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Advanced morphological processing

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Methods for Image Processing

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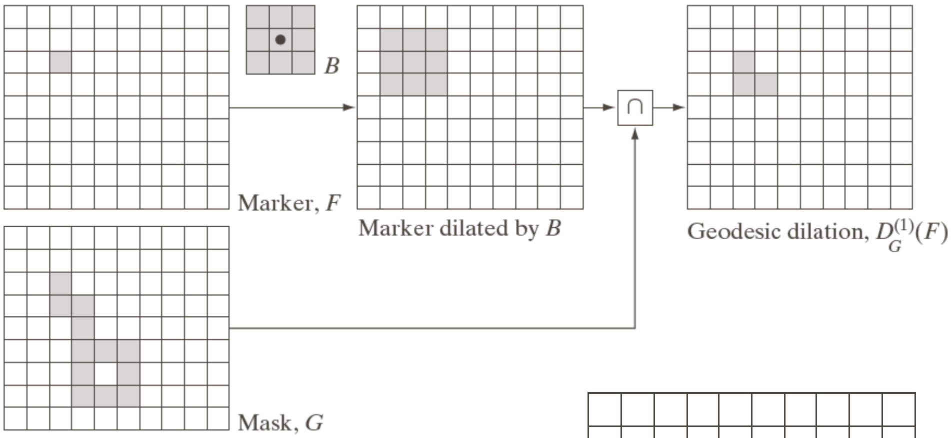
Geodesic dilation

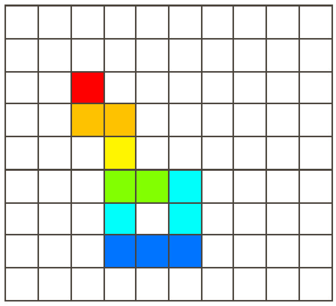
The *geodesic dilation* is an iterative morphological transformation requiring:

- marker image, F: starting points;
- mask image, G: constraint;
- structuring element, B.
- $F \subseteq G$
- $D_G^{(n)}(F)$: geodesic dilation of size n of F with respect to G.

- $D_G^{(0)}(F) = F$
- $D_G^{(1)}(F) = (F \oplus B) \cap G$
- $D_G^{(n)}(F) = D_G^{(1)}\left(D_G^{(n-1)}(F)\right)$

Geodesic dilation (2)





Geodesic erosion

The *geodesic erosion*, similarly to the geodesic dilation, is defined as:

- $E_G^{(0)}(F) = F$
- $E_G^{(1)}(F) = (F \ominus B) \cup G$
- $E_G^{(n)}(F) = E_G^{(1)}\left(E_G^{(n-1)}(F)\right)$

- Geodesic dilation and erosion are dual with respect to the complementation.

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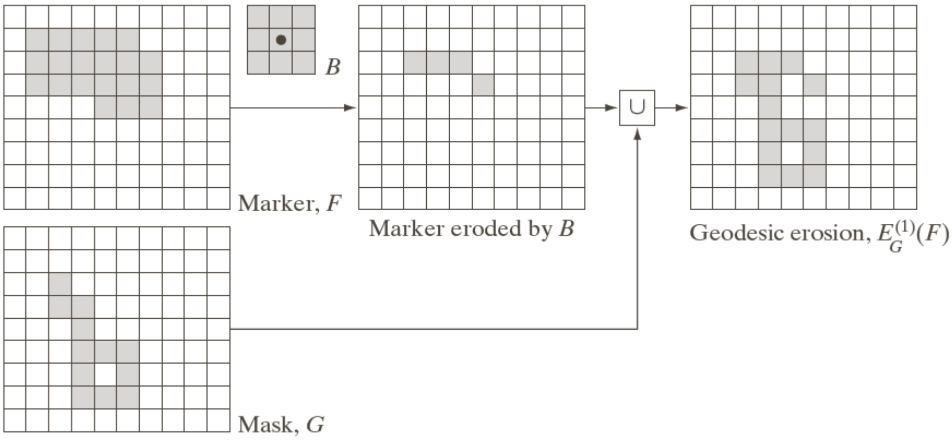
Geodesic erosion (2)



