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Advanced morphological processing

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Methods for Image Processing

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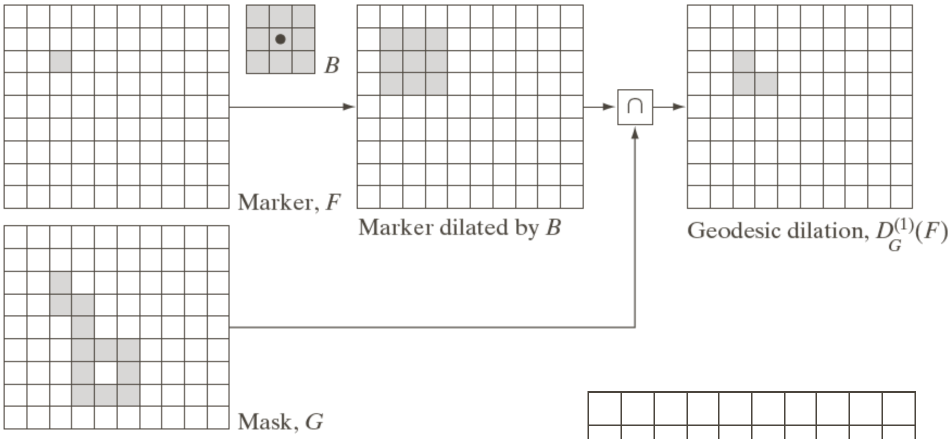
Geodesic dilation

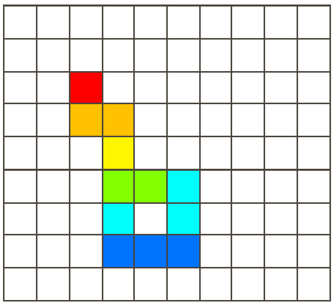
The *geodesic dilation* is an iterative morphological transformation requiring:

- marker image, F: starting points;
- mask image, G: constraint;
- structuring element, B.
- $F \subseteq G$
- $D_G^{(n)}(F)$: geodesic dilation of size n of F with respect to G.

- $D_G^{(0)}(F) = F$
- $D_G^{(1)}(F) = (F \oplus B) \cap G$
- $D_G^{(n)}(F) = D_G^{(1)}\left(D_G^{(n-1)}(F)\right)$

Geodesic dilation (2)





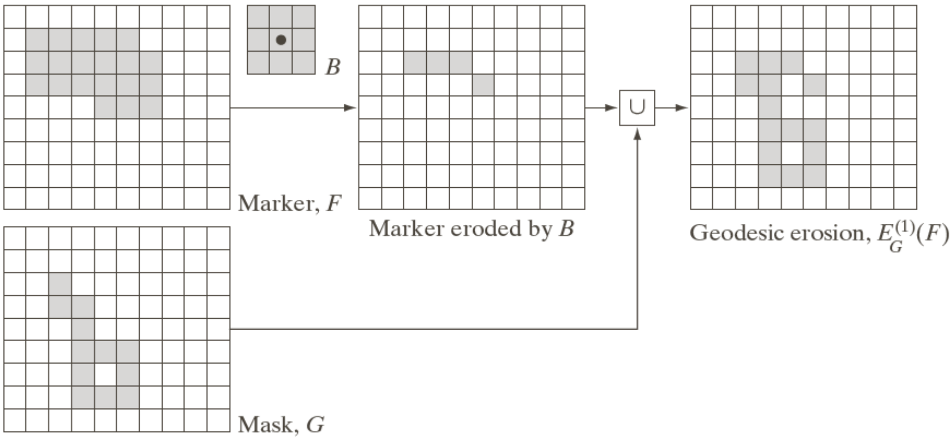
Geodesic erosion

The *geodesic erosion*, similarly to the geodesic dilation, is defined as:

- $E_G^{(0)}(F) = F$
- $E_G^{(1)}(F) = (F \ominus B) \cup G$
- $E_G^{(n)}(F) = E_G^{(1)}\left(E_G^{(n-1)}(F)\right)$

- Geodesic dilation and erosion are dual with respect to the complementation.

