

Invited Review

Research on warehouse operation: A comprehensive review

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Abstract

An extensive review on warehouse operation planning problems is presented. The problems are classified according to the basic warehouse functions, i.e., receiving, storage, order picking, and shipping. The literature in each category is summarized with an emphasis on the characteristics of various decision support models and solution algorithms. The purpose is to provide a bridge between academic researchers and warehouse practitioners, explaining what planning models and methods are currently available for warehouse operations, and what are the future research opportunities.

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1. Introduction

Warehouses are an essential component of any supply chain. Their major roles include: buffering the material flow along the supply chain to accommodate variability caused by factors such as product seasonality and/or batching in production and transportation; consolidation of products from various suppliers for combined delivery to customers; and value-added-processing such as kitting, pricing, labeling, and product customization.

Market competition requires continuous improvement in the design and operation of produc-

tion-distribution networks, which in turn requires higher performance from warehouses. The adoption of new management philosophies such as Just-In-Time (JIT) or lean production also brings new challenges for warehouse systems, including tighter inventory control, shorter response time, and a greater product variety. On the other hand, the widespread implementation of new information technologies (IT), such as bar coding, radio frequency communications (RF), and warehouse management systems (WMS), provides new opportunities to improve warehouse operations. These opportunities include, but are not limited to: real-time control of warehouse operation, easy communication with the other parts of the supply chain, and high levels of automation.

A number of warehouse operation decision support models have been proposed in the literature, but there remains considerable difficulty in applying

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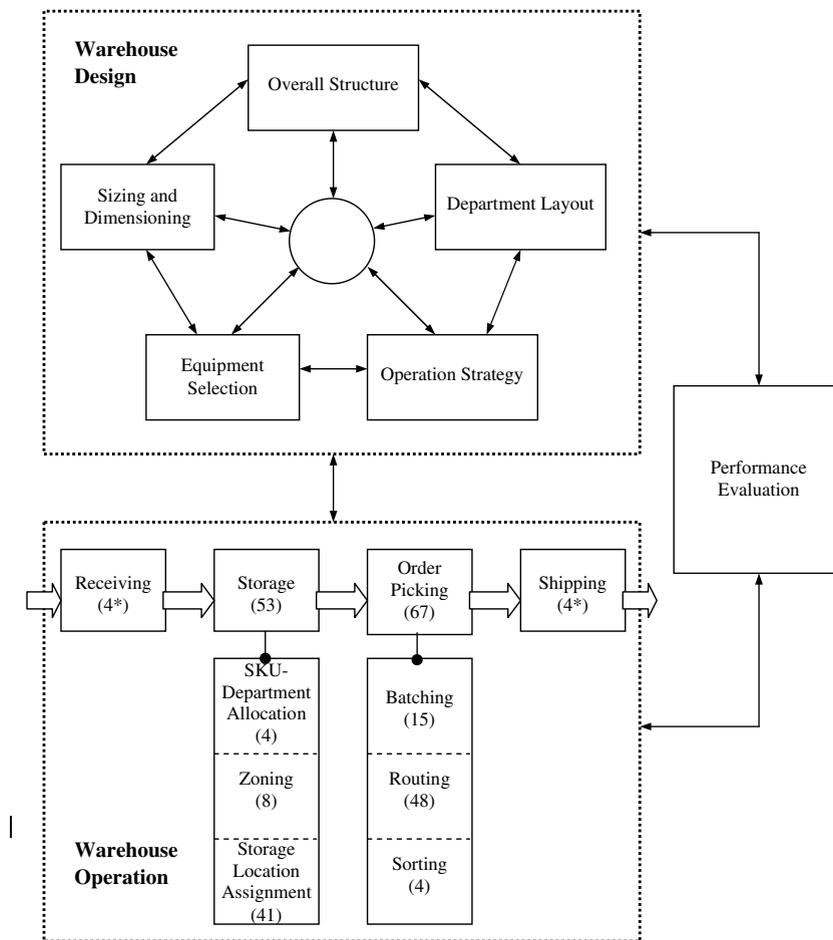
these models to guide warehouse operations. The objective of this paper is to classify and summarize the prior research results, and to identify research opportunities for the future. The intended outcome is both a guide to practitioners on the analytical methodologies and tools available to support better warehouse operation planning, and a roadmap for academic researchers to future research opportunities.

This paper presents a comprehensive review of the state-of-the-art in research on warehouse operation planning. We first present a unifying framework to classify the research on different but related warehouse problems. Within this framework, historical progress and major results are summarized with an emphasis on how the research on these problems evolved and the relationships between various problems. Future research direc-

tions are identified and discussed. The scope of this paper is restricted to warehouse operation-planning methods. There are a lot of related results on performance evaluation, which we believe deserve a separate discussion since it is a key issue in warehouse design and operation that provides the basis for intelligent decision-making. The companion paper (Gu et al., 2005) provides a detailed discussion on this topic together with warehouse design, computational systems, and case studies. Readers may also refer to Rowenhorst et al. (2000) for a recent survey on the overall warehouse design and operation problems.

2. Framework

The basic requirements in warehouse operations are to receive Stock Keeping Units (SKUs) from sup-



* This number represents papers on both receiving and shipping.

Fig. 1. Framework for warehouse design and operation problems.

pliers, store the SKUs, receive orders from customers, retrieve SKUs and assemble them for shipment, and ship the completed orders to customers. There are many issues involved in designing and operating a warehouse to meet these requirements. Resources, such as space, labor, and equipment, need to be allocated among the different warehouse functions, and each function needs to be carefully implemented, operated, and coordinated in order to achieve system requirements in terms of capacity, throughput, and service at the minimum resource cost.

A scheme to classify warehouse design and operation planning problems and the corresponding literature is shown in Fig. 1 (the numbers in parentheses represent the numbers of papers reviewed in this document for each operation planning problem) and a more detailed description of each problem category identified is given in Table 1. This paper will focus on the operation planning

problems, while warehouse design and performance evaluation are discussed in Gu et al. (2005).

Receiving and shipping are the interface of a warehouse for incoming and outgoing material flow. Incoming shipments are brought to the warehouse, unloaded at the receiving docks, and put into storage. Orders are picked from storage, prepared, and shipped to customers through shipping docks. Receiving and shipping operations involve, for example, the assignment of trucks to docks and the scheduling of loading and unloading activities. Research on receiving and shipping is very limited, and will be reviewed together in Section 3.

