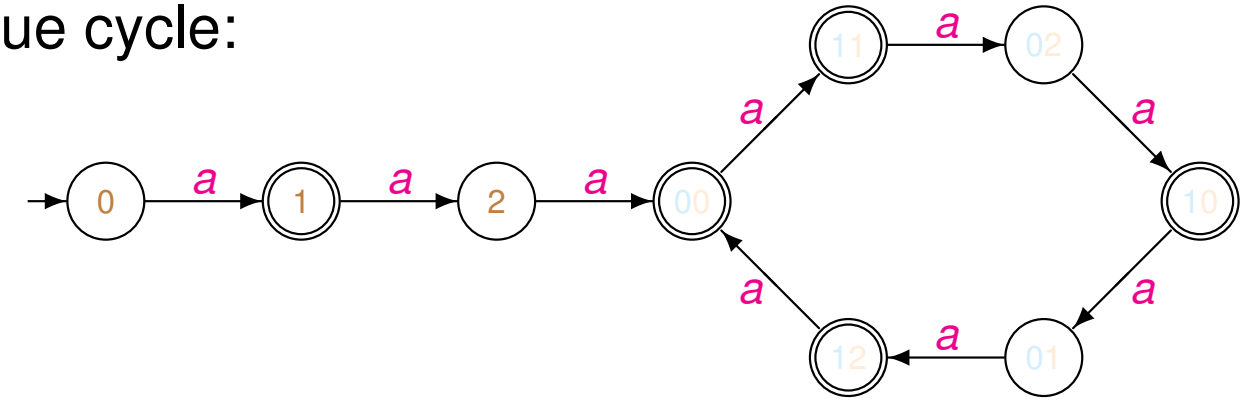

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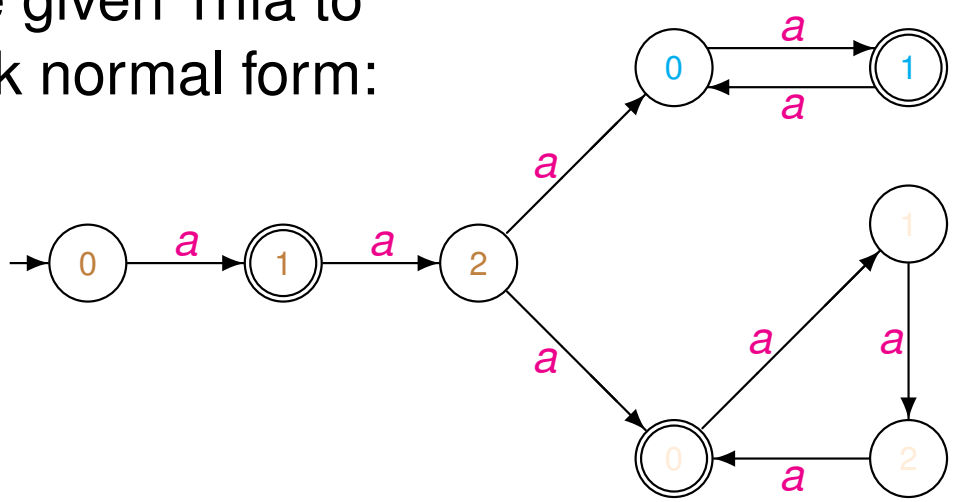
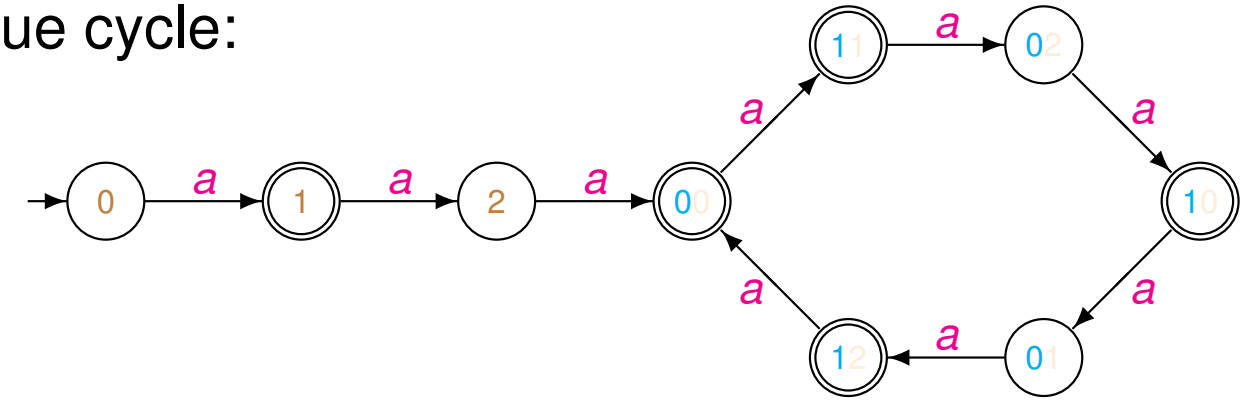
How to eliminate the nondeterminism from a unary 1nfa?