Geodesic erosion (2)



Morphological reconstruction

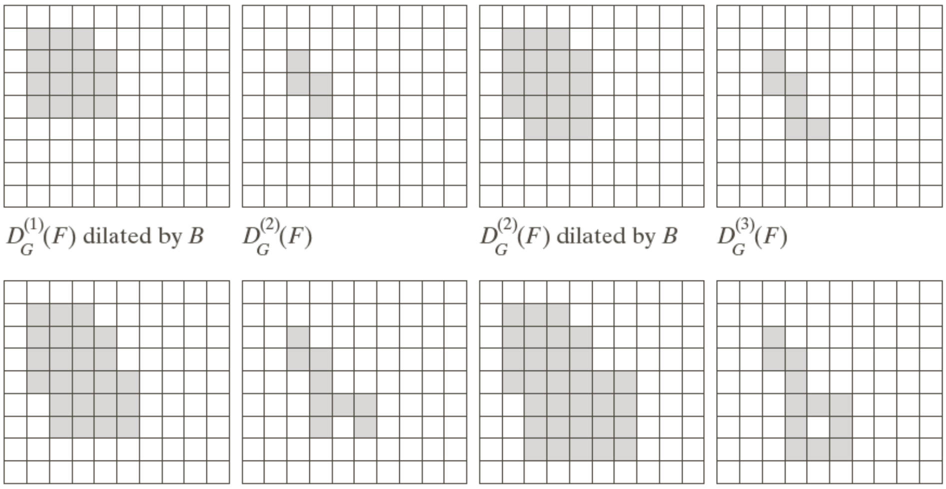
- The reconstruction through the geodesic operations are defined as the results of such operation at the stability:
  - morphological reconstruction through dilation:

$$R_G^D(F) = D_G^{(k)}(F), \text{ where } D_G^{(k)}(F) = D_G^{(k-1)}(F)$$

  - morphological reconstruction through erosion:

$$R_G^E(F) = E_G^{(k)}(F), \text{ where } E_G^{(k)}(F) = E_G^{(k-1)}(F)$$

Morphological reconstruction (2)



$D_G^{(1)}(F)$ dilated by B — $D_G^{(2)}(F)$ — $D_G^{(2)}(F)$ dilated by B — $D_G^{(3)}(F)$

$D_G^{(3)}(F)$ dilated by B — $D_G^{(4)}(F)$ — $D_G^{(4)}(F)$ dilated by B — $D_G^{(5)}(F) = R_G^D(F)$

Opening through reconstruction

- The effects of the opening:
  - details are eliminated by the erosion;
  - the dilation operates the reconstruction.
- The quality of the results depends by the shape of the structuring element.

Opening through reconstruction of size n:

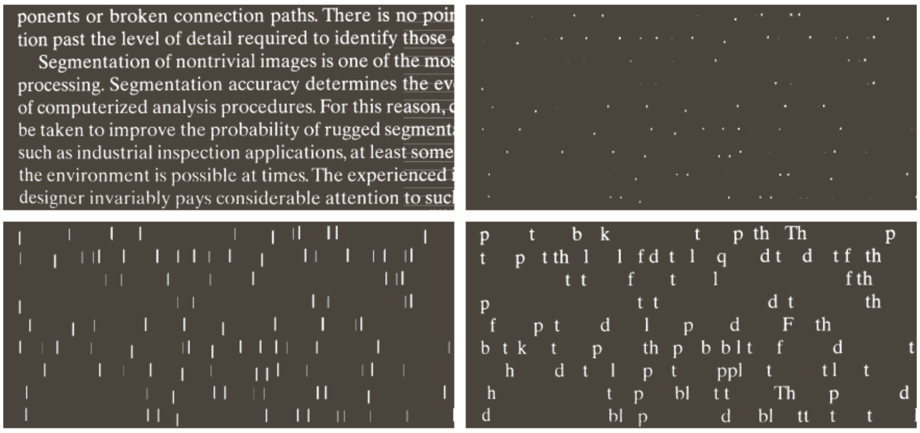
$$O_R^{(n)}(F) = R_F^D[F \ominus nB]$$

- erosion for eliminating the details (n times);
- reconstruction through the geodesic dilation.
- Note: the reconstruction is *bounded* by the mask.

Opening through reconstruction (2)

Selection of the "long" characters:

1. erosion with a 51×1 structuring element;
2. opening through reconstruction.



opening with a 51×1 SE — opening through reconstruction

Hole filling

The opening through the reconstruction allows to state a hole filling procedure that does not need of a starting "seed".