Storage is concerned with the organization of goods held in the warehouse in order to achieve high space utilization and facilitate efficient material handling. Goods in storage can be organized into different departments. The drivers of department organization may be physical characteristics of the

Table 1
Description of warehouse design and operation problems

Design and operation problems		Decisions	
Warehouse design	Overall structure	<ul style="list-style-type: none"> • Material flow • Department identification • Relative location of departments 	
	Sizing and dimensioning	<ul style="list-style-type: none"> • Size of the warehouse • Size and dimension of departments 	
	Department layout	<ul style="list-style-type: none"> • Pallet block-stacking pattern (for pallet storage) • Aisle orientation • Number, length, and width of aisles • Door locations 	
	Equipment selection	<ul style="list-style-type: none"> • Level of automation • Storage equipment selection • Material handling equipment selection (order picking, sorting) 	
	Operation strategy	<ul style="list-style-type: none"> • Storage strategy selection (e.g., random vs. dedicated) • Order picking method selection 	
Warehouse operation	Receiving and shipping	<ul style="list-style-type: none"> • Truck-dock assignment • Order-truck assignment • Truck dispatch schedule 	
	Storage	SKU-department assignment	<ul style="list-style-type: none"> • Assignment of items to different warehouse departments • Space allocation
		Zoning	<ul style="list-style-type: none"> • Assignment of SKUs to zones • Assignment of pickers to zones
		Storage location assignment	<ul style="list-style-type: none"> • Storage location assignment • Specification of storage classes (for class-based storage)
	Order picking	Batching	<ul style="list-style-type: none"> • Batch size • Order-batch assignment
		Routing and sequencing	<ul style="list-style-type: none"> • Routing and sequencing of order picking tours • Dwell point selection (for AS/RS)
Sorting		<ul style="list-style-type: none"> • Order-lane assignment 	

goods (e.g., pallet storage vs. case storage); management considerations such as a dedicated storage area for a specific customer; or material handling considerations such as a forward area for fast picking. Within departments, goods may be further organized into pick zones. A pick zone is a set of storage locations that are often arranged in close physical proximity. A particular pick zone holds a limited subset of the SKUs, and pickers may be dedicated to one or more zones to pick the required items. Because of the limited physical size of the zone, the picker achieves a high ratio of SKU extracting time to traveling time between locations and an increased familiarity with the SKUs in the zone. Within a department/zone, goods are assigned to storage locations, and the storage location assignment has significant impact on storage capacity, inventory tracking, and order picking. Different storage strategies can be used such as random, class-based, and dedicated storage. The selection of which storage strategy to use is considered a design problem and therefore is discussed in Gu et al. (2005). However, the implementation of each storage strategy is an operational issue (e.g., using a particular rule to assign SKUs to storage locations for dedicated storage), and therefore is discussed in Section 4.3.

Order picking is generally recognized as the most expensive warehouse operation, because it tends to be either very labor intensive or very capital intensive (Frazelle, 2002). Managing the order picking process requires the organization of the orders to be picked and of the material handling operations of the picking. There are different order picking methods, for example, single-order picking, batching with sort-while-pick, batching with sort-after-pick, sequential zone picking with single order, sequential zone picking with batching, concurrent zone picking without batching in the zones, and concurrent zone picking with batching in the zones. The selection of an order picking method is a strategic decision since it has a wide impact on many other decisions in warehouse design and operation. For example, a downstream sortation system is needed if sort-after-pick is used. The topic of order picking system selection is covered more thoroughly in Gu et al. (2005). The focus here is at the operational planning level of order picking. Several basic decisions need to be made at the operational planning level, which include pick wave sizing, batching, routing, and sorting. The planning of batching, routing, and sorting will be discussed in

detail in Section 5. Research on pick wave sizing is very limited, and therefore will not be further discussed.

3. Receiving and shipping

Goods arrive to a warehouse in a carrier and are unloaded at the receiving docks. Later they are loaded into a carrier and leave the warehouse through the shipping docks. For cross-docking warehouses, received goods are sent directly from the receiving docks to the shipping docks. For traditional warehouses that hold inventory, received goods are put away into storage and later picked and shipped through shipping docks. In this case, the receiving and shipping operations are more complex to manage since they are coupled with the storage and order picking function. For example, the scheduling of shipping trucks may depend on how orders are batched and assigned to picking waves and vice versa.

The basic decisions in receiving/shipping operations can be described as

Given:

- (1) Information about incoming shipments, such as their arrival time and contents.
- (2) Information about customers demands, such as orders and their expected shipping time.
- (3) Information about warehouse dock layout and available material handling resources.

Determine:

- (1) The assignment of inbound and outbound carriers to docks, which determines the aggregate internal material flows.
- (2) The schedule of the service of carriers at each dock. Assuming a set of carriers is assigned to a dock, the problem is similar to a machine-scheduling problem, where the arriving carriers are the jobs to be scheduled.
- (3) The allocation and dispatching of material handling resources, such as labor and material handling equipment.

Subject to performance criteria and constraints such as:

- (1) Resources required to complete all shipping/receiving operations.
- (2) Levels of service, such as the total cycle time and the load/unload time for the carriers.

- (3) Layout, or the relative location and arrangement of docks and storage departments.
- (4) Management policies, e.g., one customer per shipping dock.
- (5) Throughput requirements for all docks.

Decision making in receiving and shipping is limited by the level of prior knowledge about incoming and outgoing shipments, for which the following scenarios can be distinguished:

- No knowledge, other than warehouse layout.
- Partial statistical knowledge of arriving and departing processes, such as the average level of material flow from an incoming carrier to an outgoing carrier.
- Perfect knowledge of the content of each arriving carrier and each departing carrier.

In the first scenario, not only do we have no basis for assigning carriers to docks, we also have no basis for assigning goods to storage locations. It is not clear in this case if any storage assignment rule is preferred to any other. Public warehouses may operate under this set of conditions. The second scenario is most common in company-owned or dedicated distribution warehouses and is the basis for most of the decision models in the literature. The third scenario is becoming increasingly common through the application of advanced information technologies such as RFID, GPS, and advanced shipping notices (ASN).

The research on receiving and shipping has been focused on the carrier-to-dock assignment problem for cross-docking warehouses, assuming statistical knowledge of incoming and outgoing shipments. The cross-docking warehouse is operated as follows: inbound trucks arrive in the yard of the warehouse and proceed to the assigned receiving doors (or strip doors) for unloading; the unloaded goods are sorted according to their destinations, and then loaded onto outbound trucks at shipping doors (or stack doors) for delivery to customers. Often, each stack door is designated to a particular destination, and once established, the designations of stack doors generally do not change. The decisions for a cross-docking warehouse manager are then to designate the doors as either strip or stack doors, assign destinations to stack doors, and assign inbound trucks to strip doors in order to minimize the total operational cost.

Assuming the designations of doors as either strip or stack doors have already been made, Tsui

and Chang (1990, 1992) formulate a bilinear model to assign inbound and outbound trucks to strip and stack doors, respectively. Gue (1999) proposes a model to estimate the operational cost by optimally assigning inbound trucks to strip doors given the specification of doors as either strip or stack doors and the assignment of destinations to stack doors. Based on the cost model, he uses a local search procedure to find an efficient door layout. Bartholdi and Gue (2000) consider the cross-docking warehouse door layout problem with the objective of minimizing the total travel time and waiting time incurred due to congestion. They model the total travel time and waiting time for a fixed door layout using transportation and queuing models and then embed the cost model in a simulated annealing algorithm to find an efficient door layout.

In summary, very few formal models have been developed for the management of shipping and receiving operations. Most of the literature that is available in this area addresses shipping and receiving operations and truck-to-dock assignment strategies for cross-docking warehouses.

