

Topological structure of images

Stefano Ferrari

Università degli Studi di Milano
stefano.ferrari@unimi.it

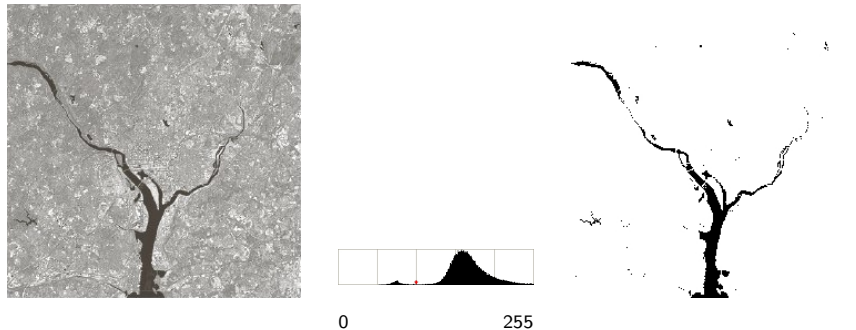
Methods for Image Processing

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Use of simple relationships between pixels

- ▶ The structure of a digital image allows to state some basic relationships between pixels that can be useful in some practical cases.
 - ▶ The pixels are organized in a regular structure and can have a limited number of values.
- ▶ Some operations consider groups of pixels that share the same features or are related by some peculiar characteristics.

Example: estimation of the area covered by the river



- ▶ identify the pixels which have the color of the river;
 - ▶ several shades for the river;
- ▶ select among these the connected pixels;
 - ▶ other “water” regions are not of interest;
- ▶ consider the regions that are “naturally” aligned;
 - ▶ sections of the rivers parted by bridges, bottlenecks, or covered.

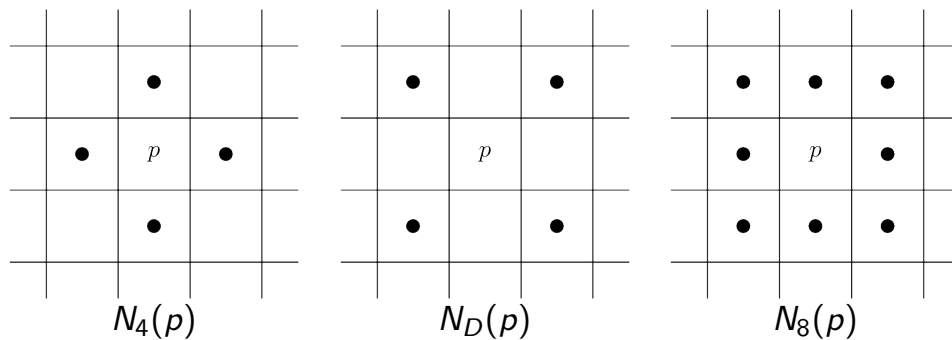
Relationships between pixels

- ▶ Some tools can be very useful:
 - ▶ a neighborhood relationship for pixels;
 - ▶ an adjacency relationship for pixels;
 - ▶ a connectivity relationship for pixels;
 - ▶ a distance measure on the image domain.
- ▶ The same concepts can be easily extended to subset of pixels.

Neighborhood

The pixel p , positioned in (x, y) has the following *neighbors*:

- ▶ N_4 : $(x + 1, y)$, $(x - 1, y)$, $(x, y + 1)$, $(x, y - 1)$;
- ▶ N_D : $(x + 1, y + 1)$, $(x - 1, y + 1)$, $(x + 1, y - 1)$, $(x - 1, y - 1)$;
- ▶ N_8 : $N_4 \cup N_D$.