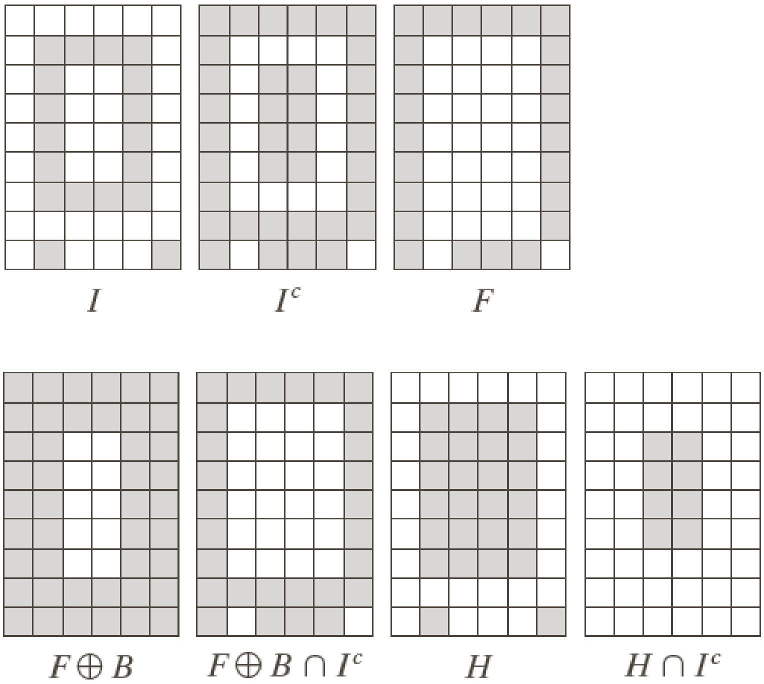
- $I(x, y)$, binary image (with an hole)
- $F(x, y)$, marker image, generated as a matrix of 0s, but in those border pixels where I is 0:

$$F(x, y) = \begin{cases} 1 - I(x, y), & \text{if } (x, y) \text{ is a border pixel} \\ 0, & \text{otherwise} \end{cases}$$

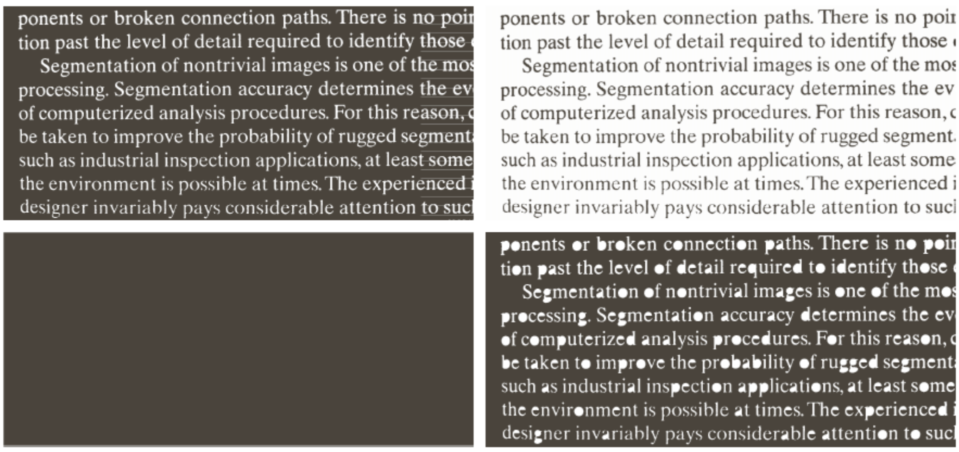
- $H(x, y)$, hole filled version of I:

$$H = \left[R^D_{I^c}(F) \right]^c$$

Hole filling (2)



Hole filling (3)


ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
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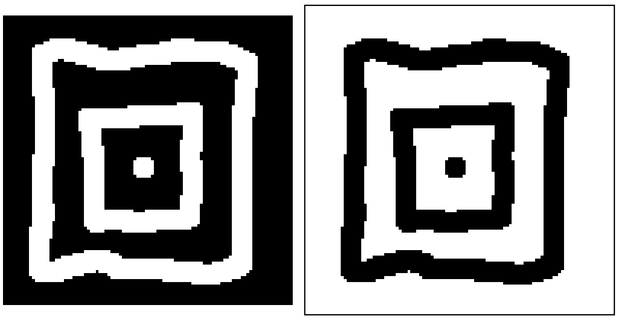
I	I^c
F	H

Hole filling: a simple exercise



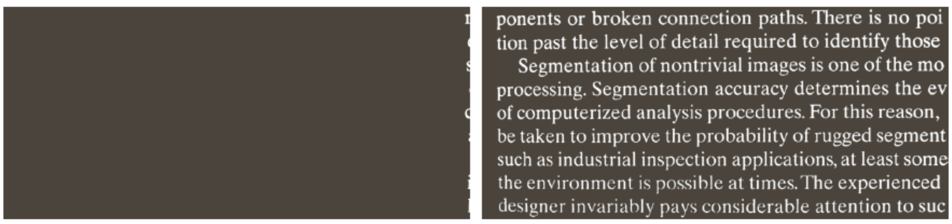
What happen to the inner holes?