4. Storage

Storage is a major warehouse function. Three fundamental decisions shape the storage function, i.e., how much inventory should be kept in the warehouse for an SKU; how frequently and at what time should the inventory for an SKU be replenished; and where should the SKU be stored in the warehouse and distributed and moved among the different storage areas. The first two questions lead to the lot sizing and staggering problems, respectively, which belong to the traditional inventory control area and are not further discussed here. The readers may refer to Gallego et al. (1996) and Hariga and Jackson (1996) for a detailed review. This section will focus on the storage assignment question, which includes the decisions of assigning SKUs to various storage departments and scheduling of inventory moves between the departments, of assigning SKUs to different zones (zoning), and of the storage location assignment within a department/zone. The two major criteria in making these decisions are the storage efficiency, which corresponds to the holding capacity, and the access efficiency, which corresponds to the resources consumed by the insertion (store) and extraction (order picking) processes.

4.1. Assigning SKUs across departments

A SKU may be stored in more than one warehouse department. The specification of departments is a design decision. Once the departments are specified, one needs to determine which SKU should be stored in which department, in what quantity, and what are the corresponding inter-departmental moves for that SKU. In some cases, this decision is straightforward. For example, if a department is dedicated to a certain customer, then all SKUs for that customer are assigned to that department; or if a SKU will be stored and picked only in units of pallets, then it will be assigned only to the pallet storage department. In other cases, a SKU could be assigned to multiple departments. These departments are usually different in terms of their storage and material-handling capability. Therefore, a careful decision needs to be made in order to balance the tradeoff between storage and material handling cost and capacities.

The forward-reserve problem belongs to this category and is a well-researched problem. It is a common practice in warehousing to create a separate physically compact forward (or “fast pick”) area for picking high-demand, fast-moving products. This reduces order picking costs but at the expense of requiring additional material handling to restock the forward area from a reserve area, and additional space as storage is less efficient in the forward area than in the reserve area. Furthermore, since the size of the forward area usually is limited, one needs to determine which SKUs should be stored in the forward area and in what quantity.

Bozer (1985) first introduces the problem of splitting a pallet rack into an upper reserve area and a lower forward picking area. Hackman and Rosenblatt (1990) treat the problem of deciding which SKUs to assign to the forward area, and how to allocate space among the assigned SKUs, given the forward area has a fixed capacity. The objective is to minimize the total material handling costs of order picking and replenishing. They propose a knapsack-based heuristic to solve this problem and provide sufficient conditions for optimality of this heuristic. Frazelle et al. (1994) extend the problem and solution method of Hackman and Rosenblatt (1990) by treating the size of the forward area as a decision variable. The costs in their model include the equipment cost of the fast pick area (modeled as a linear function of its size), and the material handling cost for order picking and replenishment.

The above models assume the replenishment of a SKU can be done in a single trip. van den Berg et al. (1998) consider the problem for unit-load replenishments, i.e., only one unit can be replenished per trip. Assuming the forward area can be replenished instantaneously there is no need to assign more than one unit to the forward area. They consider warehouses that have busy and idle periods, so it is possible to reduce the number of replenishments during busy periods by performing replenishments in the preceding idle periods. A knapsack-based heuristic is proposed to find the set of SKUs to put in the forward area that minimizes the expected total labor-time related to order-picking and replenishing during a busy period.

4.2. Assigning SKUs across zones (zoning)

The zoning problem is to specify different storage zones within a department and assign SKUs to the specified zones. It can be both a “hard” and a “soft” decision; it is a hard decision if it leads to zone-specific storage technology selection and physical arrangement, but it is a soft decision if it is simply an organization of similar storage locations. Thus, zoning decisions fall in between warehouse design decisions and warehouse operation decisions.

A primary reason for dividing a storage department into zones is to organize order picking activities (i.e., zone picking). The fundamental advantages of zone picking are the limited space the picker has to traverse to pick an order, the increased familiarity of the picker with a subset of the SKUs, and the reduced order picking time span for an order if zones are picked in parallel. On the other hand, additional costs may be incurred in zone picking, caused by sorting in parallel zone picking and by the queuing in sequential zone picking. Storage needs to be planned for zone picking to determine the specification (the number, size, and shape) of the zones and to assign SKUs to zones in such a way that minimizes the total order picking cost and balance the workloads across zones. The literature on the storage planning for zone picking is very limited. Gray et al. (1992) present a hierarchical framework for designing warehouses with zone picking to determine the number of zones and pickers, zone sizes (storage spaces per zone), storage assignment across and within zones, and order batch size. The effects of zone shape (i.e., the number of aisles per zone and the length of aisles) on operational cost is

investigated by Petersen (2002) with simulation. It is shown that zone shape has a substantial impact on the operational cost depending on factors such as the zone size and the batch size. Algorithms for assigning SKUs to zones can be found in Jane (2000) and Jewkes et al. (2004). Jane (2000) proposes a simple heuristic approach that assigns SKUs to zones to balance the workloads of pickers. Jewkes et al. (2004) consider a specific sequential zone picking method where pickers work at home bases within their zones and are required to return to their home bases after each pick. An optimal approach is proposed to determine the zones, the assignment of SKUs to zones, and the base locations in order to minimize the expected total order picking cost.

4.3. Storage location assignment

The storage location assignment problem (SLAP) is to assign incoming products to storage locations in storage departments/zones in order to reduce material handling cost and improve space utilization. Different warehouse departments might use different SLAP policies depending on the department-specific SKU profiles and storage technology. The storage location assignment problem is formally defined as follows:

Given:

- (1) Information on the storage area, including its physical configuration and storage layout.
- (2) Information on the storage locations, including their availability, physical dimensions, and location.
- (3) Information on the set of items to be stored, including their physical dimensions, demand, quantity, arrival and departure times.

Determine:

The physical location where arriving items will be stored.

Subject to performance criteria and constraints such as:

- (1) Storage capacity and efficiency.
- (2) Picker capacity and efficiency based on the picker cycle time.
- (3) Response time.
- (4) Compatibility between products and storage locations and the compatibility between products.

- (5) Item retrieval policy such as FIFO (first-in, first-out), LIFO (last-in, first-out), BFIFO (batch first-in, first-out). When using the BFIFO policy, items that arrived in the same replenishment batch are considered to be equivalent.

In typical warehouse operations, the physical storage infrastructure and its characteristics are known when planning the storage location assignment. The availability of storage locations is always known in automated warehouses and often known in mechanized warehouses. SLAP can be divided into three classes depending on the amount of information known about the arrival and departure of the products stored in the warehouse: (1) item information, (2) product information, or (3) no information. Different operational policies exist for each of these classes, and their implementation and performance have been discussed extensively in the literature. Most of the research has focused on unit-load warehouses. Of course, these SLAP policies can be applied to non unit-load warehouses as well, but it is usually much more difficult to provide analytical results because of the complexity of computing the associated material handling times and cost involved in a non unit-load warehouse (e.g., when batching and routing are used).

