Using Crowd sourcing for Local Topology Discovery in Wireless Networks

Alessandro Checco[1] and Carlo Lancia[2,3]

[1] *Hamilton Institute, NUI Maynooth*
[2] *University of Rome Tor Vergata*
[3] *Technische Universiteit Eindhoven*

December 14, 2012

Abstract

Interference is a basic feature of modern wireless networks, and the network interference topology is of fundamental importance. In modern wireless networks, the problem of topology discovery is difficult, because a central intelligence with full knowledge of the location and parameters of the wireless devices is absent. In this paper we introduce the exploitation of user reports to estimate the local topology. We formulate a crisp mathematical discussion of the problem, and present some useful bounds for effectively determining those scenarios which can actually benefit from making use of user reports.

1 Introduction

Modern wireless networks are deployed in an unplanned manner. The problem of determining, for each access points, the neighbours that cause interference is thus a difficult problem. An example is the growing field of small cell networks (femtocells), where cells are being deployed in an ad-hoc manner, but these need to respect strict constraints on the scrambling code used by neighbouring cells, because handover between two adjacent cells having same scrambling code can jeopardize the functionality of the whole network [1]. These simple, inexpensive cells are not able to sense effectively the code confusion level in their whole coverage area, and so exploiting user reports is essential.

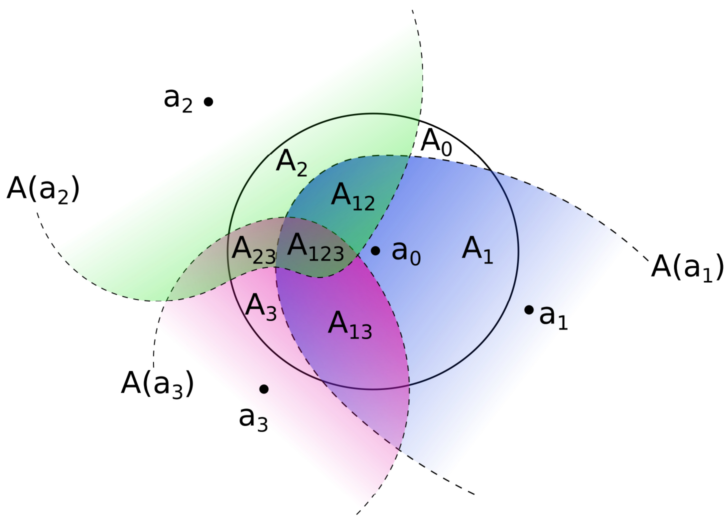

Figure 1: Example of a scenario in which the access point a_0 has three interfering neighbours: a_1, a_2, a_3. The coverage area $A(a_0)$ can be tessellated with the sets $A_1, A_2, A_3, A_{12}, A_{13}, A_{23}, A_{123}$.

Our main question is how much knowledge we can expect to obtain from user reports given a reasonable model on user mobility. Estimating the minimal density of users that can guarantee full knowledge of the local topology is fundamental to determine the class of network deployments that can effectively benefit from exploiting user reports.

In this paper we state the problem of user reports-based local topology discovery, and then introduce a number of bounds, which are useful for gaining insight into those situations where user reports function is likely to give the greatest benefit.

1.1 Related Work

Although topology discovery has been extensively studied for geographical position discovery [6], routing protocols study [3], and in general for ad-hoc networks configuration [5], the effect of user reports function on interfering neighbours discovery capability have never been studied in the literature.

This function is already available on commercial femtocells [7] and its implementation for interference reduction is recommended in [2] and [1].

1.2 Problem Statement

Given a set of wireless access points $\mathcal{A}$, we let $A(a_i) \subset \mathbb{R}^2$ denote the coverage area of access point $a_i \in \mathcal{A}$, that in general depends on the transmission power of a_i and on the radio propagation properties. We focus on access point a_0, and let $\mathcal{B} = \{a_i \in \mathcal{A}, i > 0\colon A(a_0) \cap A(a_i) \neq \emptyset\}$ denote the set of neighbouring access points, i. e. the access points whose coverage areas overlap with $A(a_0)$. We introduce a tessellation of the area $A(a_0)$ based on the lexicographic indexing of the intersections and we map it on the 2^N vertices of a N-dimensional hypercube. So the tessellation element $A_{\mathbf{i}}$, where $\mathbf{i} = \{i_1, i_2\}$ and $i_1 < i_2$, is the portion of $A(a_0)$ which is covered by $A(a_{i_1})$ and $A(a_{i_2})$ only and the intersection of $A_{\mathbf{i}}$ with $A(a_k)$ is empty for all $k \neq i_1, i_2$. More precisely,