Morphological reconstruction

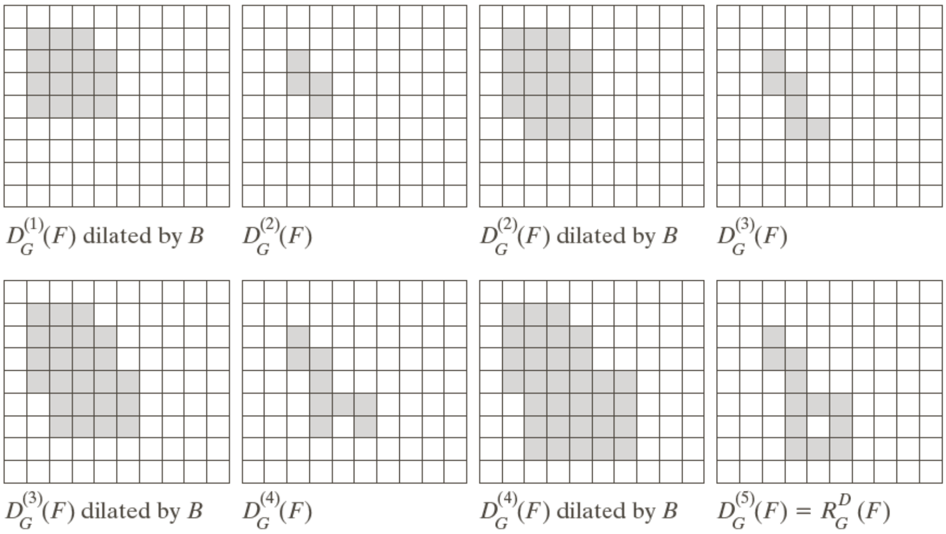
- The reconstruction through the geodesic operations are defined as the results of such operation at the stability:
  - morphological reconstruction through dilation:

$$R_G^D(F) = D_G^{(k)}(F), \text{ where } D_G^{(k)}(F) = D_G^{(k-1)}(F)$$

  - morphological reconstruction through erosion:

$$R_G^E(F) = E_G^{(k)}(F), \text{ where } E_G^{(k)}(F) = E_G^{(k-1)}(F)$$

Morphological reconstruction (2)



Opening through reconstruction

- The effects of the opening:
  - details are eliminated by the erosion;
  - the dilation operates the reconstruction.
- The quality of the results depends by the shape of the structuring element.

Opening through reconstruction of size n:

$$O_R^{(n)}(F) = R_F^D[F \ominus nB]$$

- erosion for eliminating the details (n times);
- reconstruction through the geodesic dilation.
- Note: the reconstruction is *bounded* by the mask.

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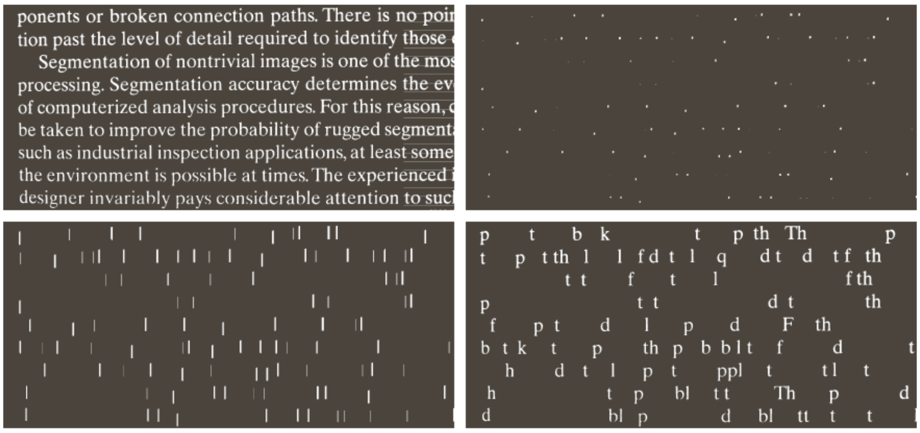
Opening through reconstruction (2)

Selection of the "long" characters:

1. erosion with a 51×1 structuring element;
2. opening through reconstruction.



opening with a 51×1 SE　　opening through reconstruction

Hole filling

The opening through the reconstruction allows to state a hole filling procedure that does not need of a starting "seed".