4.3.1. Storage location assignment problem based on item information (SLAP/II)

In the SLAP/II problem, it is assumed that complete information is known about the arrival and departure time of the individual items. The resulting problem is a specially structured Assignment Problem (AP), where items are assigned to storage locations. The special structure derives from the property that two items can occupy the same storage location, provided they do not occupy it at the same time. This problem has been called the Vector Assignment Problem (VAP), since the occupation is no longer expressed as a single binary status variable but as a vector over the different time periods (Goetschalckx, 1998). The optimal solution of this problem for typical warehousing operations is computationally impractical because of the very large problem instances. The problem is of interest in academic research on warehouse operations because it provides a cost lower bound or performance upper bound. An example of a heuristic SLAP/II policy is the Duration-of-Stay (DOS) policy of Goetschalckx and Ratliff (1990). In DOS-based policies,

the expected DOS of the i th unit of a SKU with replenishment lot size Q is i/λ for $i = 1, 2, \dots, Q$, where λ is the demand rate of that SKU. Then the items of all the different products having the shortest DOS are assigned to the closest locations. Hence, the items of a single replenishment batch of a single product may not be stored together in the warehouse.

4.3.2. Storage location assignment problem based on product information (SLAP/PI)

Often only product information is known about the items to be stored, and items are instances of products. Products may be classified into product classes, e.g. by size or usage rate. The assignment problem now assigns an individual item to a product class based on its product characteristics, and assigns a product class to storage locations. The location of an item in its class is most often done using some simple rule, such as nearest location, or randomly. If the number of classes is equal to the number of products, then this policy is called Dedicated Storage. If the number of classes is equal to one, it is called Random Storage. Otherwise, it is called Class-Based Storage, which may have any number of storage classes ranging from two to the number of products minus one (2–5 storage classes are commonly used in warehouse operations).

Different criteria can be used to assign a product (class) to storage locations. The three most frequently used criteria (see also Frazelle, 2002) are

- (1) Popularity (defined as the number of storage/retrieval operations per unit time period). For the popularity policy, product classes are ranked by decreasing popularity and the classes with the highest popularity are assigned the most desirable locations.
- (2) Maximum inventory (defined as the maximum warehouse space allocated to a product class). For the maximum inventory policy, product classes are ranked by increasing maximum inventory and the classes with the lowest maximum inventory are assigned the most desirable locations.
- (3) Cube-Per-Order Index (COI, which is defined as the ratio of the maximum allocated storage space to the number of storage/retrieval operations per unit time). The COI policy takes into consideration both a SKU's popularity and its storage space requirement. Product

classes are ranked by increasing COI value and the classes with the lowest COI are stored in the most desirable locations.

The implementation of the above policies depends on the types of warehouse systems and therefore may have different variations, for example:

- (1) If storage space is measured in units (e.g., shelves and bays), each unit can be treated as an individual product by appropriately apportioning demand. This is most commonly used in unit load warehouses (e.g., Hausman et al., 1976) and sometimes in less-than-unit-load warehouses (e.g., Jarvis and McDowell, 1991). Since each unit load occupies the same amount of storage space, the popularity policy based on the apportioned popularity is essentially the same as the COI policy. However, it is different from the popularity policy without apportioning. For example, suppose product A has three unit loads and a popularity of three picks per day, and product B has one unit load and a popularity of two picks a day. The popularity policy without apportioning will rank product A ahead of product B. On the other hand, if product A is treated as three products (denoted as A1, A2, and A3), each of them will have an apportioned popularity of 1 pick per day. So the popularity policy based on the apportioned popularity will now rank product B ahead of product A1, A2, and A3, which can be easily verified to be equivalent to the COI policy.
- (2) The definition of “the most desirable locations” depends on the system as well as the travel pattern. For example, if traversal routing policy is used for traveling in a conventional multi-parallel-aisle system, the desirability of locations are measured in terms of aisles where the most desirable locations are in the aisle that is closest to the I/O point. This leads to the so-called organ pipe storage location assignment, for example, see Jarvis and McDowell (1991).

The above three policies are simple and flexible enough to be implemented in different warehouse systems. Among them, the COI policy has been the most comprehensively studied one. The COI policy was first described by Heskett (1963, 1964)

without a proof of its optimality. [Kallina and Lynn \(1976\)](#) discussed the implementation of the COI policy in practice. It has been proved that the COI policy is optimal in minimizing the material handling cost in dedicated storage when some assumptions are satisfied:

- (1) The objective is to minimize the long-term average order picking cost.
- (2) The travel cost depends only on locations. Examples that do not satisfy this assumption include the case when the travel cost is item dependent or when there are multiple I/O points, and products have different probability of moving from/to the I/O points, i.e., it does not satisfy the factoring assumption as defined in [Mallette and Francis \(1972\)](#).
- (3) When dual or multi-command order picking is used, there is no dependence between the picked items in the same picking tour.
- (4) Certain routing policies are assumed for multi-command order picking, e.g., [Jarvis and McDowell \(1991\)](#) assume the traversal routing policy for the conventional multi-aisle order picking system.

- (5) There are no compatibility constraints that limit the storage location assignment, e.g., certain items must and/or cannot be put together.

[Table 2](#) summarizes the results on COI-based dedicated storage and its optimality in different order picking systems based on the above assumptions; [Table 3](#) provides a group of related heuristic algorithms for dedicated storage when these assumptions cannot be satisfied, and therefore the COI rule is not directly applicable.

Comparing dedicated storage with random storage, the former has the advantage of locating fast-moving and compact SKUs close to the I/O points, and therefore is beneficial for efficient material handling. However, it also requires more storage space since sufficient storage locations must be reserved for the maximum inventory of each product. Class-based storage provides an alternative that is in between and has the benefits of both dedicated and random storage. The implementation of class-based storage (i.e., the number of classes, the assignment of products to classes, and the storage locations for each class) has significant impact on the required storage space and the material handling

Table 2
COI-based dedicated SLAP policy and its optimality in different systems

	Single-command	Dual-command	Multi-command	Carousel
COI rules and its variants	Mallette and Francis (1972) , Harmatuck (1976)	Malmborg and Krishnakumar (1987) , Malmborg and Krishnakumar (1990)	Malmborg and Krishnakumar (1989) , Jarvis and McDowell (1991)	Bengu (1995) , Vickson (1996) , Vickson and Lu (1998)

Table 3
Other dedicated SLAP policies with different complications

Citation	Problem summary	Algorithm
Montulet et al. (1998) Lee (1992) , Rosenwein (1994) , Brynzer and Johansson (1996) , van Oudheusden and Zhu (1992) , Liu and Lu (1999) Malmborg (1995)	The objective is to minimize the peak operations cost Items are not independent such that some items are more likely to appear on the same order	Branch and Bound Cluster analysis; Space filling curve based heuristics
Lai et al. (2002) , Zhang et al. (2000) , Zhang et al. (2002)	All items of any SKU must be located in the same aisle in a multi-aisle AS/RS system Storage location assignment is constrained by product size; all items of the same product must be placed at adjacent locations; and travel costs are item dependent	Random search plus simulated annealing Simulated annealing; Genetic algorithms
Hwang et al. (2003)	Product weight is considered and the objective is to minimize the work (a function of weight and distance) involved in order picking	A heuristic similar to COI

cost in a warehouse. Research on this problem has been largely focused on AS/RS, especially single-command AS/RS. Hausman et al. (1976) show that for single-command AS/RS with the Chebyshev metric, the ideal shape of storage regions is L-shaped. For such systems, the problem reduces to determining the number and boundaries of the classes. Explicit analytical solutions for the class boundaries can be derived for the case with 2 or 3 classes, as shown by Hausman et al. (1976), Kouvelis and Papanicolaou (1995), and Eynan and Rosenblatt (1994). For the general n -class case, Rosenblatt and Eynan (1989) and Eynan and Rosenblatt (1994) suggest a one-dimensional search procedure to find the optimal boundaries. The implementation of class-based storage in multi-command AS/RS is discussed in Guenov and Raeside (1992).

