## 1 Optimal picking in automated warehouses

Automated Storage and Retrieval Systems (AS/RS) are commonly used to speed-up warehouse operations in industrial production plants. At their core, a crane operates to pickup boxes at specific positions in the warehouse shelves and to bring them to an output port, where a conveyor belt is placed.

In this paper we consider an automated warehouse where all components are stored in identical (standardized) boxes, called items in the remainder. The items are stored in suitable locations along a single aisle. All the required items are collected by a single crane equipped with $q$ shuttles, that moves along a rail. The crane is initially empty at an idle point that also acts as the output location. The idle point (called origin in the remainder) is at a given intermediate position along the rail. Therefore, the aisle can be represented by two lines with a common origin, as in Fig. [1. In general each required item can be picked up on either line, because multiple identical copies of the same materials or components can be stored in the automated warehouse. Without loss of generality, we consider the case where each required item can be picked up from two distinct locations, one on each line. At each cycle the crane starts from the origin, it moves along one of the two lines, it collects at most $q$ required items and it returns to the origin where it unloads the collected items. Hence, the total distance travelled in a cycle is twice the distance between the origin and the farthest collected item. The objective is to minimize the total distance travelled by a crane of capacity $q$ to collect a set of required items.


Figure 1: Schematic top-view of the 1-dimensional automated warehouse considered in this paper. The real elements (crane, rail, items, in/out site) are depicted in gray, while their abstract representation (origin, lines and points) in black.

## 2 Problem definition

We indicate by $N=\{1,2, \ldots, n\}$ the set of items to be picked-up from a 1-dimensional warehouse. The origin is indicated by $O$. Each line holds one copy of each item $i \in N$ at a given location. From now on, $d_{\ell}(i)$ denotes the distance of $i \in N$ from $O$ on line $\ell \in\{1,2\}$. Multiple items can be stored at a same location. The crane capacity is a given positive integer $q$.

Definition 1 (Trips). $A$ trip $T$ is a subset of $N$ of cardinality at most $q$. The cost of a trip $T$ on line $\ell \in\{1,2\}$ is $C_{\ell}(T)=\max _{i \in T}\left\{d_{\ell}(i)\right\}$, i.e. half the distance travelled by the crane.

The problem $q / 1 / P / V$ requires to find a pair $\left(\mathcal{T}^{1}, \mathcal{T}^{2}\right)$ of sets of non-empty trips such that the trips in $\mathcal{T}^{1} \cup \mathcal{T}^{2}$ partition $N$ and such that the total $\operatorname{cost} C(\mathcal{T})=$ $\sum_{\ell=1}^{2} \sum_{T \in \mathcal{T}^{\ell}} C_{\ell}(T)$ is minimum.