$$A(a_0) = A_0 \cup \bigcup_{k=1}^{N} \bigcup_{\substack{\mathbf{j} \subseteq \{1,N\} \\ |\mathbf{j}|=k}} A_{\mathbf{j}}.$$

where

$$A_{\mathbf{j}} = A(a_0) \cap \bigcap_{j_m \in \mathbf{j}} A(a_{j_m}),$$

$$A_0 = A(a_0) \setminus \bigcup_{\substack{\mathbf{j} \subseteq \{1,N\}^k \\ |\mathbf{j}|=k}}^{N} A_{\mathbf{j}}.$$

In Figure 1 an example of such tessellation is presented.

We define $\mathcal{U} = \cup_{i=1}^{K} u_i$, the set of K users served by access point a_0. We assume that a user u located in $(x_u, y_u) \in A(a_0)$ can instantly communicate the set of neighbouring access points that are covering that position to the serving access point a_0. Moreover, to keep the model as more conservative as possible, and to encompass the frequent case of half-duplex access points, we assume a_0 can have knowledge of the existence of a_i only relying on the aforesaid user reports.

Define the following function $\Psi : A(a_0) \ni (x, y) \mapsto \mathbf{j} \subseteq \{1, 2, \ldots, N\}$ such that $A_{\mathbf{j}}$ is the (unique) element of the tessellation containing (x, y).

Definition 1 (Full Knowledge)**.** Given a set of users $\mathcal{U}$, access point a_0 is said to have *Full Knowledge* of its neighbours, iff

$$\bigcup_{u \in \mathcal{U}} \Psi(x_u, y_u) = \{1, 2, \ldots, N\}$$

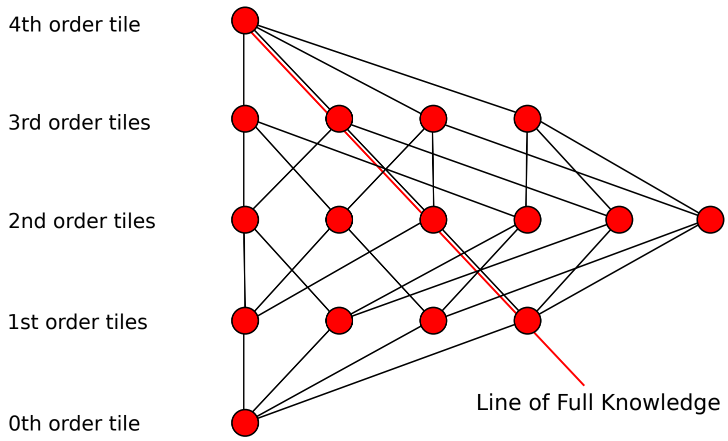

Figure 2: Hypercube representation of the tessellation for $N = 4$. There is one zeroth order tile, namely A^C, four first order tiles, A_1, A_2, A_3 and A_4, six second order tiles, $A_{12}, A_{13}, A_{14}, A_{23}, A_{24}, A_{34}$, four third order tiles, $A_{123}, A_{124}, A_{134}, A_{234}$ and a fourth order tile, i.e. A_{1234}.

Remark 1. The number of tiles $A_{\mathbf{j}}$ which are of k-th order, i.e. such that $|\mathbf{j}| = k$, is $\binom{N}{k}$. Figure 2 underlines the dependecies structure between tiles, a user in a k-th order tile is equivalent to k users in a different first order tile each. Full Knowledge is attained when the configurations of users is equivalent to have one user in each of the N first order tiles. In particular Full Knowledge is attained with at least a user in the N-th order tile or with two users in two distinct $(N-1)$-th tiles, etc. Modulo a rearrangement of the graph representation of the N-dimensional hypercube this idea gives rise to what we call *Line of Full Knowledge*, see Figure 2.

So we can now state the fundamental problem of dimensioning the number of users to have full knowledge.

Problem 1. *Given an access point a_0 and a set $\mathcal{B}$ of neighbours, and a set of users $\mathcal{U}$ drawn from a Poisson point process in $A(a_0)$ with intensity λ, we want to know the expected number of users required to have full knowledge of the neighbours set.*

Remark 2. The probability that a user fall in an element of the tessellation $A_{\mathbf{j}}$ conditioned to the even of falling in $A(a_0)$ is proportional to $|A_{\mathbf{j}}|$, i. e. to the sum of the areas defined by $\mathbf{j}$.