4.3.3. Storage location assignment problem based on no information (SLAP/NI)

If no information is available on the characteristics of the arriving items, only very simple storage policies can be constructed. In this case the most frequently used policies are (1) Closest-Open-Location (COL), (2) Farthest-Open-Location (FOL), (3) Random (RAN), and (4) Longest-Open-Location (LOL). The first two policies pick an open location based on its distance to the receiving dock; the last policy picks the location that has been vacant for the longest time. It is not known if there is any significant performance difference between them.

4.3.4. SLAP summary

In practice, SLAP/PI is much more common than SLAP/II and SLAP/NI. Random, dedicated, and class-based storage are three popular used storage strategies, and each of them has its advantages and disadvantages. The selection of storage strategy is a strategic decision, which affects warehouse design and has long-term effects. For example, if random storage is used instead of dedicated storage, the warehouse might have a smaller size but require more effort to accurately track the inventory. This topic is further discussed in Section 2.5 of Gu et al. (2005).

Once a storage strategy is selected, its implementation is an operational problem. The implementation of random storage is relatively straightforward. For dedicated and class-based storage, the implementation involves assigning products/classes to storage location. The COI policy has been extensively studied in the literature and is considered as more effective

than the other two policies. In class-based storage, additional decisions are to determine the number of classes and to assign products to classes. Current results on these decisions have been focused mainly on AS/RS and need to be further developed for other storage technologies.

All of the above research on SLAP assumes that replenishment lot sizes of the SKUs are given. However, Wilson (1977) demonstrates that the lot sizing problem and the SLAP should be considered simultaneously in order to achieve an optimal total cost including both inventory cost and material handling cost. Algorithms for the integrated lot sizing and SLAP problem can be found in Wilson (1977), Hodgson and Lowe (1982), Malmborg et al. (1986), Malmborg and Deutsch (1988), and Malmborg et al. (1988).

The version of the SLAP problem studied in the literature is most often static, i.e., it assumes that the incoming and outgoing material flow patterns are stationary over the planning horizon. In reality, the material flow changes dynamically due to factors such as seasonality and the life cycles of products. Therefore, the storage location assignment should be adjusted to reflect changing material flow requirements. One possibility is to relocate those items whose expected retrieval rate has increased (decreased) closer to (farther from) the I/O point. Such relocations are only beneficial when the expected saving in order picking outweighs the corresponding relocation cost. Therefore, decisions must be made carefully concerning which set of items to be relocated, where to relocate them, and how to schedule the relocations. Another type of relocation might take place as a result of the uncertainty in incoming shipments. For example, Roll and Rosenblatt (1987) describes the situation when the storage area is divided into separate zones and any incoming shipment must be stored within a single zone. It might happen that none of the zones has sufficient space to accommodate an incoming shipment. In such cases, it is advisable to free some space in a certain zone to accommodate the incoming shipment by shifting some stored products in that zone to other zones. Table 4 gives a summary of the literature on various dynamic storage location assignment problems.

5. Order picking

Different order picking methods can be employed in a warehouse, for example, single-order picking,

Table 4
Dynamic storage location assignment problem

Citation	Problem statement	Method
Christofides and Colloff (1972)	The set of items to be relocated and their destinations are given, and the problem is to route the relocation tour to minimize the total relocation cost	Two-stage heuristics that is optimal in a restricted case
Muralidharan et al. (1995)	The set of high-demand items to be relocated and their destinations are given, and the problem is to route the relocation tour to minimize the total relocation cost	A nearest-neighbor heuristic and an insertion heuristic
Jaikumar and Solomon (1990)	Determine the items to be relocated and their destinations with the objective to find the minimum number of relocations that results in a throughput satisfying the throughput requirement in the following busy periods	Optimal ranking algorithm
Sadiq et al. (1996)	Determine the relocation schedule in face of the dynamically changing order structure, i.e., relocate items that are more likely to appear in the same order in clusters	Rule of thumb procedure based on cluster techniques
Roll and Rosenblatt (1987)	Using zone storage without splitting, it might happen that none of the zones has sufficient space to accommodate an incoming shipment. The problem is how to shift some stored products in a certain zone to other zones in order to free space for the incoming shipment	Rule of thumb procedure

batching and sort-while-pick, batching and sort-after-pick, single-order picking with zoning, and batching with zoning (Yoon and Sharp, 1996). Each order picking method consists of some or all of the following basic steps: batching, routing and sequencing, and sorting.

5.1. Batching

The batching problem is part of planning for order picking. Orders are received and subsequently released for fulfillment. Given a set of released orders, the problem is to partition the set into batches, where each batch will be picked and accumulated for packing and shipping during a specific time window, or “pick wave.” The time required to pick the items in any batch should not exceed the time window or pick wave duration. If zone picking is employed, the batch should balance pick effort across the zones to achieve high picker utilization, while minimizing pick time so that the number of pickers required is minimized.

The batching problem can be stated as

Given:

- (1) Warehouse configuration.
- (2) Pick wave schedule.
- (3) A set of orders to pick during a shift.

Determine:

A partition of orders for assignment to waves and pickers.

Subject to performance criteria and constraints such as:

Picker effort, imbalance among pickers, time slots, picker capacity, and order due dates.

In creating an abstract statement of the problem, there are potentially two levels of partitioning: (1) partitioning in time (into pick waves); and (2) partitioning among pickers in a wave or zone. Constraints include the picker capacity during the time interval associated with a pick wave, and perhaps time constraints on when an order should be completed.

Partitioning into time slots is essentially a “bin packing” type problem, where the goal is to balance the pick time among the time slots or pick waves. The difficulty, of course, is that the time required to pick a batch is not known until the batch has been determined, partitioned among individual picker, and the pickers have been routed through the warehouse.

Partitioning of the orders among the pickers is a variation of the classical vehicle routing problem (VRP), in which “stops” are assigned to routes and the objective is to minimize the total route distance or time. However, in the order-batching problem, assigning an order to a picker’s route implies that all the picking locations for the SKUs in this order are assigned to this route. This is similar to the pick-up and delivery vehicle routing problem, or the dial-a-ride problem, where a service request consists of a pick-up location and a drop-off location

with time precedence. In the order partitioning problem, there may be many stops (SKUs) associated with a single service request (order) but there are no precedence constraints.