- $I(x, y)$, binary image (with an hole)
- $F(x, y)$, marker image, generated as a matrix of 0s, but in those border pixels where I is 0:

$$F(x, y) = \begin{cases} 1 - I(x, y), & \text{if } (x, y) \text{ is a border pixel} \\ 0, & \text{otherwise} \end{cases}$$

- $H(x, y)$, hole filled version of I:

$$H = \left[R_{I^c}^{D}(F) \right]^c$$

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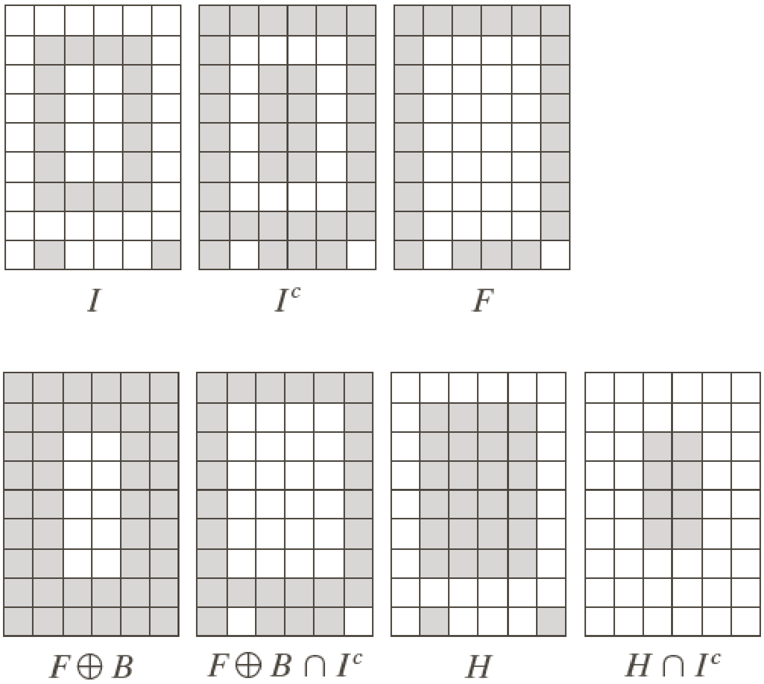
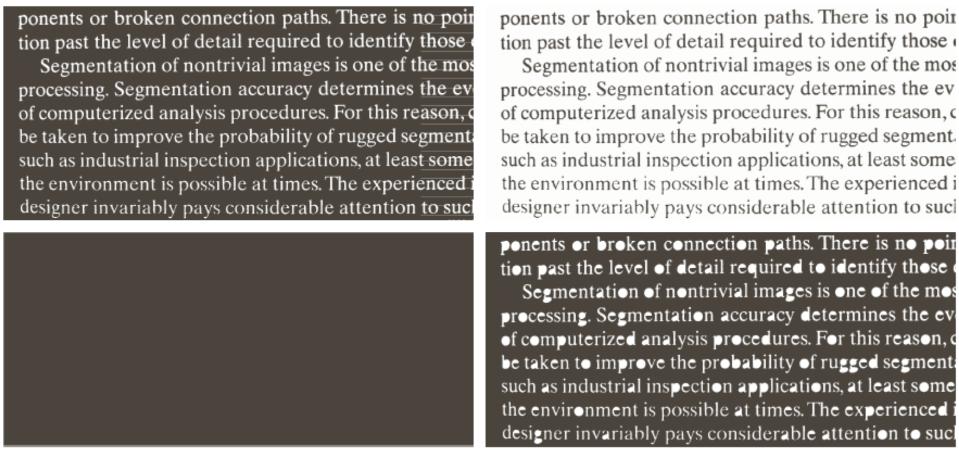
Hole filling (2)