1. Convert the given 1nfa to the Chrobak normal form:



2. Copy the initial path
3. Replace the set of cycles with a unique cycle:



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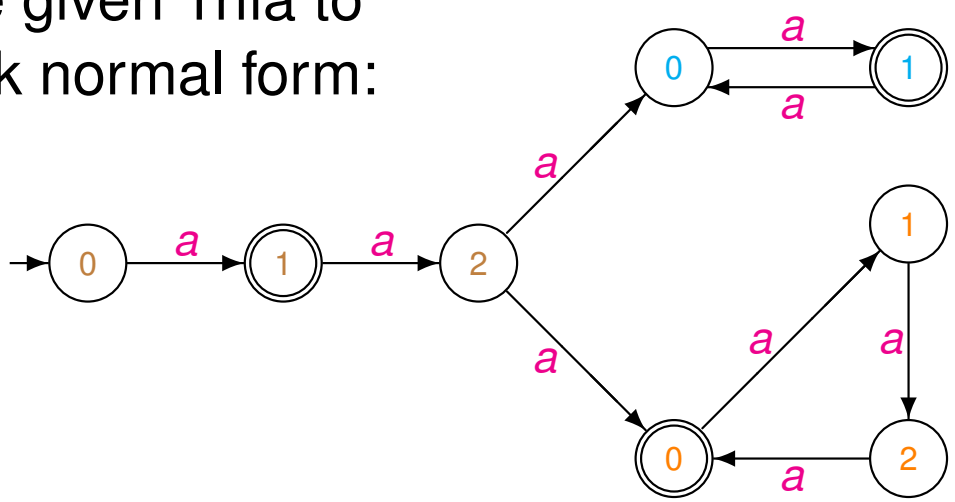

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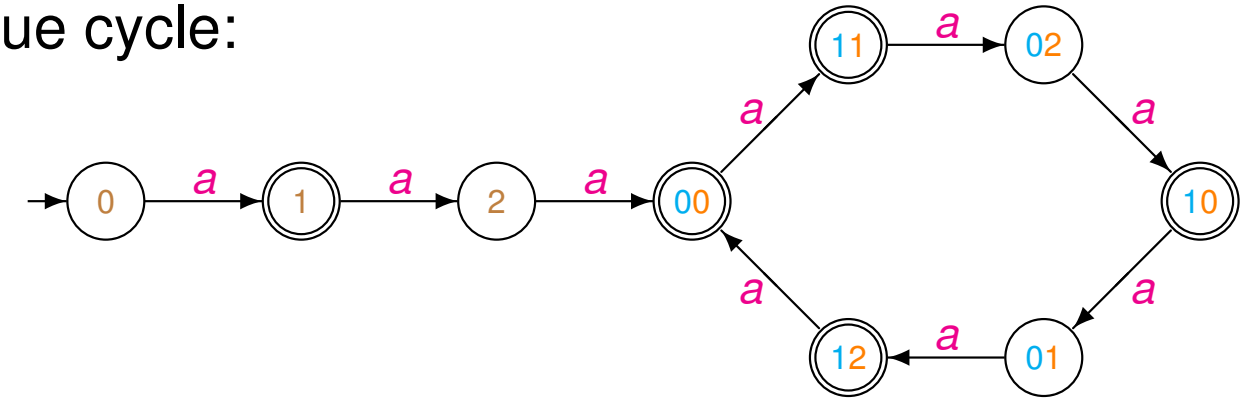

Simulation of unary 1nfa by 1dfa

Given an n-state 1nfa:

- Convert it in Chrobak normal form:
  - Initial path of $O(n^2)$ states
  - Cycles of lenghts $\lambda_1, \ldots, \lambda_k$, with $\lambda_1 + \ldots + \lambda_k \leq n$
- The cycles are replaced by a unique cycle of length $\mathrm{lcm}(\lambda_1, \ldots, \lambda_k)$

Hence:

- The number of states in the resulting cycle is bouned by $F(n) = \max\{\mathrm{lcm}(x_1, \ldots, x_k) \mid x_1 + \ldots + x_k = n\}$

and:

- $F(n) = e^{O(\sqrt{n \log n})}$ [Landau 1903]

Theorem ([Chrobak 1986])

Each unary n-state 1nfa can be simulated by a 1dfa with $e^{O(\sqrt{n \log n})}$ states.