2 Model Description

We consider the follow discrete-time model, that can be used to represent easily Problem 1. The set of access points $a_i, i = 0, \ldots, N$ are given, and therefore the corresponding areas and the resulting tessellation are fixed. At each time slot t, a user u_t is generated u.a.r. inside $A(a_0)$.

Problem 1 can be equivalently expressed as the following

Problem 2. *Given an access point a_0 and a set $\mathcal{B}$ of neighbours, and a sequence of users $\{u_t\}_{t\geq 0}$, each one drawn u.a.r. in $A(a_0)$, we want to know the expected first time of full knowledge T, i. e.*

$$T = \min\left\{ t \geq 0 \ : \ \bigcup_{s=0}^{t} \Psi(x_{u_s}, y_{u_s}) = \{1, 2, \ldots, N\} \right\}.$$

Remark 3. When the sequence of users is modeled as a Markov Chain on $A(a_0)$ the time T is recognized to be a *stopping time*.

In the simplest scenario the coverage areas $A(a_i)$ have nonempty intersection only with $A(a_0)$, that is to say $A(a_0) \cap A(a_i) \cap A(a_j) = \emptyset$ whenever $i \neq j$. In this case an algorithm can be designed for the topology discovery where a sequence of users is generated uniformly at random inside $A(a_0)$. In this case the first time T after which it is possible to reconstruct the network topology can be immediately mapped into the hitting time of the state 0 of a generalized Coupon Collector chain [4].

In a more general framework we face also intersections of higher order. If a user is in the intersection $A(a_0) \cap A(a_i) \cap A(a_j)$, which is now nonempty. A description in terms of a Markov chain is still possible but the state space is composed of all the 2^N vertices of the N-dimensional hypercube. If the neighbours of a_0 are displaced around it with a regular pattern then it may be the case that the state space of the Markov chain can be projected onto a more handy set of states. Let us imagine that a_0 has four neighbours a_1, a_2, a_3 and a_4, at the same distance from a_0 and equally spaced in angle. Then the intersections of the same order have the same extension, that is to say the probability of a user falling into a tessel conditioned to the order of the tile is uniform. If we denote by A_k the area of the k-th order tiles, we have that the state space can be projected into a set of lines parallel to the Full Knowledge line, see Figure 3.

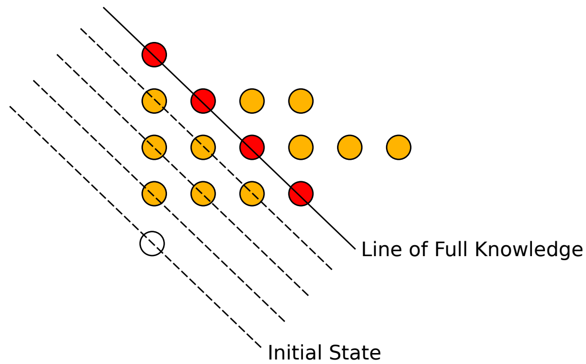

Figure 3: Projection of the state space.

The transition matrix of the chain is the following

$$M = \begin{bmatrix} A_0 & 4A_1 & 6A_2 & 4A_3 & A_4 \\ 0 & A_0+A_1 & 3A_1+3A_2 & 3A_2+4A_3 & A_4 \\ 0 & 0 & A_0+2A_1+A_2 & 2A_1+4A_2+A_3 & A_2+3A_3+A_4 \\ 0 & 0 & 0 & A_0+3A_1+3A_2+A_3 & A_1+3A_2+3A_3+A_4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Figure 4 sketches the behavior of the algorithm and the means by which Full Knowledge is achieved. It is clear that the problem has a rich combinatorial structure which must be exploited to give a broader representation of the algorithm in terms of a Markov chain and to achieve a better understanding of the mechanisms underlying its convergence.

References

[1] A. Checco, R. Razavi, D.J. Leith, and H. Claussen. Self-configuration of scrambling codes for wcdma small cell networks.

[2] J. Edwards. Implementation of network listen modem for wcdma femtocell. In *Cognitive Radio and Software Defined Radios: Technologies and Techniques, 2008 IET Seminar on*, pages 1–4. IET, 2008.

[3] S. Marwaha, C.K. Tham, and D. Srinivasan. A novel routing protocol using mobile agents and reactive route discovery for ad hoc wireless networks. In *Networks, 2002. ICON 2002. 10th IEEE International Conference on*, pages 311–316. IEEE, 2002.