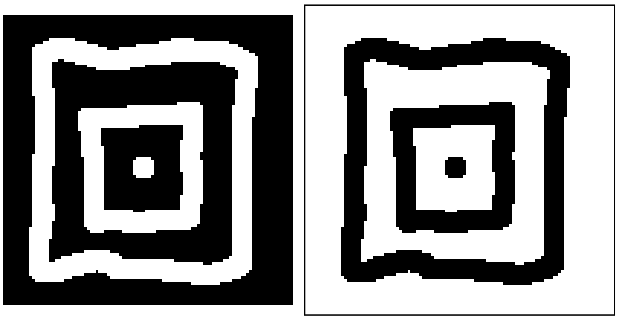


Hole filling (3)



I	I^c
F	H

Hole filling: a simple exercise



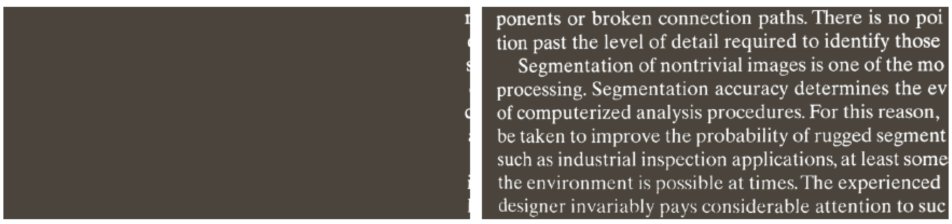
What happen to the inner holes?

Partial objects elimination

- $I(x, y)$, binary image (containing partial objects)
- $F(x, y)$, marker image, generated as a matrix of 0s, but in those border pixels where I is 1:

$$F(x, y) = \begin{cases} I(x, y), & \text{if } (x, y) \text{ is a border pixel} \\ 0, & \text{otherwise} \end{cases}$$

- $R_I^D(F)$, image with only the objects that touch the border
- $I - R_I^D(F)$, original image without objects touching the border

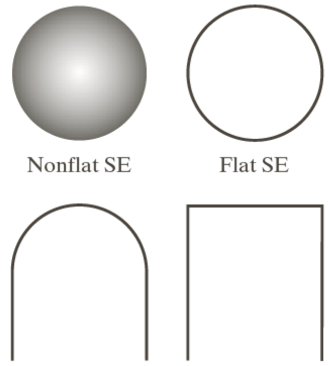


Gray levels morphology

- The morphological operations can be extended for the gray levels images.

- Here the images cannot be considered as sets anymore, but as (non-binary) functions.

- Two types of structuring elements can be devised:
  - flat;
  - non-flat.



Flat erosion and dilation

The morphological operations with *flat* structuring element, b, on the image f can be defined as follows:

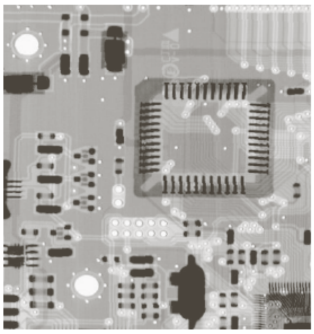
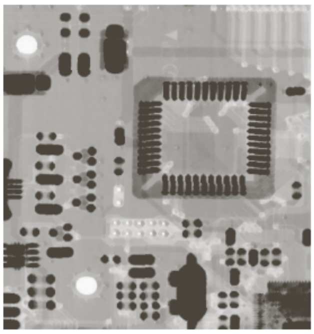
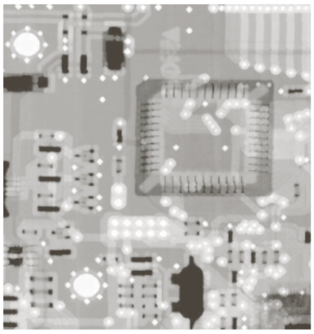
- Erosion:

$$[f \ominus b](x, y) = \min_{(s,t) \in b} \{f(x+s, y+t)\}$$

- Dilation:

$$\begin{aligned} [f \oplus b](x, y) &= \max_{(s,t) \in \hat{b}} \{f(x+s, y+t)\} \\ &= \max_{(s,t) \in b} \{f(x-s, y-t)\} \end{aligned}$$