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Each unary n-state 1nfa can be simulated by a 1dfa with $e^{O(\sqrt{n \log n})}$ *states.*

Sakoda&Sipser question: 1nfa vs 2dfa

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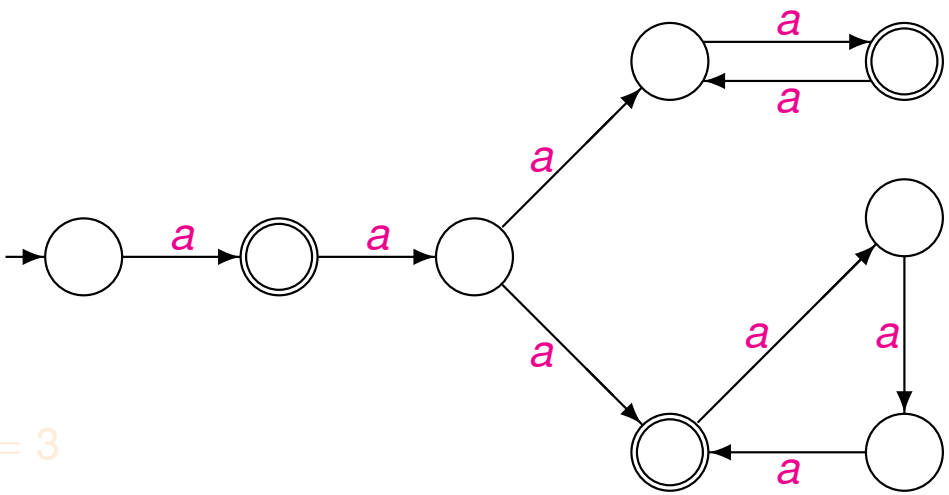

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On input a^m:

- First scan – simulation of the initial path:
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Sakoda&Sipser question: 1nfa vs 2dfa

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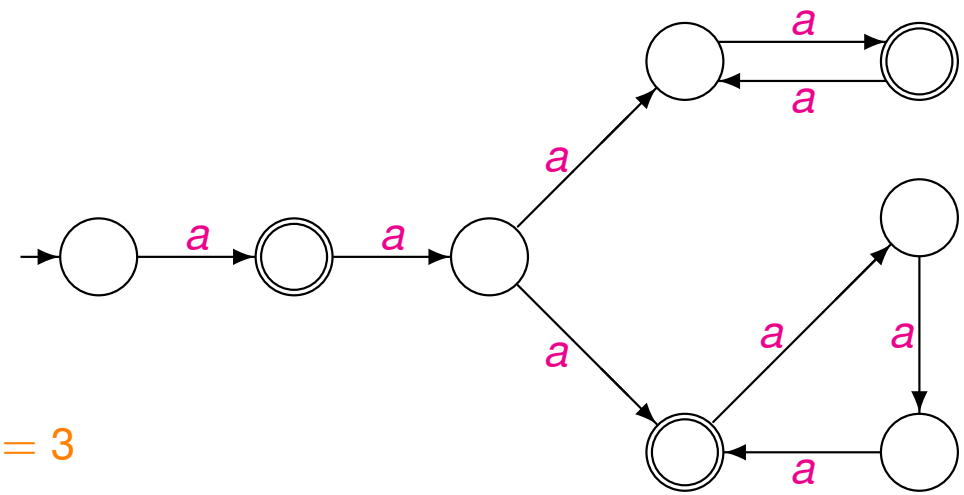

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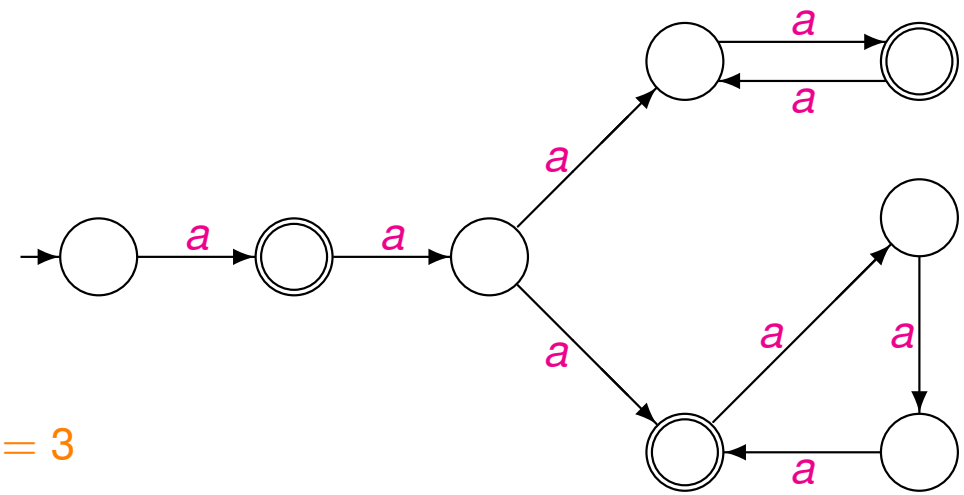

