Privacy-Preserving Location-based Services

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Location based service (LBS)

Location based service:
- request includes location information
- (service) information response is a function of the given location

Example
“Give me the closest vegetarian restaurant to this location”

where “this location” can be filled in by a GPS device and/or the cell operator.
Generally speaking

**Objective:**

- Allow the use of location service without giving up (too much) privacy

**Two privacy concerns:**

- Location of the user is private information
- The request (or the fact there is a request from the user) is private information

**Two methods:**

- “Obfuscate” private information
- “Disown” private information
Basics first: near/nearest neighbor search

Problem:

▶ Given an $n$-dimensional point $q$, find the nearest neighbor among a set $S$ of ($n$-dimensional) points.
▶ We assume $n = 2$ here.

Distance function: Usually Euclidean.

▶ Each point is a vector: $X = \langle x_1, \ldots, x_n \rangle$.
▶ Given two points $X$ and $Y$, $D(X, Y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$.
▶ Triangulation property, $D(X, Y) \leq D(X, Z) + D(Z, Y)$, holds for all points $X, Y, Z$.

Two versions

▶ Near neighbors of $X$ within distance $\delta$: $\{Y \in S \mid D(X, Y) \leq \delta\}$
▶ Nearest neighbors of $X$: $\{Y \mid \forall Y' \in S : D(X, Y') \geq D(X, Y)\}$
PR-Quadtree and KD-tree

PR-Quadtree
The PR-quadtree partitions the 2-dimensional region (a node) into four equal quadrants (4 children nodes), recursively, until the region for each node (leaf) contains no greater than the prescribed number of points.
Online demo: http://donar.umiacs.umd.edu/quadtree/points/prquad.html

KD-tree
A kd-tree is similar to PR-Quadtree, i.e., it recursively split the region (subregions form the children nodes). Kd-tree, however, only uses a splitting plane that is perpendicular to one of the coordinate system axes. Each time it chooses a particular axes so that the splitting is most even in terms of the number of points in the two subregion.
Nearest neighbor search

NN search algorithm

- Given a query point $q$ and a PR-quadtree, go down the tree from the root to the most promising branch (depth first search)
- Once in a leaf node, find the nearest point (called candidate NN) to $q$, and remember the point and the distance $\delta$ from $q$ to this point.
- Now go on with the depth first search but prune the search space by the distance $\delta$ to $q$, i.e., if a region is farther away from $q$ than $\delta$, then we do not need to go into that region.
- Every time we reach a leaf node, we try to modify the candidate NN and $\delta$ and continue the search.
Privacy preserving NN search – disowning the request

The idea is to obtain some kind of $k$-indistinguishability, or $k$-anonymity.
- Remove the obvious ID values from request, but can’t remove location information
- HOWEVER, Location information may be used to link back to the user! \(^1\)

Then?
- Find $k - 1$ other users around you, and use their location to ask for nearest neighbors for each of for them. (Group nearest neighbor search.)
- Then from all the answers, find the real target
- Note that the actual NN must be within the returned set
- The service provider would not just have location of one user, but locations of $k$ users, then $k$-anonymity is obtained.

But wait... which other $k - 1$ users to choose (or “use”)?

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\(^1\)Adversary model! In this case, we assume the adversary can somehow link the location to user.
If the adversary does not have any idea how the other \( k - 1 \) users are chosen, then any other \( k - 1 \) users work. For efficiency, perhaps just choose the nearest \( k - 1 \) neighbors. (Why more efficient this way?)

However, if the adversary knows the algorithm\(^2\), then the adversary may be able to figure out the location of the user who issued the request!

Perché? Let’s try use the “nearest \( k - 1 \) neighbors” method as an example. Consider \( k = 2 \). Give an example when the privacy protection fails.

\(^2\)Adversary model!
The lesson from the above example: under the assumption that the adversary knows the algorithm of choosing the other $k - 1$ users,

Then if we use a group $G$ of $k$ users for $k$-anonymity for query point $q$ and a particular algorithm $f(q) = G$ to choose this group, then we need to make sure that $f(p) = G$ as well for each $p$ in $G$. 
So we need a method to find this $f(q) = G$.

A remark is that it's always the better if the points in $G$ are all close to $q$.

This is why we tried to use $(k-1)$-NN for $q$... but it does not satisfy the above condition (namely, $f(p) = G$ for each $p \in G$)

Sergio Mascetti is an expert in finding this $f$ function!
The Grid algorithm

(a) First iteration

(b) Second iteration

(c) Third iteration
Achieving location uncertainty

▶ What if I am only concerned with my location information being revealed?
  ▶ That is, I don’t really care if adversary knows that I issued the request...
  ▶ But I DO care if they find out from WHERE I issued the request...
▶ No anonymity is needed because in this case, they can know that it’s me who issued the request
▶ We need location uncertainty

First attempt

▶ What about just randomly pick a position near me as the location information in my request? Like in the next building, or next block, or next town, depending on the user privacy concern.
▶ What if the adversary knows the algorithm to choose the fake location? (Homework for you! *Hint: randomization.*)
Achieving location uncertainty (2)

- Fake location information works to hide the location of the issuer...
- But the NN query will not give a right answer
- Dilemma:
  - To get more precise answer, we need to choose a location near the original/true location... but this provides less uncertainty
  - To provide more uncertainty, less precise query result will be given
- Any method to have best of both?
Here is an idea (SpaceTwist algorithm):

- I use a fake point to ask to server for nearest neighbor
- But keep on asking for the next nearest neighbor, and next, and next...
- Until I see the nearest neighbor of my original query point.