[4] R. May. Coupon collecting with quotas. *the electronic journal of combinatorics*, 15(31):1, 2008.

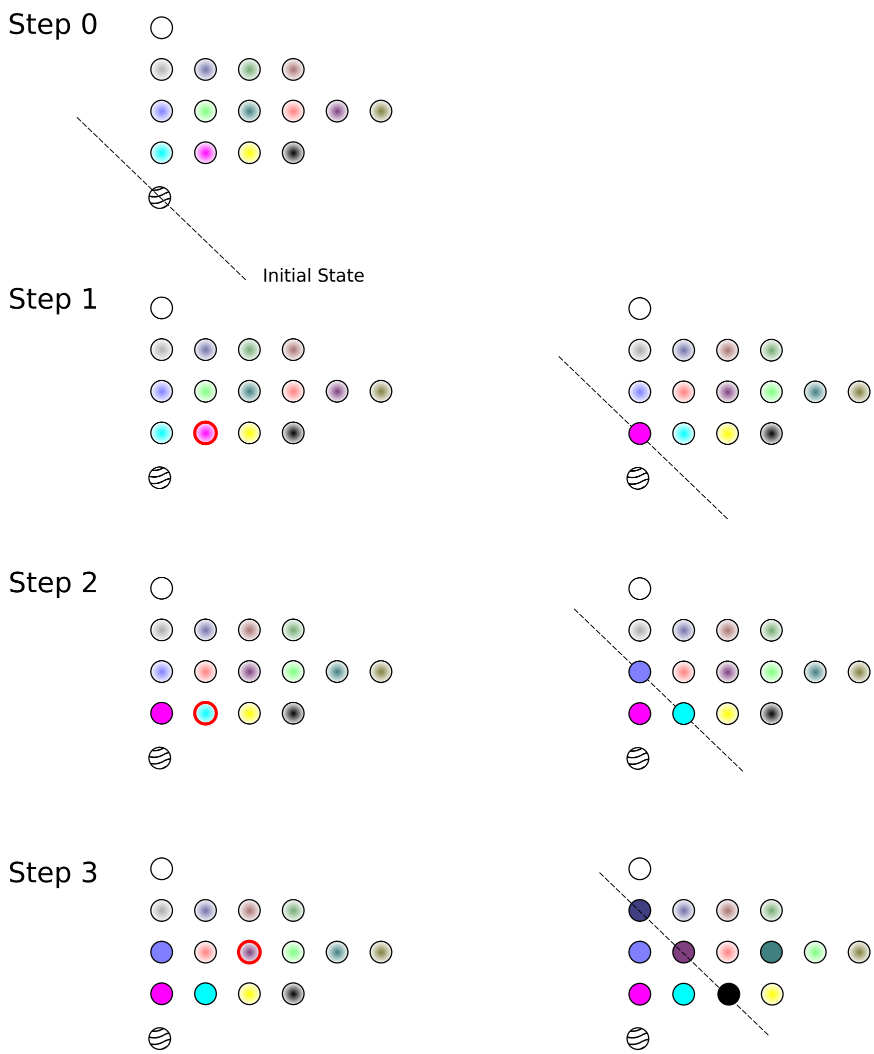

Figure 4: A possible evolution for the algorithm in the case of $N = 4$. Shaded circles have not been visited yet, solid have. The neighbouring structure is represented according to the CMYK system: first order tiles have colors Cian, Magenta, Yellow and Black, a user in the light green second order tile is equivalent to a user in the cyan tile and one in the yellow tile. Note that in Step 2 after Cyan is visited, we have a user in Magenta and one in Cyan, which is equivalent to have a single user in Indigo (2nd order). The same happens in Step 3, a user in Purple (2nd order) induces a user in Black which, together with Cyan and Magenta, is equivalent in turn to a single user in Dark Blue (3rd order).

[5] R. RoyChoudhury, S. Bandyopadhyay, and K. Paul. A distributed mechanism for topology discovery in ad hoc wireless networks using mobile agents. In *Proceedings of the 1st ACM international symposium on Mobile ad hoc networking & computing*, pages 145–146. IEEE Press, 2000.

[6] R. RoyChouldhury, S. Bandyopadhyay, and K. Paul. A mobile agent based mechanism to discover geographical positions of nodes in ad hoc wireless networks. In *6th Asia-Pacific Conference on Communications (APCC2000), Seoul, Korea*, volume 30, 2000.

[7] Philip Sapiano. Discovering neighbouring femto cells, August 2010.