Flat erosion and dilation (2)

f | $f \ominus b$ | $f \oplus b$

- f, 448×425 pixels;
- b, flat disk of radius 2.

Non-flat erosion and dilation

The morphological operations with *non-flat* structuring element, b_N, on the image f can be defined as follows:

- Erosion:

$$[f \ominus b_N](x, y) = \min_{(s,t) \in b_N} \{f(x+s, y+t) - b_N(s, t)\}$$

- Dilation:

$$[f \oplus b_N](x, y) = \max_{(s,t) \in b} \{f(x-s, y-t) + b_N(s, t)\}$$

Like in the binary domain:

- $(f \ominus b_N)^c = f^c \oplus \hat{b_N}$
- $(f \oplus b_N)^c = f^c \ominus \hat{b_N}$

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Opening and closing

Similarly to the binary case, the operations of opening and closing can be defined.

- Opening

$$f \circ b = (f \ominus b) \oplus b$$

- Closing

$$f \bullet b = (f \oplus b) \ominus b$$

The duality properties hold as well:

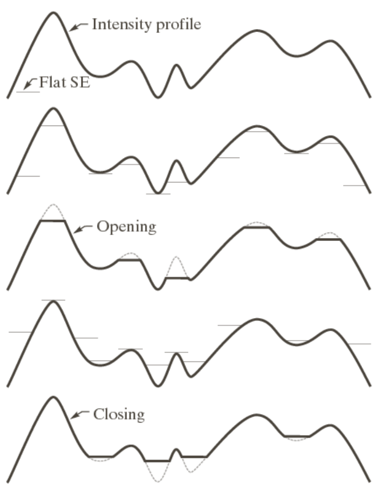
- $(f \bullet b)^c = f^c \circ \hat{b}$
- $(f \circ b)^c = f^c \bullet \hat{b}$

Opening and closing (2)

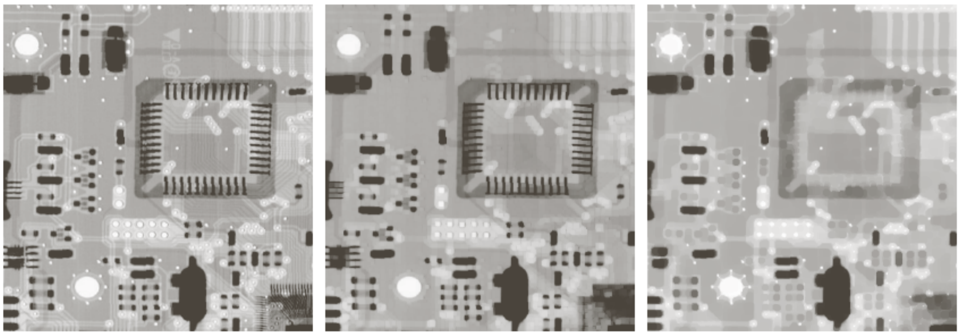
- The opening shaves the peaks.
- The closing fills the valleys.
- The following properties hold:
  - $f \circ b \leq f$
  - if $f_1 \leq f_2$, then $f_1 \circ b \leq f_2 \circ b$
  - $(f \circ b) \circ b = f \circ b$

  and
  - $f \leq f \bullet b$
  - if $f_1 \leq f_2$, then $f_1 \bullet b \leq f_2 \bullet b$
  - $(f \bullet b) \bullet b = f \bullet b$

  where $f \leq g$ if the domain of f is a subset of the domain of g and if $f(x, y) \leq g(x, y)$ for every points of the domain of f.