The published research has focused primarily on the problem of partitioning among pickers. There are two major types of batching heuristics that attempt to minimize total picking effort and are based on VRP heuristics. A seed algorithm selects initially a single seed order in the batch. More orders are then added according to a route closeness criterion until no more orders can be added due to a capacity constraint. The capacity constraint can be based on total pick time, number of orders in the batch, or weight. A savings heuristic starts by assigning each order to a separate batch. The algorithm then iteratively selects a pair of batches to be combined based on the savings of combining them until no more batches can be combined due to the capacity constraint.

Central to both types of algorithms is an order-to-route closeness metric, which defines the order addition rule in the seed algorithms and the combination rule in the saving algorithms. Table 5 summarizes closeness metrics proposed in the literature. The seed and savings algorithms proposed in the literature are similar in terms of their general procedure, but differ in the closeness metric used. Table 6 shows the different algorithms and the close-

Table 5
Order closeness metrics for batching

Index	Closeness metric	Example
1	Number of common locations between two orders	Elsayed (1981)
2	Combined number of locations of two orders	Elsayed and Stern (1983)
3	Sum of the distance between each location of one order and the closest location on the other order	Elsayed and Stern (1983)
4	Difference of the order-theta values of two orders defined based on space-filling curves	Gibson and Sharp (1992)
5	The number of additional aisles to travel when two orders are combined	Rosenwein (1996)
6	Savings in travel when two orders are combined	Elsayed and Unal (1989)
7	Center of gravity metric	Rosenwein (1996)
8	Economic convex hull based metric	Hwang and Lee (1988)
9	Common covered regions or areas	Hwang et al. (1988)

Table 6
Order batching heuristics by type

Seed algorithm	Saving algorithm
Elsayed (1981) (1)	Rosenwein (1996) (5, 7)
Elsayed and Stern (1983) (1, 2, 3)	Hwang and Lee (1988) (8)
Elsayed and Unal (1989) (6)	Elsayed and Unal (1989) (6)
Gibson and Sharp (1992) (3, 4)	de Koster et al. (1999) (6)
Hwang and Lee (1988) (8)	
Hwang et al. (1988) (9)	
Pan and Liu (1995) (1, 3, 4, 6, 8)	
de Koster et al. (1999) (3, 5, 6, 7)	

ness metrics they used as shown by the bold number after each citation.

Many of the papers listed in Table 6 also provide performance evaluation of the different batching algorithms using simulation. It is however difficult to draw general conclusions since the performance depends heavily on factors such as storage location assignment policies, routing policies, the structure of orders, storage systems, and the maximum batch size. A comprehensive study that considers all the above factors and the various batch construction heuristics has not been published at this time. A few results have been published where two policy classes are studied jointly, for example, de Koster et al. (1999) evaluate batching and routing algorithms together, and Ruben and Jacobs (1999) evaluate batching algorithms with different SLAP policies.

The majority of literature has been focused on the objective of minimizing the total order picking time. In practice, there are might be other important criteria, for example, lead time and tardiness. Elsayed et al. (1993) present a heuristic for batching orders that have due dates with the objective to minimize earliness and tardiness penalties. Elsayed and Lee (1996) consider batching and sequencing of both storage and retrieval orders such that the total tardiness of the retrieval orders is minimized. Cormier (1987) propose a heuristic for batching and sequencing orders to minimize the weighted sum of order picking time and tardiness in an AS/RS. Won and Olafsson (2005) develop mathematical models and heuristics that solve the joint problem of order batching and picking considering both picking efficiency and order lead time.

Very few papers have developed optimal order batching algorithms. [Armstrong et al. \(1979\)](#) present a mixed-integer formulation for batching in a semi-automated order-picking system with the objective to minimize the total picking time. The model was solved using Bender’s decomposition. [Gademann et al. \(2001\)](#) consider the order batching problem with the objective of minimizing the maximum lead time of the batches and solve the formulation optimally using a branch-and-bound algorithm. [Gademann and van de Velde \(2005\)](#) solve the batching problem to minimize the total order picking time by formulating it as a set partitioning problem, and then solving it with a branch-and-price algorithm and an approximation algorithm. [Chen and Wu \(2005\)](#) use a clustering approach to batch orders that are highly associated, i.e., orders sharing a large number of common items. They propose a method to calculate the association between orders, which is used in an integer-programming model to maximize the total association measure.

5.2. Sequencing and routing

The sequencing and routing decision in order picking operations determines the best sequence and route of locations for picking and/or storing a given set of items. The objective is typically to minimize the total material handling cost. This problem

is a warehouse-specific Traveling Salesman Problem (TSP), where the picking/storing location of an item is given. The problem where there are several candidate locations for the retrieval or storage of an item is more complex and few research results are available, although it is often found in practice. The TSP in the warehouse is special because of the aisle structure of the possible travel paths. The published research focuses on four classes of warehouse systems, i.e., conventional multi-parallel-aisle systems, man-on-board AS/RS systems, unit-load AS/RS systems, and carousel systems.

5.2.1. Sequencing and routing for conventional multi-parallel-aisle systems

In a conventional multi-parallel-aisle system, the aisle structure limits the TSP state space, which greatly simplifies its solution. [Ratliff and Rosenthal \(1983\)](#) propose a polynomial-time dynamic programming algorithm to optimally solve this problem. The algorithm depends on the following assumptions: parallel, narrow and equal aisles, a single I/O point for the picker in the warehouse, the aisles connected by a cross aisle at each end, and the SKU locations given. Other authors have relaxed some of these assumptions and proposed different algorithms to deal with these complications. These related results are summarized in [Table 7](#), where [Ratliff and Rosenthal \(1983\)](#) is listed first

Table 7
Algorithmic routing approaches for conventional multi-parallel-aisle warehouses

Citation	Problem setting	Algorithm	Optimal or not
Ratliff and Rosenthal (1983)	Narrow aisles; A tour starts and ends at the central depot; Only two cross aisles located at the ends of picking aisles; Picking locations are given	A dynamic programming based algorithm	Optimal with computational time linear in the number of aisles
Goetschalckx and Ratliff (1988a,b)	Routing in wide aisles	A shortest path algorithm and a set-covering based algorithm with the consecutive ones property	Optimal for the routing within a single aisle
de Koster and van der Poort (1998)	A tour can start and end at the head of any picking aisle	An extension of Ratliff and Rosenthal (1983)	Optimal
Roodbergen and de Koster (2001b)	There are three cross aisles	An extension of Ratliff and Rosenthal (1983)	Optimal
Vaughan and Petersen (1999) , Roodbergen and de Koster (2001a)	There are arbitrary number of cross aisles	Dynamic programming based heuristics	Heuristics
Daniels et al. (1998)	Picking locations need to be selected before routing	TSP based heuristics with local search methods	Heuristics

with the assumptions they made and followed by the other results that relax some of the restrictive assumptions (see the problem setting column in Table 7).

