Devlin's Angle

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In Math You Have to Remember, In Other Subjects You Can Think About It

The title of this month's rant is a statement made by a female high school student. I'll come back to her in due course.

The US ranks much worse than most of our economic competitors in the mathematics performance of high school students. Many attempts have been made to improve this dismal performance, but none have worked. To my mind (and I am by no means alone in thinking this), the reason is clear. Those attempts have all focused on improving basic math skills. In contrast, the emphasis should be elsewhere.

Mathematics is a way of thinking about problems and issues in the world. Get the thinking right and *the skills come largely for free.*

Numerous studies over the past thirty years have shown that when people of any age and any ability level are faced with mathematical challenges that arise naturally in a real-world context that has meaning for them, and where the outcome directly matters to them, they rapidly achieve a high level of competence. How high? Typically 98 percent, that's how high. I describe some of those studies in my book *The Math Gene* (Basic Books, 2000). I also provide an explanation of why those same people, when presented with the very same mathematical challenges in a traditional paper-and-pencil classroom fashion, perform at a lowly 37 percent level.

The evidence is clear. It's not that people cannot think mathematically. It's that they have enormous trouble doing it in a de-contextualized, abstract setting.

So why the continued focus on skills? Because many people, even those in positions of power and influence, not only are totally unaware of the findings I just mentioned, they don't even understand what mathematics is and how it works. All they see are the skills, and they think, wrongly, that is what mathematics is about. Given that for most people, their last close encounter with mathematics was a skills-based school math class, it is not hard to see how this misconception arises. But to confuse mathematics with mastery of skills is the same as thinking architecture is about bricklaying, or confusing music with mastering the musical scale.

Of course basic skills are important. But they are merely the tools for mathematical thinking. In the pre-computer era, an industrial society like the United States needed a large workforce of people with mastery of basic math skills who could carry out tasks assigned to them by others. But in today's workplace, the coin of the realm is creative problem solving, usually in collaborative groups, making use of mathematical thinking when it is required. How well are we preparing today's students for life in that environment? How do we compare with our competitor nations?

The answer is, not well. In an international survey conducted in 2003, students from forty countries were asked whether they agreed or disagreed with the statement: "When I study math, I try to learn the answers to the problems off by heart." Across all students, an average of 65 percent disagreed with this statement - which is encouraging since it is a hopeless way to learn math - but 67 percent of American children agreed with it! [Learning for Tomorrow's World: First results from PISA 2003, OECD]

So what are we doing wrong?

People disagree over some details, but there seems to be a wide (though not universal) consensus over three main causes:

- Our mathematics curriculum (actually, it's curricula, since each state has its own, but the complaint applies to all) contains far too many required topics, each of which can be taught only to a shallow degree - often referred to as the "mile wide, inch deep" problem;
- Far too many math teachers do not have a good understanding of the subject they teach;
- In the majority of classrooms, mathematics is taught in a rigid, rule-based fashion.

That caveat I made about the agreement on these three factors not being universal applies primarily to the last item in this list. Walk into any US high-school math classroom, and you will be greeted by one of two very different scenes.

In the first scene, by far the most common, you will see the students sitting in neatly organized rows, facing the teacher, who stands at the front. On the desk in front of each student you will likely see a textbook, a notebook, a pen or pencil, and perhaps a calculator. At the start of each class, the teacher will spend some time at the whiteboard, explaining some new rule or technique and working through one or two examples. Then the students will open their textbooks and proceed to work through a number of assigned examples whose solutions require the technique they have just been shown. They will for the most part work alone, and in silence. When they run into a problem, they will call on the teacher for help, not each other. When they have completed the task, the cycle begins over again. This teaching method is general known as "the traditional approach." It's an appropriate name, since it has been used since the beginnings of mathematics, some three thousand years ago.

The other, less common scene appears much more chaotic. Groups of students sit around circular tables discussing how to solve a particular problem, or standing at the whiteboard arguing about the best way to proceed. The teacher moves around the room talking with the different groups in turn, making suggestions as to how to proceed, or pointing out possible errors in a particular line of reasoning the students are following. Occasionally, the teacher will call the entire group to order and ask one group to explain their solution to the rest of the class, or to give a short, mini-lecture about a particular concept or method. This is sometimes called "the progressive approach."