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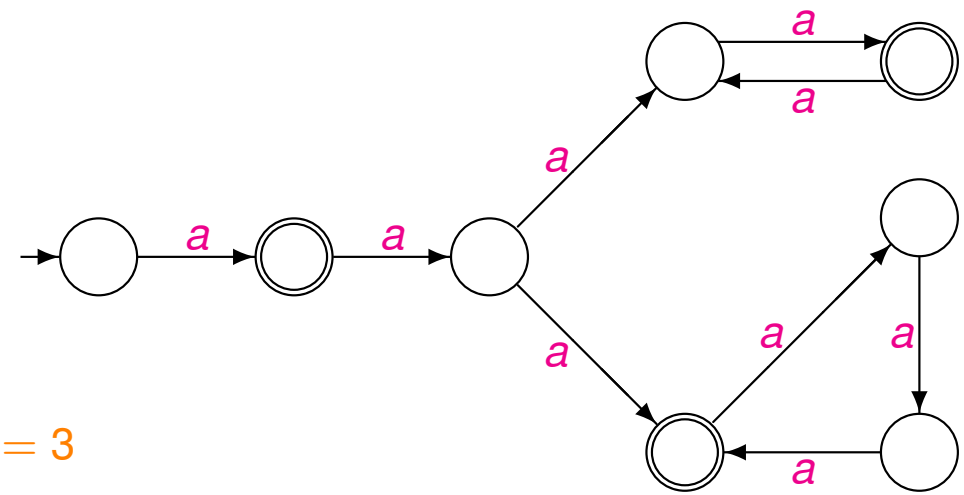

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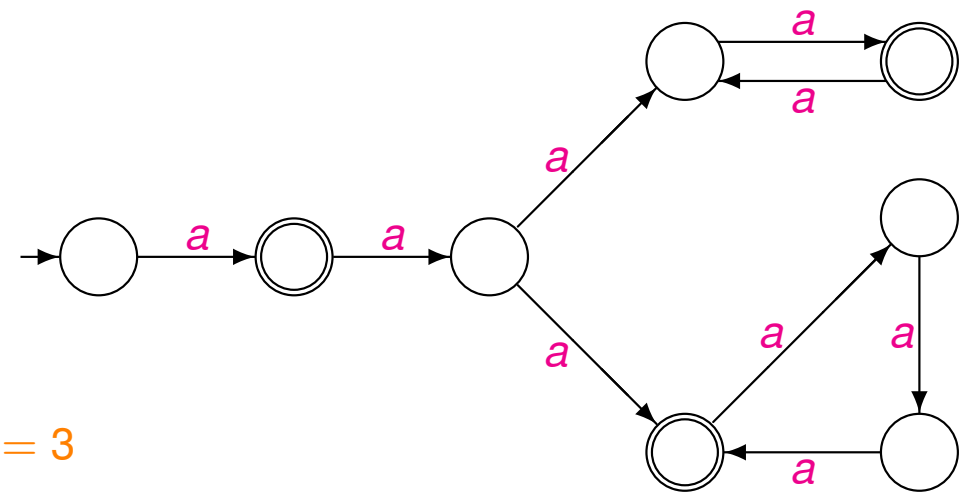

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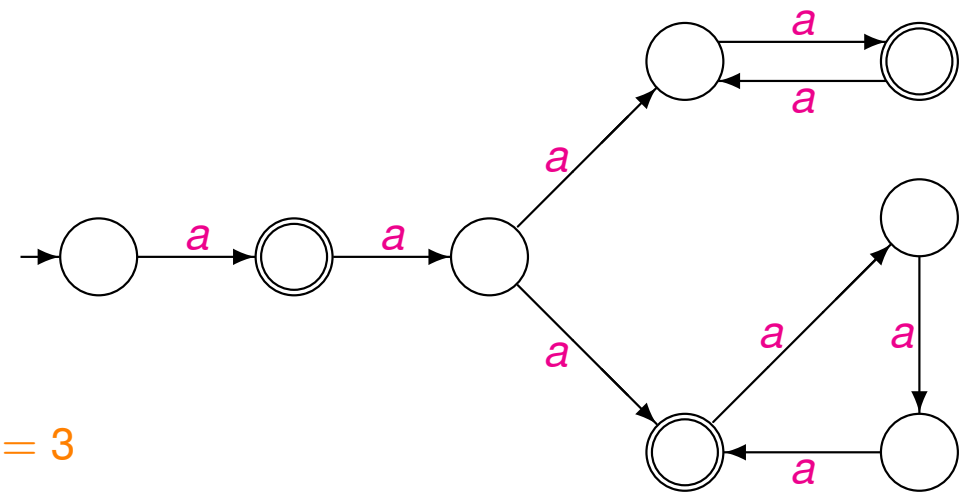