Although it is possible to construct optimal routing algorithms efficiently, simple heuristics such as the traversal and return policies are widely used in practice because they are easy to understand and the resulting routes are more consistent. For the traversal policy, the picker will cross through the whole aisle that contains at least one pick, and therefore always enters at one end of the aisle and exits at the other end. For the return policy, the picker always enters and exits at the same end of the aisle, and the same aisle may be entered twice from its two ends. Detailed description of these routing policies and their variations can be found in Hall (1993) and Caron et al. (1998). Performance evaluation of these different routing algorithms can be found in Hall (1993) and Caron et al. (1998, 2000) based on analytical models, and Petersen (1997, 1999), Petersen and Schmenner (1999), and Petersen and Aase (2004) based on simulations.

5.2.2. Sequencing and routing for man-on-board AS/RS

The routing problem for man-on-board AS/RS is a TSP with a Chebyshev distance metric. The literature on this problem has been focused primarily on efficient heuristics. Gudehus (1973) describes the band heuristic, which divides the rack into two equal height horizontal bands; the points in the lower band are visited in the increasing x -coordinate direction, while the points in the upper band are visited in the opposite direction. If the tour must visit many points, the rack may be divided into several pairs of horizontal bands. Goetschalckx and Ratliff (1988c) propose a convex hull algorithm based on the property of Chebyshev metric that some points not on the convex hull can be inserted into it without incurring additional travel distance. The algorithm constructs the convex hull of all the picking locations, then those free insertion locations for each segment of the convex hull are identified and inserted into the convex hull, and then the remaining points are sequentially inserted into the tour in a way that minimizes the increase in tour length for each insertion. The band algorithm is easy to implement and computationally efficient, but might give inferior solutions in some cases. On the other hand, the convex hull algorithm is effective in finding short tours, but is difficult to implement (to find

the convex hull and free insertion points) and less computationally efficient.

Bozer et al. (1990) propose the 1/2 band insertion heuristic, which is a combination of the band and convex hull heuristics. The heuristic first divides the rack into three equal width horizontal bands, all the points in the first and third band are routed in the same way as in the band heuristic to obtain a partial tour, and the points in the middle band are then inserted as in the final stage of the convex hull algorithm. Other heuristics in the literature include the center sweep heuristic (Bozer et al., 1990), the space-filling curve based heuristic (Bartholdi and Platzman, 1988), and the combined convex hull heuristic for a variation of the man-on-board systems (Hwang and Song, 1993). Local improvement procedures (Bozer et al., 1990; Makris and Giakoumakis, 2003) can be used together with all the above heuristics to further reduce the tour length.

Bozer et al. (1990) give a comprehensive comparison of these heuristics, and conclude that the convex hull and 1/2 band insertion heuristics consistently outperform the others, and suggest the use of the 1/2 band heuristic because it achieves performance close to that of the convex hull algorithm, but is very simple to implement and runs very efficiently. Bachers et al. (1988) provide a comparison of several traditional TSP heuristics, such as the nearest-neighbor method, the successive insertion method, and the local search method, through simulation.

Kim et al. (2005) study a special AS/RS system that is similar to the man-on-board system in the sense that it picks multiple items from the rack in each cycle. The difference is that after each pick, the picked item must be put into a drop buffer that is vertically below the picking location. The routing problem for this system can also be formulated as a special TSP, and an x -coordinate based heuristic and a clustering based heuristic are proposed to solve it.

5.2.3. Sequencing and routing for unit-load AS/RS

The routing problem for unit-load AS/RS (also called the interleaving problem) pairs a storage operation with a retrieval operation for a dual command cycle. Graves et al. (1977) demonstrate that careful interleaving can effectively reduce the total travel distance by reducing the unproductive travel between storage and retrieval locations. The algorithms reported in the literature are either static or

Table 8
Static sequencing algorithms for dual-command AS/RS

	Citation	Problem setting	Algorithm	Optimal or not
Randomized storage	Han et al. (1987)	Unit-load AS/RS	Nearest-neighbor heuristic	Heuristic
	Lee and Schaefer (1996)	Unit-load AS/RS	Assignment-based algorithm	ϵ -optimal
	Mahajan et al. (1998)	Miniloan end-of-aisle AS/RS	Nearest-neighbor heuristic	Heuristic
	Keserla and Peters (1994)	Unit-load dual shuttle AS/RS	Minimum-perimeter heuristic	Heuristic
	Sarker et al. (1991)	Unit-load dual shuttle AS/RS	Nearest-neighbor heuristic	Heuristic
Dedicated storage	van den Berg and Gademann (1999)	Unit-load AS/RS	Transportation problem	Optimal
	Lee and Schaefer (1997)	Unit-load AS/RS	Assignment problem	Optimal
Class-based storage	Eynan and Rosenblatt (1993)	Unit-load AS/RS	Nearest-neighbor heuristic	Heuristic
	Sarker et al. (1994)	Unit-load dual shuttle AS/RS	Nearest-neighbor heuristic	Heuristic

dynamic. Static algorithms fix a block of storage and retrieval requests, sequence the requests in the block, and execute the resulting schedule ignoring new storage and retrieval requests. Dynamic algorithms re-sequence the storages and retrievals whenever new requests arrive. The static sequencing problem for randomized and class-based storage is believed to be NP-hard, and most algorithms for this problem use a nearest-neighbor heuristic or one of its variations. Han et al. (1987) proposed a match of a storage location with a retrieval location that has the minimum travel distance between them. Lee and Schaefer (1996) developed an assignment formulation and can find an optimum or near-optimum solution for problems of moderate size. The static case for dedicated storage policies can be solved in polynomial time by formulating it as a transportation or assignment problem (van den Berg and Gademann, 1999; Lee and Schaefer, 1997). Table 8 summarizes the static algorithms for different systems and storage policies. Dynamic algorithms in the literature are mainly direct extensions of the static algorithms that re-sequence the requests whenever a new request arrives in the system as reported by Lee and Schaefer (1997), Eben-Chaime (1992), and Ascheuer et al. (1999). Seidmann (1988) proposes a different dynamic control approach based on artificial intelligence techniques.

In some cases, Just-In-Time performance of the AS/RS is more important than minimizing the total operational cost. For example, if the AS/RS is used to feed a production line, it is important that the requested materials are retrieved at the time determined by the production schedule. Lee and Kim (1995) and Linn and Xie (1993) develop heuristics to sequence the storage and retrieval requests in order to improve the due date related performance.

Several authors have studied the dwell point selection problem in a unit-load AS/RS. The dwell point is the position where the S/R shuttle stops when the system is idle. The dwell point can be selected to minimize the expected travel time to the position of the first transaction after an idle period, and thus improve system response. Research results on this topic can be found in Bozer and White (1984), Egbelu (1991), Egbelu and Wu (1993), Chang and Egbelu (1997), Hwang and Lim (1993), Peters et al. (1996), and van den Berg (2002).

Simulation studies of the operational policies for an unit-load AS/RS can be found in Linn and Wysk (1987) and van den Berg and Gademann (2000), which compare different sequencing rules, dwell point selection rules, and storage location assignment rules under various conditions of the product mix and the traffic intensity.