The question which is the better method lies at the heart of the infamous "math wars" that have raged in California and New York (in particular). Though you can find the same two pedagogic approaches in many countries, only in the US has the issue become an issue for fierce and often angry debate. As an outsider, coming from the UK fairly late in my career, what struck me from the start was that much of the debate comprised the setting up and knocking down of straw men. Traditionalist teachers would be accused of doing nothing but drill their students mercilessly in basic procedural skills, paying no attention to the development of mathematical concepts, while the progressives would be burned in effigy for not valuing basic skills (or seeing the need to practice them) and presenting students with a wishy-washy, watered down pap that had no real mathematical meat. Doubtless you can find examples of each, but for the most part these caricatures exist only in the minds of the critics who make those claims, and neither is remotely correct. Rather, what you find are thousands of teachers doing the best they can, trying to balance the need for conceptual understanding with the need to practice basic skills, but unsure of what is the best way to proceed, particularly when it comes to motivating their students. In the meantime, absent any clear evidence as to how best to proceed, the majority of teachers quite understandably default to more or less the same teaching methods that they themselves experienced. Overwhelmingly that is the traditional method, though the fact that no one has been able to make this approach work (for the majority of students) in three-thousand years does make some wonder if there is a better way.

Now, at last, there is evidence, and more is being gathered. This means that raw belief and blind faith can finally start to be replaced by a reasoned choice, based on the evidence. This will surely happen, but how long it will take, after such a bitter battle, remains to be seen. Most likely the conflict will be fully put to rest only after the same has happened to some of the more prominent proponents. Meanwhile, expect to see gradual change as more teachers, parents, and politicians become aware of the rising mass of hard data.

One of the researchers who has been patiently gathering that data is Jo Boaler, and she recently published a short, readable summary of some of her findings in the form of a book: What's Math Got To Do With It? Though written for parents, I believe Boaler's book should be read by everyone involved in mathematics education. You may question some of her conclusions. Indeed, some did when her work first became known in the US, around 2003, though her subsequent research has, in my view, answered some of the questions raised. But in a field like mathematics education, where conclusive hard evidence is so hard to come by, and where the majority of claims made about the efficacy of various pedagogies are based on nothing more than an extrapolation from personal experience (of the teacher, not the student), any in-depth study such as hers deserves to be seriously considered. Not least because Boaler focuses her study not on the teachers but on the students being taught. Her recent book forms the basis for the remainder of this essay. The quote I took for my title can be found on page 40.

Boaler began her career as a math teacher in her native UK, transferred to academia (London University), and was for several years a professor of mathematics education (and hence a colleague of mine) at Stanford. Then, in 2006, she returned to the UK to take up the newly established Marie Curie Chair in Mathematics Education at the University of Sussex in England, This summer, she is returning to Stanford, where we are most eager to have her back.

Over many years, Boaler conducted interviews with hundreds of students from both traditionally taught math classes and those with a more progressive approach. One of the questions she asked them what it took to be successful in math. By far the most common answer she received from students taught in a traditional fashion was to *pay careful attention*.

Among other answers Boaler received in schools with a traditional pedagogy, which she quotes in her book, are [p.41]:

"I'm just not interested in, first, you give me a formula, I'm supposed to memorize the answer, apply it, and that's it."

"You have to be willing to accept that sometimes things don't look like - they don't see that you should do them. Like they have a point. But you have to accept them."

Another traditionally-taught student Boaler interviewed, called Rebecca, was conscientious, motivated, and smart, and regularly attained A+ grades in mathematics. She was able to follow the methods her teacher demonstrated in class, and could reproduce them perfectly. But she did not understand what she was doing, and as a result she regarded herself as not good at math. When Boaler asked her why she thought that, she replied, "Because I can't remember things well and there is so much to remember." [p.164.]

The school by the tracks

Over a four-year period, Boaler followed the progress of seven hundred students through their high school careers at three high schools. One of the three was "Railside High". Not its real name, this school was in an urban setting, close by a railway line. She first visited the school in 1999, having heard that they seemed to be achieving remarkable results, despite the poor location and run-down appearance of the school buildings.