Opening and closing (3)

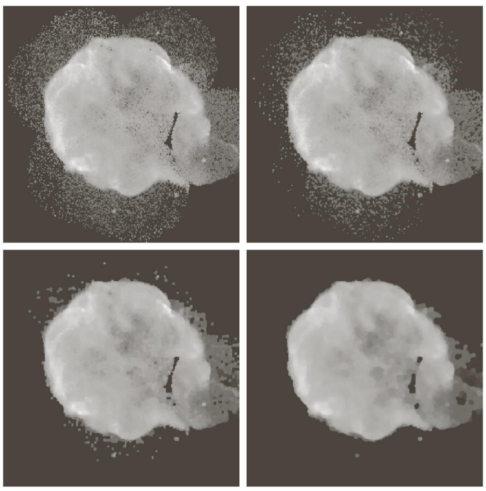


f $\quad$ $f \circ b_1$ $\quad$ $f \bullet b_2$

- f, 448×425 pixels;
- b_1, flat disk of radius 3;
- b_2, flat disk of radius 5.

Morphological smoothing

- opening + closing = smoothing

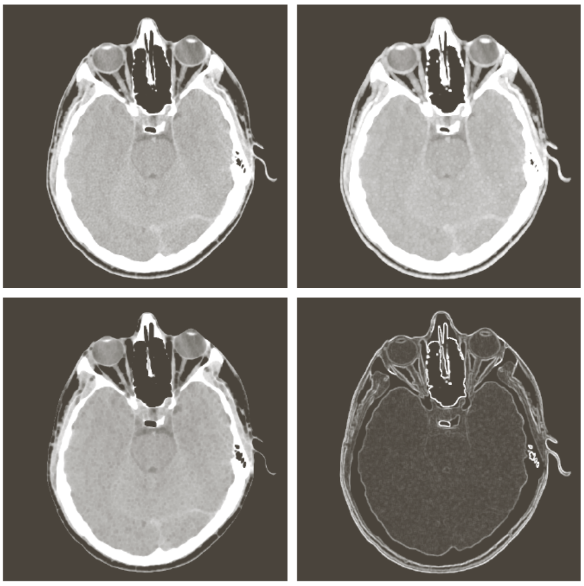


Morphological smoothing with disks of 1, 3, and 5 pixels of radius.

Alternatively, smoothing with the same SE can be iterated.

Morphological gradient

- $g = (f \oplus b) - (f \ominus b)$



f	$(f \oplus b)$
$(f \ominus b)$	g

Top-hat and *bottom-hat* transformations

- *Top-hat* transformation:

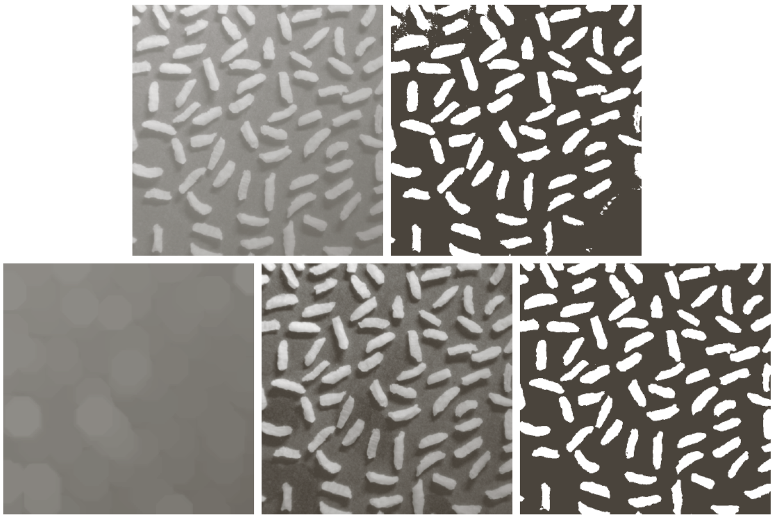
$$T_{\text{hat}} = f - (f \circ b)$$

- *Bottom-hat* transformation:

$$B_{\text{hat}} = (f \bullet b) - f$$

- These transformations preserve the information removed by the opening and closing operations, respectively.
- They are often cited as *white top-hat* and *black top-hat*.