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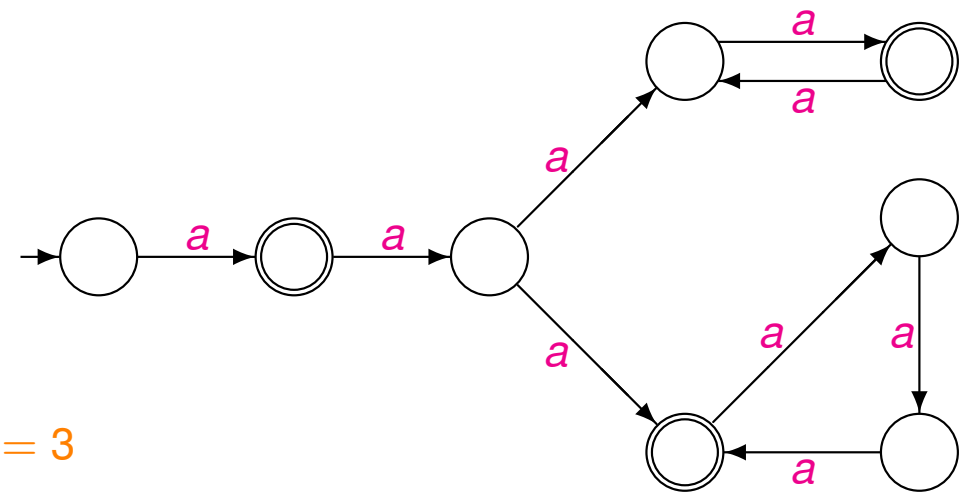

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Sakoda&Sipser question: 1nfa vs 2dfa

How to reduce unary 1nfa to 2dfa?

Given a 1nfa in Chrobak normal form with:

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Sakoda&Sipser question: 1nfa vs 2dfa

Cost of the simulation of unary 1nfa by 2dfa

- Each unary 1nfa in Chrobak normal form with:
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  can be simulated by a 2dfa with $\mu + \lambda_1 + \ldots + \lambda_k$ states.
- Each unary n-state 1nfa can be simulated by a 1nfa in Chrobak normal form with $O(n^2)$ states.

Hence:

Theorem ([Chrobak 1986])

Each unary n-state 1nfa can be simulated by a 2dfa with $O(n^2)$ states.

This gives a *quadratic upper bound* for the simulation of 1nfa by 2dfa in the unary case.

Sakoda&Sipser question: 1nfa vs 2dfa

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  - an initial path of μ states, and
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  can be simulated by a 2dfa with $\mu + \lambda_1 + \ldots + \lambda_k$ states.
- Each unary n-state 1nfa can be simulated by a 1nfa in Chrobak normal form with $O(n^2)$ states.

Hence:

Theorem ([Chrobak 1986])

Each unary n-state 1nfa can be simulated by a 2dfa with $O(n^2)$ states.

This gives a *quadratic upper bound* for the simulation of 1nfa by 2dfa in the unary case.

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What about the lower bound?

For each $n \geq 2$ consider the language

$$L_n = \{a^m \mid m = \alpha n + \beta(n-1), n, m \in \mathbf{N}\}$$

It is possible to prove that:

- L_n is accepted by a 1nfa with n states
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Hence, even the lower bound is *quadratic*.

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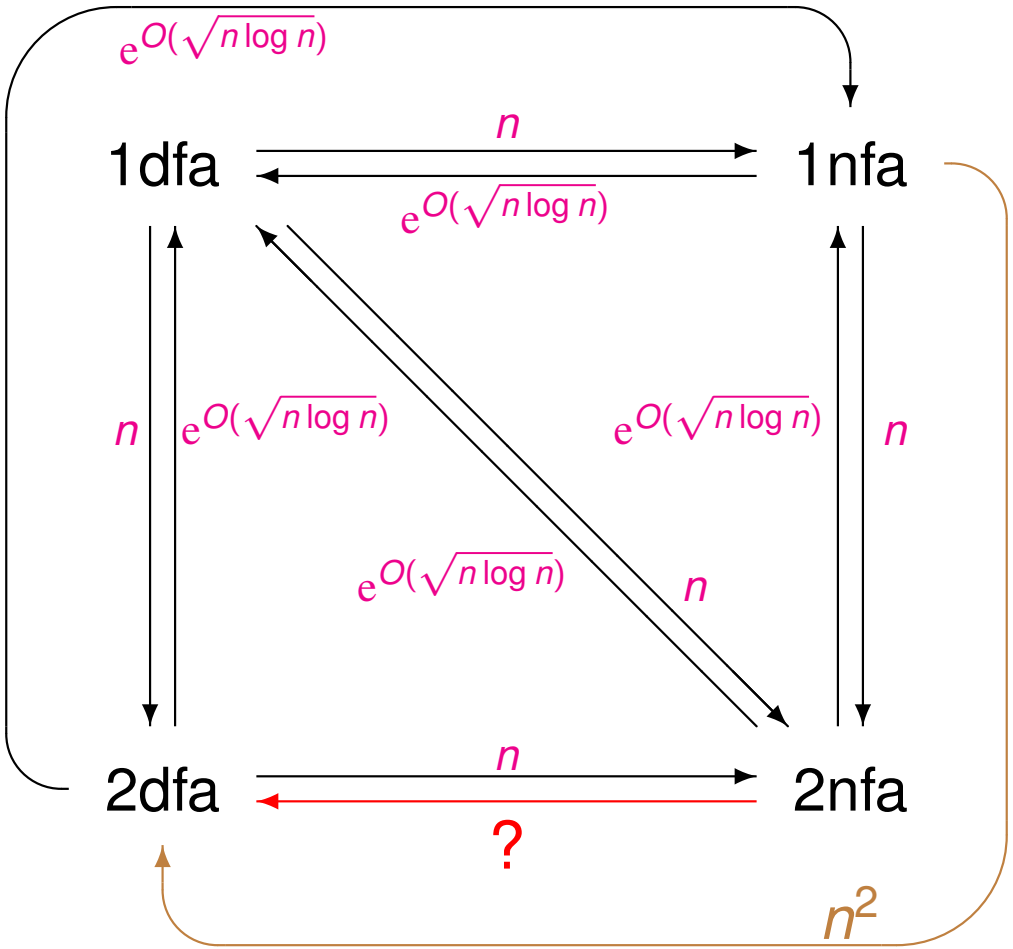
Simulations between unary automata