A number of features singled out Railside. First, the students were completely untracked, with everyone taking algebra as their first course, not just the higher attaining students. Second, instead of teaching a series of methods, such as factoring polynomials or solving inequalities, the school organized the curriculum around larger themes, such as "What is a linear function?" The students learned to make use of different kinds of representation, words, diagrams, tables symbols, objects, and graphs. They worked together in mixed ability groups, with higher attainers collaborating with lower performers, and they were expected and encouraged to explain their work to one another. [pp.58-68]

Parents whose own math education was more traditional, with the students sitting in rows, in ability-streamed classes, being shown methods by the teacher and then working silently on their own - and that is practically all parents - often find it hard to believe that the Railside approach could work. They believe the loose structure will mean the kids won't master skills well enough to pass tests, and that the presence of weaker students will drag down the better ones. Often they maintain this belief despite freely admitting that the traditional approach did not work for them, and contrary to their own experiences every day at work, where over many years they have come to know that collaborative working is highly effective, and that when someone who knows how to do something assists someone who does not, *both* learn and benefit from the experience.

In the nineteenth century and for much of the twentieth, most industrial workers *did* work silently on their own, in large open offices or on production lines, under the supervision of a manager. Schools, which have always been designed to prepare children for life as adults, were structured similarly. An important life lesson was to be able to follow rules and think *inside* the box. But today's world is very different - at least for those of us living in highly developed societies. Companies long ago adopted new, more collaborative ways of working, where creative problem solving is the key to success - the ones that did not went out of business - but by and large the schools have not yet realized they need to change and start to operate in a similar fashion.

Of course, it may, as many parents seem to assume, be different in schools. After all, they will argue, what works for adults may not be successful for children. That's a fair concern. It's a concern that is addressed head on by Boaler's findings. The other two schools Boaler studied along with Railside were in more affluent suburban settings, and the students started out with higher mathematics achievements than did those at the urban Railside school. Since those two schools adopted a traditional form of instruction, Boaler was able to compare student outcomes over the entire four years of high school. By the end of the first year, she found that the Railside students were achieving at the same levels as the suburban students on tests of algebra. By the end of the second year, the Railside students were outperforming their counterparts in the two suburban schools in both algebra and geometry tests. By their senior year, 41 percent of Railside students were in advanced classes of precalculus and calculus, compared to only 23 percent of students from the other two schools in more affluent neighborhoods.

What's more, the Railside students learned to enjoy math, and saw it as useful. When Boaler and her team interviewed 105 students (mainly seniors) about their future plans, 95 percent of the students from the two suburban school said they did not intend to pursue mathematics as a subject any further, even those who had been successful. At Railside, 39 percent said they planned to take further math courses.

When Boaler would visit a class being taught in a Railside-like fashion and ask students what they were working on, they would describe the problem and how they were trying to solve it. When she asked the same questions of students being taught the traditional way, they would generally tell them what page of the book they were on. When she asked them, "But what are you actually doing?" they would answer "Oh, I'm doing number 3." [p.98]

The Brits make the same mistake

Prior to coming to Stanford, while she was still working in her native UK, Boaler had begun a similar longitudinal study, comparing two very different schools that she called Phoenix Park (in a working class area) and Amber Hill (located in a more affluent neighborhood). The former adopted a collaborative, project-based approach, similar to Railside, the latter a more traditional pedagogy. [pp.69-83]

Boaler had chosen these two schools because, despite being in different social regions, their student intakes were demographically very similar, their entering students at age thirteen had all experienced the same educational approach, and the teachers at both schools were well qualified.

One difference between the English schools and those in California is that the UK does not follow the idiotic US practice of dividing mathematics up into separate sub-subjects, such as Algebra I, Algebra II, or Geometry; rather they just learn math (or "maths" as it's called in Britain). But other than that, this was very much like the study she would subsequently conduct in California, and the results were strikingly similar.

At Phoenix Park, students were given considerable freedom in math classes. They were usually given choices between different projects to work on, and they were encouraged to decide the nature and direction of their work. One student explained to Boaler how they worked with these words: [p.70]

"We're usually set a task first and we're taught the skills needed to do the task, and then we get on with the task and we ask the teacher for help."

Another described the process like this: [p.70]

"You're just set the task and then you go about it ... you explore the different things, and they help you in doing that ... so different skills are sort of tailored to different tasks."