Top-hat transformation for granulometry



A non uniform illumination can prevent thresholding effectiveness. Top-hat with a 40 pixels radius disk (on a 600 $\times$ 600 pixels image) solves the problem.

Advanced morphological operations on gray levels images

- Geodesic operations and morphological reconstruction can be defined also for grayscale images:

  geodesic dilation of size 1
  $$D_g^{(1)}(f) = (f \oplus b) \wedge g$$
  geodesic erosion of size 1
  $$E_g^{(1)}(f) = (f \ominus b) \vee g$$
  morphological reconstruction by dilation/erosion

  geodesic operations at stability

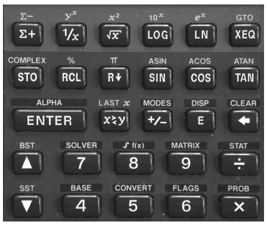
  where $\wedge$ and $\vee$ are the pointwise minimum and maximum operators.
- Opening and closing by reconstruction can be obtained similarly as in the binary case:

$$O_R^{(n)}(f) = R_f^D\,(f \ominus n\,b) \qquad \text{and} \qquad C_R^{(n)}(f) = R_f^E\,(f \oplus n\,b)$$

Top-hat by reconstruction

- Top-hat by reconstruction makes use of opening by reconstruction instead of just opening:

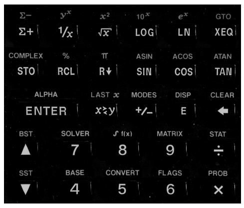
$$T_{\mathrm{hat}R}(f) = f - (R_f^D(f \ominus b))$$



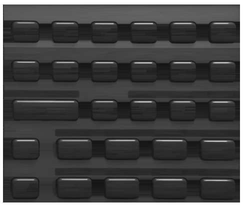
$f : 1360 \times 1134$

$b : 1 \times 71$

$R_f^D(f \ominus b)$



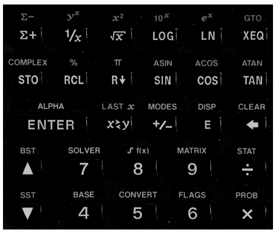
$T_{\mathrm{hat}R}(f)$



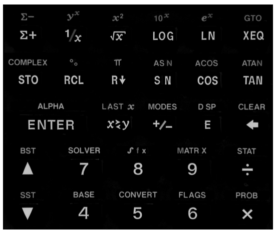
$f \circ b$

$T_{\mathrm{hat}}(f)$

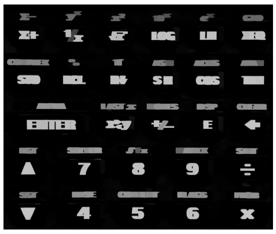
Top-hat by reconstruction (2)



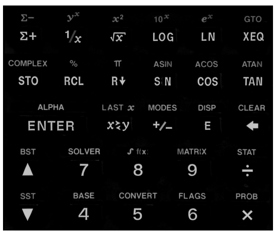
$T_{\mathrm{hat}R}(f)$



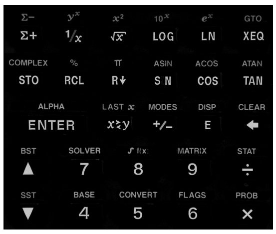
$O_R^1(T_{\mathrm{hat}R}(f))$
$b_2 : 1 \times 11$



$g = O_R^1(T_{\mathrm{hat}R}(f)) \oplus b_3$
$b_3 : 1 \times 21$



$h = T_{\mathrm{hat}R}(f) \wedge g$



$R_h^D(g)$

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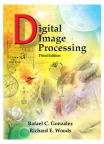
Homeworks and suggested readings



DIP, Sections 9.5.9–9.6

- pp. 656–679