Costs ot the optimal simulations between unary automata

[Chrobak 1986, Mereghetti and Pighizzini 2001]



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What about the simulation of unary 2nfa by 2dfa?

- Exponential upper bound: $e^{O(\sqrt{n \log n})}$ (simulation of 2nfa by 1dfa)
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It is possible either to increase the lower bound or to decrease the upper bound?

Theorem ([Geffert, Mereghetti, Pighizzini 2003])

Each unary n-state 2nfa can be simulated by a 2dfa with $n^{O(\log n)}$ states.

Remark: The function $n^{O(\log n)}$ is not polynomial, but it grows less than any exponential function $2^{n^{O(1)}}$.

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Definition

Two finite automata are *almost equivalent* iff their accepted languages coincide, with the possible exception of a finite number of strings.

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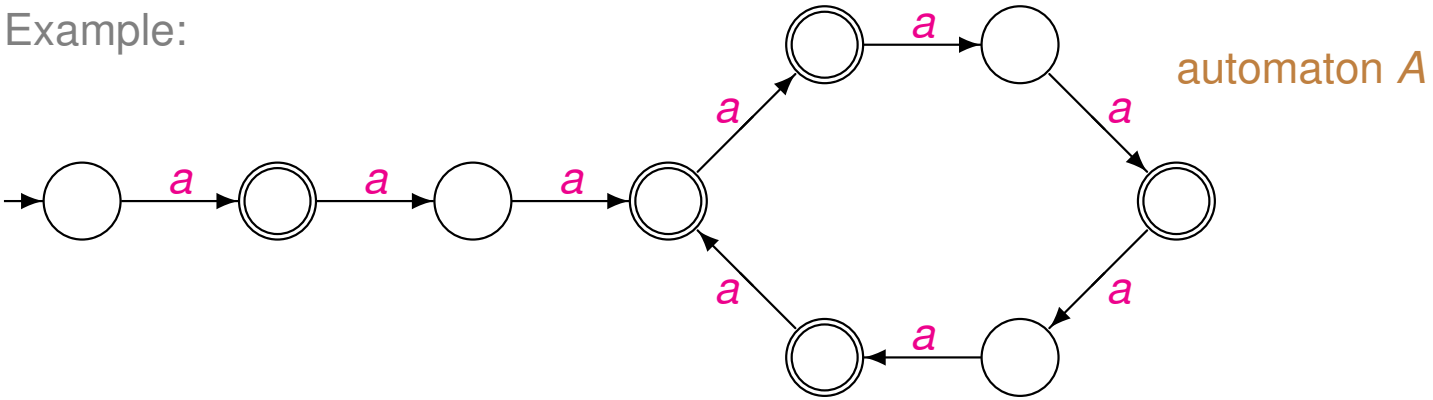