In one task Boaler describes, the students were simply told that a certain object had volume 216, and asked to describe what it might look like. In another, the students were told that a farmer had thirty-six meter-long lengths of fencing and asked to find the largest area the fences could enclose.

If you think either of these is "shallow" or "not real math" then almost certainly you are living, walking proof that traditional math instruction deadens the mind to see the many possibilities each task offers, and the amount of mathematical thinking required to carry out the investigations. In her book, Boaler sketches some of the creative thinking the Phoenix Park students brought into the two tasks, and the mathematics learning that resulted. To my mind, what she describes is the early development of the creative, collaborative, problem solving skills that are essential in today's world. As one student explained to her: [p.74]

"If you find a rule or a method, you try to adapt it to other things."

While the Phoenix Park students were discovering that math is challenging and fun, providing an excellent outlet for their natural human curiosity, things were going very differently over at Amber Hill. There, the students worked hard, but most of them disliked mathematics. They came to believe that math was a subject that only involved memorizing rules and procedures. As one student put it: [p.75]

"In maths, there's a certain formula to get to, say from A to B, and there's no other way to get it. Or maybe there is, but you've got to remember the formula, you've got to remember it."

It was at Amber Hill that a student provided Boaler with the quotation that forms the title of this month's column.

Though the Amber Hill students spent more time-on-tasks than their counterparts at Phoenix Park, they thought math was a set of rules to be memorized. The ones that were successful did so not by understanding the mathematical ideas but by learning to follow cues. The biggest cue telling them how to solve a problem was the method the teacher had just explained on the board, or the worked example that immediately preceded it in the textbook. Another cue was to use all the information provided in the question, but nothing else.

That strategy can be made to work well until the examination at the end of the year, when those cues are not present. Predictably, even those Amber Hill students who did well during the term did poorly in those exams. And, in the national exams that all British students take at age sixteen, the Phoenix Park students easily outperformed them. Faced with a problem they did not recognize as being of a familiar type, an Amber Hill student might freeze, or struggle in vain to remember the right formula, whereas Phoenix Park students tried to make sense of it, and adapt a method they thought could be made to work.

In addition to her classroom studies at the two schools, Boaler also interviewed the students about their use of mathematics out of school. By then, many of them had weekend jobs. All forty of the Amber Hill students she interviewed declared they would never, ever make use of their school-learned methods in any situation outside school. To them, what they had been taught in the math class was a strange sort of code that can be used in only one place, the math classroom. In contrast, the Phoenix Park students were confident they would make use of the methods they had learned at school, and they gave her examples of how they had already made use of their school-learned math in their weekend jobs.

In a follow up study she conducted some years later, Boaler surveyed the then twenty-four year old graduates from Phoenix Park and Amber Hill. When they had been at school, their social class, as determined by their parents' jobs, were the same at both schools. But eight years later, the young adults from Phoenix Park were working in more highly skilled or professional jobs than the Amber Hill adults. Demonstrating how good education can lead to upward social mobility, 65 percent of the Phoenix Park adults were in jobs more professional than their parents, compared with 23 percent of Amber Hill adults. In fact, 52 percent of Amber Hill adults were in *less* professional jobs than their parents, compared with only 15 percent of the Phoenix Park graduates. [pp.80-83]

Of course, you won't get this information from reading the computer-generated scores from the standardized tests so-beloved of the US education system. Boaler does not find her data by gazing at a computer screen. She goes out and talks to the people education is all about: the students and those who were students. I ask you, which is the more important information: the score on a standardized, written test taken at the end of an educational episode, or the effect that educational episode had on the individual concerned? As a parent (if you are one), which statement would give you more pleasure?:

- "Because of good teaching, my child scored 79% on her last math test," or
- "Because of good teaching, my child has a much better job and leads a far more interesting and rewarding life than me."

Of course, teaching math in the progressive way requires teachers with more mathematical knowledge than does the traditional approach (where a teacher with a weaker background can simply follow the textbook - which incidentally is why American math textbooks are so thick). It is also much more demanding to teach that way, which makes it a job that deserves a far higher status and better pay-scale than are presently the case. And it's a lot harder to collect the data to measure the effectiveness of the education, since it means looking at the actual products of the process: real, live people. Welcome to life in the global knowledge economy of the twenty-first century. Do you want to stay in the game, America?