Two questions:

- How do I know the NN of my original point has arrived?
- How much space uncertainty does this method give?
To answer the first question, let us consider the following diagram:

The darker circle is called the *supply region*, and the lighter one is called the *demand region*.

- When the supply region covers the demand region, then my nearest neighbor has arrived.
How are the two regions defined?

- **The supply region** is the circle with the fake point \( q' \) as the center and the distance to the current next nearest neighbor of \( q' \) as radius.
  - Property: all the points in the supply region have been delivered to me.

- **The demand region** is the circle with my query point \( q \) as the center and the distance to the nearest neighbor to \( q \) among all I have received as the radius.
SpaceTwist (4)

How much location uncertainty does this provide?

- Assume the adversary can listen to the traffic, i.e., receives all the replies the server sends $p_1, \ldots, p_n$.
- Assume also that the adversary knows the algorithm.
- Let’s try to guess where $q$ could be.

Where can $q$ be?

Assume I stopped the serve at the $n$-th point it sends me, which is when the supply region covers my demand region.

- Fact: I didn’t stop at $(n - 1)$-th point. It means at that time, the supply region did not cover the demand region.

So $q$ must satisfy the following conditions:

- $D(q, q') + \min_{1 \leq i < n} D(q, p_i) > D(q', p_{n-1})$
- $D(q, q') + \min_{1 \leq i < n} D(q, p_i) \leq D(q', p_n)$
PIR-based method

Private Information Retrieval

- Assume a server maintains a database of \( n \) items.
- A user wants to query the content of the \( i \)-th item.
- However, the user doesn’t want the server know which item he’s querying!

How do we do this?

- Assume database has \( \langle x_1, \ldots, x_n \rangle \), and each \( x_i = 0, 1 \).
- Assume we have a set:

\[
QR = \{ y \in Z_N^* | \exists x \in Z_N^* : y = x^2 \mod N \}
\]

where \( Z_N^* = \{ x \in Z_N | \gcd(N, x) = 1 \} \), and \( N = p \cdot q \) (product of two large primes)
- Items in \( QR \) is called quadratic residuals.
- Denote \( QNR \) the complementary of \( QR \) (i.e., \( QNR = \overline{QR} \)).
A fact about QR and QNR: It’s computationally hard to distinguish where a number is in QR or QNR without knowing \( p \) and \( q \). Easy if \( p \) and \( q \) are known.

Now assume a query wants content (0 or 1) at the \( i^* \)-the position.

It sends over \( y_1, \ldots, y_n \), where \( y_i^* \) is in QNR and all other \( y_i \) are in QR.

The database computes and sends \( z = \prod_{i=1}^{n} w_i \), where \( w_i = y_i^2 \) if \( x_i = 0 \) and \( w_i = y_i \) otherwise.

The user looks at \( z \): if \( z \in QNR \) then \( x_i = 1 \), otherwise \( x_i = 0 \).
Examples

- database = ⟨0, 1, 1, 0⟩
- user wants 2nd item, and sends ⟨qr, qnr, qr, qr⟩, where qr ∈ QR and qnr ∈ QNR.
- database sends back \( z = (qr)^2 \times (qnr) \times (qr) \times (qr)^2 \), and hence \( z \in QNR \), and the user knows the answer is 1.

- database = ⟨0, 1, 1, 0⟩
- user wants 4th item, and sends ⟨qr, qr, qr, qnr⟩, where qr ∈ QR and qnr ∈ QNR.
- database sends back \( z = (qr)^2 \times (qr) \times (qr) \times (qnr)^2 \), and hence \( z \in QR \) and the user knows the answer is 0.
Making it more efficient

- We fold the database \( n \)-bits into \( m \times m \) matrix (if \( n \) is not a square, pad with 0s)
- User asks for content \((i^*, j^*)\), but only send \(y_1, \ldots, y_m\) as if the database only has one row.
- Database computes \( m \) \( z \) numbers as if there are \( m \) databases (rows)
- User gets these \( m \) numbers, and pick out the \( i^* \) row (throw away everything else), and then use the same trick as before to get \((i^*, j^*)\) content.
- Why more efficient? Communication is \( O(\sqrt{n}) \) instead of \( n \).

The above can be extended to ask for \( k \)-bit strings easily (just ask one bit at a time).
PIR-based nearest neighbor search

- Divide the region of interest into a Voronoi diagram.
  - Property: the nearest neighbor of any query point in a Voronoi region is in the same region.
- Superimpose a grid onto it.
- For each grid cell, store all the points of the Voronoi regions that intersect with the cell.

![Voronoi diagram](image)

- The user just (privately) asks for the cell content where the user’s position is in, and then compute the NN.
References

- Ghinita et al. Private Queries in Location Based SEnvices: anonymizers are not necessary. ICDE 2008.
- Yiu et al. SpaceTwist: Managing the Trade-Offs Among Location Privacy, Query Performance, and Query Accuracy in Mobile Services. ICDE 2008.