5.2.4. Sequencing and routing for carousel systems

The sequencing problem in carousel systems was first considered by Bartholdi and Platzman (1986). They assume that the orders are picked one at a time, which leads to two sequencing problems, i.e., the pick sequencing within an order and the sequencing of orders. The effect of the latter is not significant when the order arrival rate is small compared with the order retrieval rate, so the problem simplifies to the pick sequencing within the orders. They present a polynomial algorithm to optimally solve this problem, as well as some simple heuristics that are easier to compute and perform well when the number of picks is large relative to the total storage space. When the order arrival rate is large, the sequencing of orders must be considered in minimizing the unproductive time of traveling from the end position of one order to the start position of the

next. In this case, an efficient heuristic is proposed based on the additional assumption that each order is picked along its shortest spanning interval, which is the shortest interval that covers all the picking locations of the order. It is shown that the proposed heuristic will produce a solution that is never more than 1 revolution longer than the optimal, i.e., the more orders to be picked, the better the solution.

Ghosh and Wells (1992) and van den Berg (1996) consider the problem when the sequence of orders is fixed (but the pick sequence within the orders is to be determined), and propose efficient dynamic programming approaches to optimally solve it. van den Berg (1996) also considers the case when both in-order and between-order picking sequences are to be determined by assuming that each order is picked along its shortest spanning interval. They formulate this problem as a Rural Postman Problem on a circle and solve it to optimality. Furthermore, they show that the solution obtained with the extra constraint is at most 1.5 revolutions more than the optimal without the extra constraint. The above research treats the carousel as a one-dimensional system, i.e., the travel perpendicular to the rotation of the carousel was not considered. Wen and Chang (1988) consider a two-dimensional carousel system and propose three heuristics that are extensions of Bartholdi and Platzman's optimal algorithm. Han and McGinnis (1986), and Han et al. (1988) extend the nearest-neighbor heuristics discussed earlier for the dual-command AS/RS to carousels and rotary racks. (A rotary rack is similar to a carousel except that it has several layers, and each layer can be operated independently.)

5.2.5. Sequencing and routing summary

In summary, the sequencing and routing problem is the most studied problem in warehouse operation. Most of the research assumes that the locations to be visited are given. The problem when multiple candidate locations are available for the retrieval or storage of an SKU remains an interesting and challenging research problem (for example, see Daniels et al., 1998). Also, in a warehouse setting, batching is closely related to sequencing, and therefore those problems require a joint solution method. Furthermore, because of the confined and narrow travel paths in a warehouse, another relevant variant of the sequencing and routing problem would consider congestion when there are multiple order picking tours executed at the same time period in the same area.

5.3. Sorting

Sorting is required when multiple orders are picked together. It can be performed either during the picking process (sort-while-pick) or after the picking process (sort-after-pick). Sort-while-pick is quite straightforward and is typically modeled by inflating the item extraction time. For sort-after-pick, a separate downstream sorting system is used to perform the sorting function. A number of questions are related to the operation of the sorting system.

Sorting systems used in warehouses usually include an accumulation conveyor, a recirculation conveyor, and exit lanes, and they operate simultaneously on all the orders in a single pick-wave. Items for a pick wave arrive at the accumulation conveyor where they wait to be released into the sorting process. They are put onto the recirculation conveyor through an induction point after the items in the previous pick-wave finish their sorting process (in some cases, the items are allowed to enter the recirculation conveyor before the previous wave has totally finished its sorting). The orders are assigned to sorting lanes according to order-to-lane assignment rules. Items circulate in the recirculation conveyor and enter the assigned sorting lane if all items of the preceding order assigned to that lane have been sorted. If not, the items bypass the sorting lane and re-circulate. Eventually, sorted orders are removed from sorting lanes, checked, packed, and shipped. Therefore, the operation problem for sorting involves decisions such as wave-releasing and order-to-lane assignment so that the orders can be efficiently sorted in a given wave.

There are relatively few research results in this area. Bozer and Sharp (1985) consider a system that processes a relatively small number of large orders. In this case, each sorting lane is typically dedicated to one order. The authors use simulation to analyze the dependence of the system throughput on factors such as the induction capacity, the number of lanes, and the length of lanes. Bozer et al. (1988) consider a similar problem but with a large number of small orders. In this case, each lane is assigned several orders and an order-to-lane assignment policy determines how and when the orders enter the sorting lanes. Orders that are not yet assigned a lane are forced to recirculate. Using simulation, they compare different order-to-lane assignment rules, which include the simplest FCFS rule and priority rules based on the sizes of orders or the time that an order

has been in the system. They find that the FCFS rule consistently outperforms more elaborated rules. Johnson (1998) verifies this result with analytical models for the sorting system operated under different order-to-lane assignment rules. Meller (1997) propose an optimal order-to-lane assignment method to minimize the sorting time for a pick-wave based on a set-partitioning model.

In practice, the sorting time in an automatic sorting system might not be a critical factor as long as all orders can be sorted within a given wave. Therefore, simple heuristics would suffice in most practical cases if orders were partitioned into pick waves in a balanced way.

6. Conclusions and discussions

The distribution of the research results among the various warehouse operational problems is shown in Fig. 1, where the numbers in parentheses represent the number of papers addressing the corresponding problem. It is clear that the past research has focused strongly on storage and order picking. This is not surprising since these are the two warehouse functions that have the largest impact on the overall warehouse operational performance including storage capacity, space utilization, and order picking efficiency.

On the other hand, the development of research is not well balanced. Some problems received far more attention from the research community than others. For example, the SLAP and routing problems account, respectively, for 32% and 38% of the total surveyed literature, while zoning accounts for less than 6%. Furthermore, there is little direct evidence of collaboration of the academic research community with industry. Many of the research results are not sufficiently communicated to industry to make a significant impact on the practice of warehouse operations. More communication from both sides might help to better identify the real challenges faced in warehouse operations, to appreciate the opportunities for better operation, and to realize these opportunities by close cooperation between researchers and practitioners.

The problems discussed in this paper are at the operational level, which means that decisions need to be made quite frequently and the influence of these decisions is typically of a short duration and localized. Such decisions typically need to be made quickly without extensive computational resources. This tends to encourage the use of heuristic proce-

dures that can find a good solution reliably in a reasonable amount of time. In addition, from the management point of view, an ideal solution method should be simple, intuitive, and reliable in order to minimize the training costs in the warehouse.

Another consequence of the operational nature of the problems discussed in this paper is that the problems should be considered dynamically by constantly incorporating new information about the operating environments. Some research on the dynamic planning of warehouse operations exists, but the dynamic problems are much less studied than the equivalent static problems. Furthermore, research in the literature usually concentrates on certain standard performance measures, such as the total order picking cost. In many practical situations, different objectives such as the tardiness, or the order cycle time, are as important as the traditional aggregate performance measure.

In summary, there continues to be a need for research focusing on the operational management of warehousing systems, where the different processes in the warehouse are considered jointly, the problems are placed in their dynamic nature, and multiple objectives are considered simultaneously. Clearly, the research domain of warehouse operations is very rich and challenging. Given the prevalence of warehouses in the supply chains, such research results can have a significant economic impact.

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