Adjacency

- ▶ The *adjacency* relationship is defined considering only “similar” pixels.
- ▶ In this context, the pixel intensity is used for defining the similarity relationship: the set V is composed by the intensities that (arbitrarily) have to be considered in the definition of the adjacency.
- ▶ Three types of adjacency are used: *4-adjacency*, *8-adjacency*, and *m-adjacency (mixed)*.
- ▶ The pixels p and q having intensity values in V are:
 - ▶ 4-adjacent if $q \in N_4(p)$;
 - ▶ 8-adjacent if $q \in N_8(p)$;
 - ▶ *m-adjacent* if:
 - ▶ $q \in N_4(p)$, or
 - ▶ $q \in N_D(p)$ and none of the pixels in $N_4(p) \cap N_4(q)$ have values in V .
- ▶ *m-adjacency* is used for avoiding the ambiguities that arise using the 8-adjacency.

Adjacency (2)

0 1 1
0 1 0
0 0 1
(a)

0 1-1
0 1 0
0 0 1
(b)

0 1-1
0 1 0
0 0 1
(c)

(a) A set of pixels. (b) 8-adjacency. (c) m -adjacency.

Path

- ▶ The *path* from pixel p to pixel q having coordinates (x_p, y_p) and (x_q, y_q) is a sequence of $n + 1$ pixels having coordinates:

$$(x_0, y_0) (x_1, y_1) \dots, (x_n, y_n)$$

where:

- ▶ $(x_0, y_0) = (x_p, y_p)$ and $(x_n, y_n) = (x_q, y_q)$
- ▶ (x_{i-1}, y_{i-1}) and (x_i, y_i) are adjacent ($1 \leq i \leq n$)
- ▶ The number n is the path *length*.
- ▶ If $(x_0, y_0) = (x_n, y_n)$, the path is *closed*.
- ▶ The path definition depends from the adjacency definition:
 - ▶ 4-paths, 8-paths, or m -paths.

Connected components

- ▶ Let S a subset of pixels in an image.
- ▶ Two pixel, p and q , in S are said to be *connected in S* if there is a *path $p-q$* composed of pixels in S .
- ▶ For any pixel in S , the set of the pixels of S connected to that pixel constitutes a *connected component* of S .
- ▶ If there is only one connected component in S , then S is called *connected set*.
- ▶ In an image, any subset of pixel, R , that is a connected set is called a *region* of the considered image.

0	1	1	0	0	0	0	0	3	3	3	0	0	0	0	0
0	0	1	1	0	0	0	0	0	3	3	3	3	3	3	3
1	1	1	0	0	0	0	0	0	0	3	3	3	3	0	0
0	0	1	1	1	0	0	2	2	2	0	0	0	3	3	3
0	0	1	1	1	1	1	2	2	2	2	0	3	3	3	0
0	0	0	1	1	0	0	2	2	2	0	0	3	3	3	0

using
4-adjacency

note:
two regions
for 3

Relationship between regions

- ▶ The *adjacency* relationship can be extended to regions.
- ▶ Two regions R_i and R_j are *adjacent* if $R_i \cup R_j$ is a connected set.
- ▶ Regions that are not adjacent are called *disjoint*.
- ▶ The adjacency of regions depends on the neighborhood relationship adopted.
- ▶ The union of all the regions of an image is called *foreground*.
 - ▶ The terms *subject(s)* or *object(s)* of the image are also used.
- ▶ The complement of the foreground (i.e., the set of the pixels that do not belong to any regions) is called *background*.
- ▶ Since it is practical that the foreground objects are surrounded by the background, if required a frame of background pixels can be added to the image (*padding*).
 - ▶ Hence, one row at the top and one at the bottom and one line at the right and one at the left are added to the original image.

Boundary of a region

- ▶ The *boundary* (or *border* or *contour*) of a region R is the set of pixels of R adjacent to the complement of R .
 - ▶ In other words, the border of a region is composed of pixels that have at least one of their neighbors in the background.
- ▶ The boundary definition depends on the adjacency.

```
0 0 0 0 0
0 1 1 0 0
0 1 1 0 0
0 1 1 1 0
0 1 1 1 0
0 0 0 0 0
```

The circled pixel belongs to the boundary only if the 8-adjacency is used.

