Edge detection

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Methods for Image Processing

academic year 2015-2016









Discontinuity types

There are three main types of discontinuity that usually carry the information:

- isolated (edge) point
 - abrupt local change of intensity
- edge segment
 - connected set of discontinuity points
- line
 - thin region where the intensity changes on both the sides

Discontinuity detection

- Derivative are the most suited tools for detecting changes
 but they are defined on continuous domains.
- Since the digital images are discrete, approximations have to be used (using Taylor series):
 - first derivative:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

second derivative:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) - 2f(x) + f(x-1)$$

Hence, the value of the derivative estimated in a pixel will be considered a measure of discontinuity of the intensity in that point.



Isolated points detection • The Laplacian is the simplest derivative operator that is isotropic with respect to the four principal directions: $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ • Partial derivatives: $\frac{\partial^2 f}{\partial x^2} = f(x+1, y) - 2f(x, y) + f(x-1, y)$ $\frac{\partial^2 f}{\partial y^2} = f(x, y+1) - 2f(x, y) + f(x, y-1)$ • Hence, the Laplacian can be computed as: $\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$ • Optionally, the diagonals contribution can be added: $\nabla^2 f(x, y) + f(x-1, y-1) + f(x+1, y+1) + f(x-1, y+1) + f(x+1, y-1) - 4f(x, y)$

Thresholding

- Thresholding is the simplest technique for obtaining the pixels where a substantial discontinuity takes place.
- ▶ Hence, the image g is considered:

$$g(x, y) = \left\{ egin{array}{ll} 1, & ext{if } |
abla^2 f(x, y)| \geq T \ 0, & ext{otherwise} \end{array}
ight.$$

for a suitable threshold T.







- The Laplacian have a double response to the edges.
- Considering the absolute value, the thickness of the detected lines results doubled.
- Lines can be handled considering only the strictly positive values.





a b c d e f

- The image (a) is processed with the 45° mask producing the image (b).
- (c) and (d) shows the details in the upper-left and bottom-right corners of (b) respectively. The response in (d) is stronger then in (c), due to the different lines thickness.
- In (e), only positive values are shown.
- In (f), (e) is thresholded using its maximum value as threshold.









Gradient based edge detection

- The gradient allows to find edge strength and direction.
- The gradient ∇f of a bidimensional function, f(x, y) is:

$$abla f \equiv \operatorname{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} rac{\partial f}{\partial x} \\ rac{\partial f}{\partial y} \end{bmatrix}$$

• The magnitude of the gradient, M(x, y) is:

$$M(x, y) = \max(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

- The approximation $M(x, y) \approx |g_x| + |g_y|$ is often used.
- The gradient vector points in the directions of the greatest rate of change at the considered location:

$$\alpha(x, y) = \tan^{-1} \left[\frac{g_y}{g_x} \right]$$















Marr-Hildreth edge detector (3) In order to achieve a good approximation of the Gaussian, n should be chosen at least as the smallest odd integer greater or equal to 6σ. ∇²G can be obtained sampling the Gaussian function and then computing its Laplacian through the above illustrated operators. Due to the linearity of the operations: [∇²G(x, y)] * f(x, y) = ∇²[G(x, y) * f(x, y)] The Marr-Hildreth algorithm is composed of three steps: Filter the image with a n × n Gaussian filter with an appropriate scale, σ. Compute the Laplacian (e.g., with a 3 × 3 mask). Find the zero-crossings.









Canny edge detector (3)

- Robust detection and linking of the edge points:
 - Hysteresis thresholding:
 - two thresholds, T_H and T_L , such that: $T_L < T_H$
 - $g_{NL}(x, y) = T_L \leq g_N(x, y) \leq T_H$
 - $g_{NH}(x, y) = g_N(x, y) \ge T_H$
 - g_{NH} and g_{NL} contain respectively the strong and the weak edge points
 - After thresholding, the edges in g_{NH} are usually broken, and in g_{NL} there are some valid points.
 - The valid points in g_{NL} can be selected:
 - for every point in g_{NH}, mark as valid all the points in g_{NL} that are 8-connected to the considered points;
 - ▶ at the end, all the valid points in g_{NL} are added to g_{NH} .









after morphological thinning.