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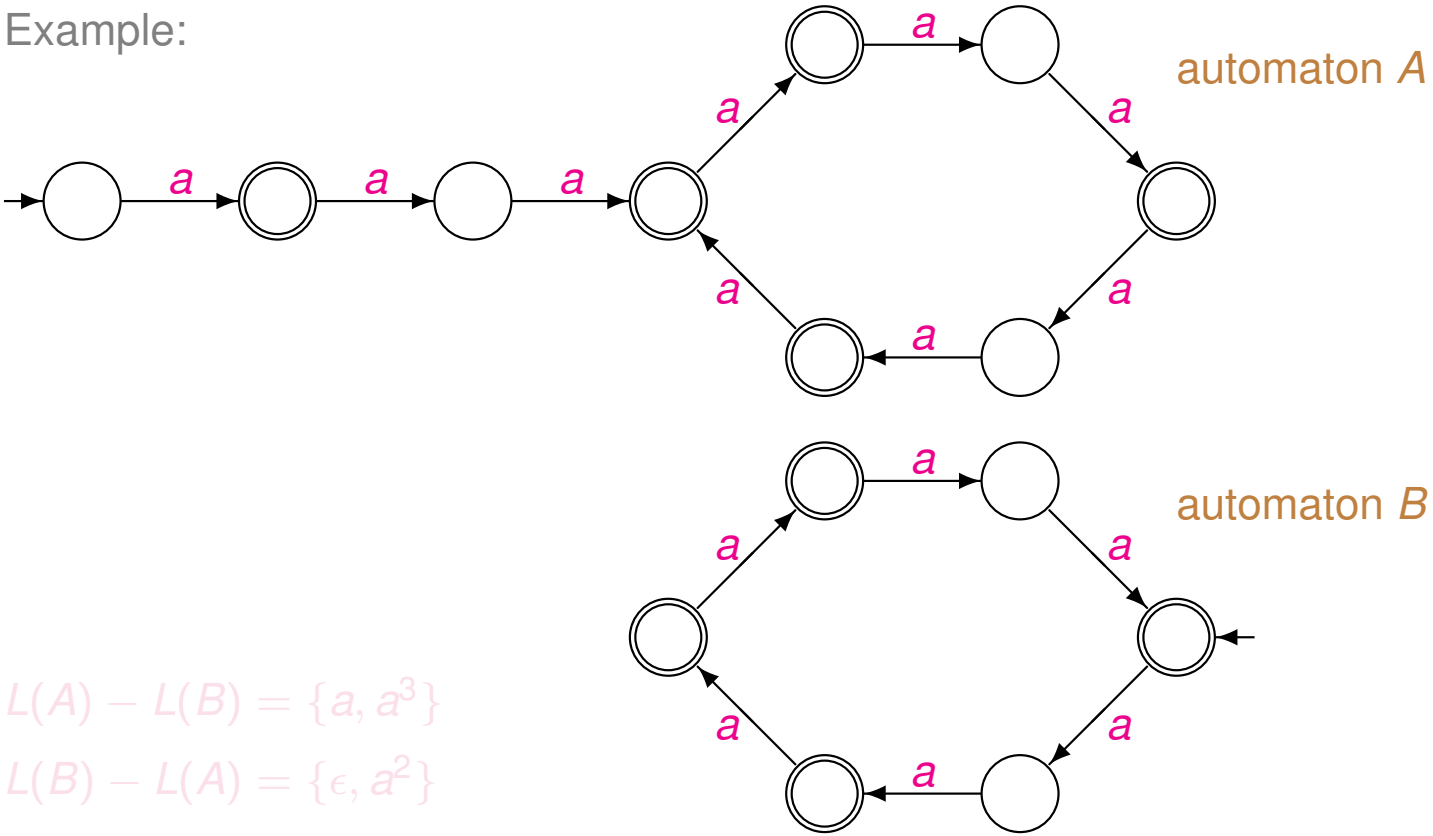

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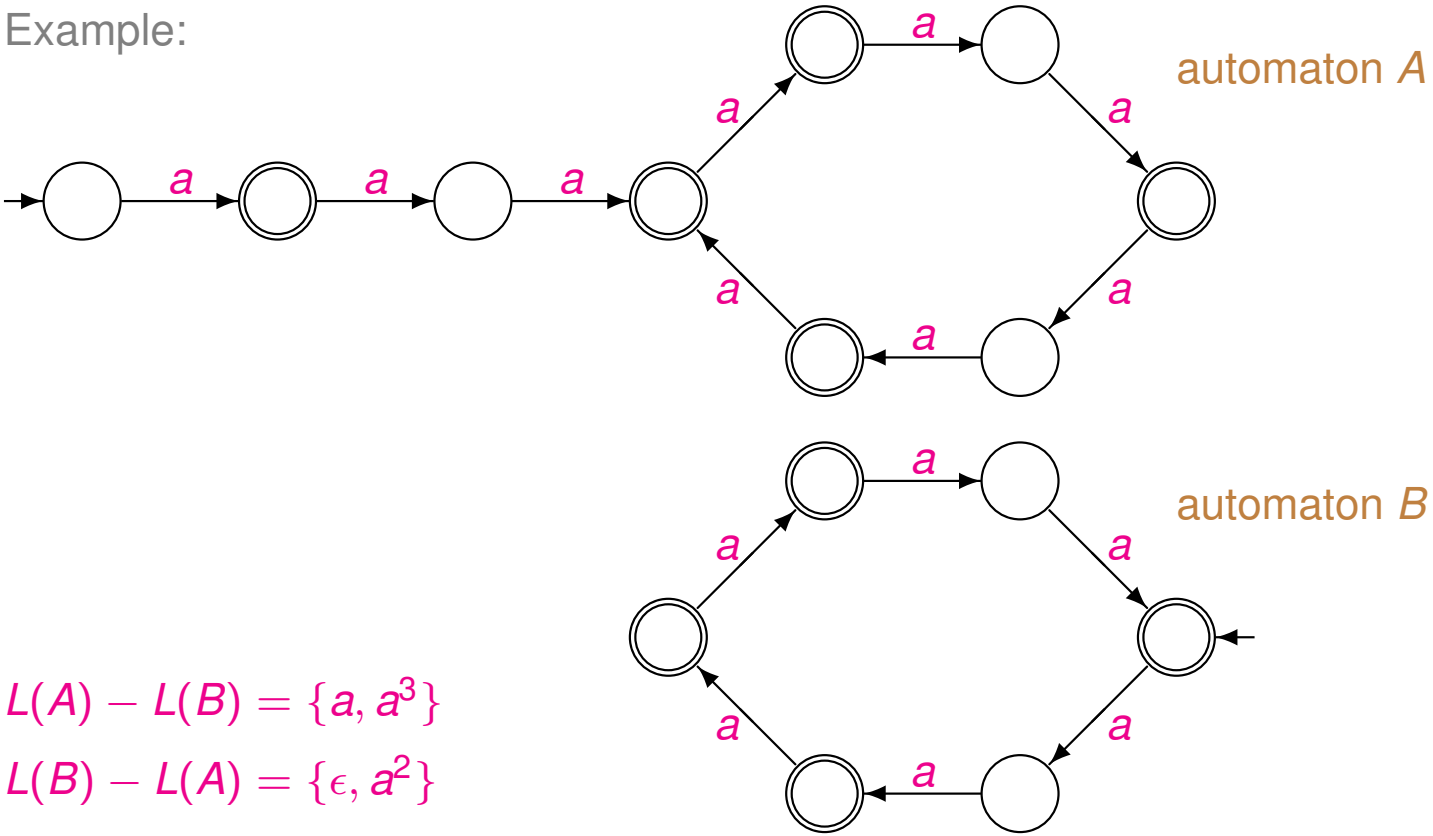