Boundary of a region (2)

- ▶ The boundary of a region is also called *inner border*, to be distinguished by the *outer border*, which is composed of background pixels that are adjacent to the inner border.
- ▶ The outer border is always a closed path.
 - ▶ The region can be an open path, and so its inner border.
 - ▶ The inner border can be identified as those foreground pixels that are adjacent to the outer border.

```
0 0 0 0 0 0 0 0 0
0 1 1 1 1 1 1 1 0
0 0 0 0 0 0 0 0 0
```

- ▶ The *edge* is an important concept that is similar to the border. It will be introduced later in the course.
 - ▶ The edge is often a subsequence of the border, but there are more complex cases.

Distance

- ▶ The *distance function* or *metric* is a real function defined on pairs of pixels such that:

1. $D(p, q) \geq 0$, $(D(p, q) = 0 \text{ iff } p = q)$
2. $D(p, q) = D(q, p)$
3. $D(p, q) \leq D(p, z) + D(z, q)$

- ▶ *Euclidean distance*, D_e , (L_2 norm):

$$D_e(p, q) = [(x_p - x_q)^2 + (y_p - y_q)^2]^{\frac{1}{2}}$$

- ▶ D_4 distance, (L_1 norm):

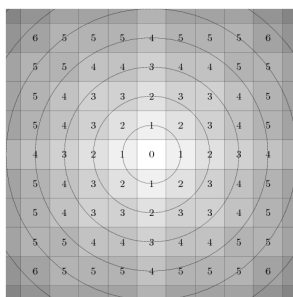
$$D_4(p, q) = \|x_p - x_q\| + \|y_p - y_q\|$$

- ▶ aka *Manhattan*, or *taxi-cab* or *city-block* distance.

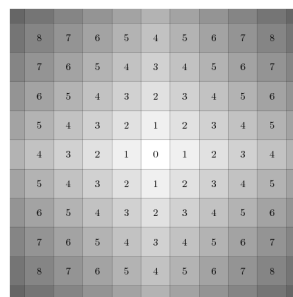
Distance (2)

- ▶ D_8 distance, or *chessboard* distance (L_∞ norm):

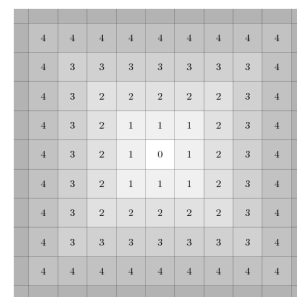
$$D_8(p, q) = \max(\|x_p - x_q\|, \|y_p - y_q\|)$$



D_e



D_4



D_8

- ▶ The pixels, q , are labeled with the integer radius, $r \in \mathbb{N}$, of the disk they belong to: $r - 1 < D(p, q) \leq r$, where p is the central pixel.

Distance (3)

- ▶ The distance can be evaluated also using the m -adjacency.
- ▶ The m -distance, D_m , differs from D_e , D_4 , and D_8 since it depends on the image content.

0	1	1	1	-	2	2	3	-	3	4	5
1	1	0	0	1	1	-	-	1	2	-	-
1	0	0	0	0	-	-	-	0	-	-	-
0	1	0	0	-	1	-	-	-	1	-	-
0	0	1	1	-	-	2	3	-	-	2	3
	(a)				(b)				(c)		

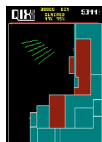
- (a) Pixel arrangement. (b) D_8 distance (only for 1-valued pixels).
(c) The corresponding D_m distance.

Homeworks and suggested readings



DIP, Section 2.5

- ▶ pp. 68–71



Flood fill, seed fill

- ▶ http://en.wikipedia.org/wiki/Flood_fill



GIMP

- ▶ Tools
 - ▶ Paint Tools
 - ▶ Bucket Fill