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Sweeping automaton:

- deterministic
- head reversals only at the endmarkers

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A two-way automaton is *quasi sweeping* iff both

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Lemma (from 2nfa to quasi sweeping 2nfa)

For each unary n-state 2nfa A there exists an almost equivalent quasi sweeping 2nfa A' with no more than $2n+2$ states.

Furthermore, the languages $L(A)$ and $L(A')$ can differ only on strings of length $\leq 5n^2$.

Lemma (from quasi sweeping 2nfa to 2dfa)

Each unary n-state quasi sweeping 2nfa can be simulated by a 2dfa with $O(n^{\lceil \log_2(n+1) \rceil + 3})$ states.

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   - B equivalent to A'
   - B almost equivalent to A
   - $n^{O(\log n)}$ states
3. $C :=$ the following dfa:

   first C checks if the input length is $\leq n^2$ and, in this case, it accepts iff A accepts

   hence if the input length is $> n^2$ then C directly simulates B
   - C is equivalent to A
   - C has $n^{O(\log n)}$ states

First lemma: outline of the proof

Lemma (from 2nfa to quasi sweeping 2nfa)

For each unary n-state 2nfa A there exists an almost equivalent quasi sweeping 2nfa A' with no more than $2n+2$ states. Furthermore, the languages $L(A)$ and $L(A')$ can differ only on strings of length $\leq 5n^2$.

- An accepting computation of a two-way machine is a sequence of *segments*
- The segments are delimited by the configurations in which the input head visits the endmarkers
- Two kinds of segments:
  - traversals
  - *U*-turns

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Traversals in 2nfa can be very complicated.
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For sufficiently long inputs ($> n^2$), the set of possible *U*-turns can be precomputed. Hence, they can be replaced by stationary moves [Geffert 1991].

By replacing

- traversals with deterministic loops
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Lemma (from quasi sweeping 2nfa to 2dfa)

Each unary n-state quasi sweeping 2nfa can be simulated by a 2dfa with $O(n^{\lceil \log_2(n+1) \rceil + 3})$ states.

We consider the following predicate *reachable*:

reachable(q, p, k) is true, for $q, p \in Q$, $k \geq 1$, iff there exists a computation path of the given 2nfa which

- starts and ends with the input head scanning the left endmarker,
- in the state q and p, respectively, and
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If an accepting computation visits the left endmarker twice in the same state, then there exists a shorter accepting computation.

Hence:

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How to evaluate *reachable*?

Divide–and–conquer technique (*reach1* is the base case):

function *reachable*(q, p, k)
if $k = 1$ **then return** *reach1*(q, p)
else begin
  for each state $r \in Q$ **do**
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This computation can be implemented by a 2dfa with $O(\text{size}(A) \cdot |Q|^3)$ states.

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Current knowledge

- Upper bounds

	1nfa $\longrightarrow$ 2dfa	2nfa $\longrightarrow$ 2dfa
unary case	$O(n^2)$ [1] optimal	$n^{O(\log n)}$ [2]
general case	exponential	exponential

[1: Chrobak 1986]
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- Lower bounds
  For all the cases, the best known lower bound is $\Omega(n^2)$ [1]

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Problem

Given a two-way automaton with n states accepting a language L, find the cost in term of states, of an automaton accepting the complement of L.

- Deterministic case:
  The cost is 4^n, for *any input alphabet*.
  [Geffert, Mereghetti, Pighizzini 2007]
- Nondeterministic case:
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Relationship with the Sakoda&Sipser question

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Moreover:

If the complementation of 2nfa requires an exponential number of states, then the gap between 2nfa and 2dfa is exponential.

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