Partial objects elimination

- $I(x, y)$, binary image (containing partial objects)
- $F(x, y)$, marker image, generated as a matrix of 0s, but in those border pixels where I is 1:

$$F(x, y) = \begin{cases} I(x, y), & \text{if } (x, y) \text{ is a border pixel} \\ 0, & \text{otherwise} \end{cases}$$

- $R_I^D(F)$, image with only the objects that touch the border
- $I - R_I^D(F)$, original image without objects touching the border

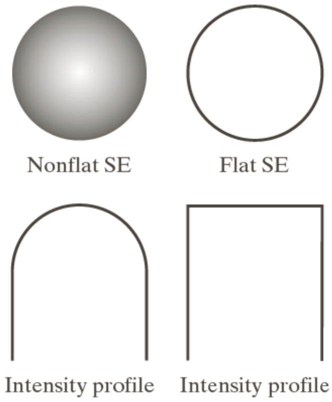


Gray levels morphology

- The morphological operations can be extended for the gray levels images.

- Here the images cannot be considered as sets anymore, but as (non-binary) functions.

- Two types of structuring elements can be devised:
  - flat;
  - non-flat.



Flat erosion and dilation

The morphological operations with *flat* structuring element, b, on the image f can be defined as follows:

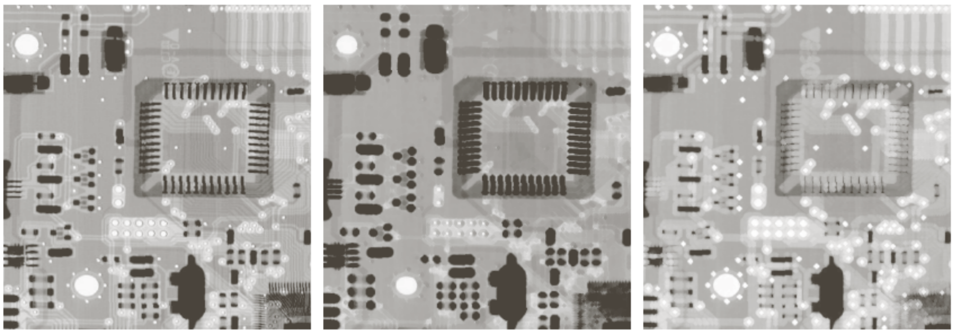
- Erosion:

$$[f \ominus b](x, y) = \min_{(s,t) \in b} \{f(x+s, y+t)\}$$

- Dilation:

$$\begin{aligned} [f \oplus b](x, y) &= \max_{(s,t) \in \hat{b}} \{f(x+s, y+t)\} \\ &= \max_{(s,t) \in b} \{f(x-s, y-t)\} \end{aligned}$$

Flat erosion and dilation (2)



f $\quad f \ominus b$ $\quad f \oplus b$

- f, 448 × 425 pixels;
- b, flat disk of radius 2.

Non-flat erosion and dilation

The morphological operations with *non-flat* structuring element, b_N, on the image f can be defined as follows:

- Erosion:

$$[f \ominus b_N](x, y) = \min_{(s,t) \in b_N} \{f(x+s, y+t) - b_N(s,t)\}$$

- Dilation:

$$[f \oplus b_N](x, y) = \max_{(s,t) \in b} \{f(x-s, y-t) + b_N(s,t)\}$$

Like in the binary domain:

- $(f \ominus b_N)^c = f^c \oplus \hat{b_N}$
- $(f \oplus b_N)^c = f^c \ominus \hat{b_N}$

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Opening and closing

Similarly to the binary case, the operations of opening and closing can be defined.

- Opening
$$f \circ b = (f \ominus b) \oplus b$$
- Closing
$$f \bullet b = (f \oplus b) \ominus b$$

The duality properties hold as well:

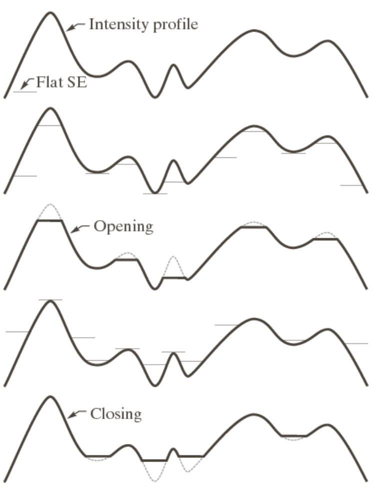
- $(f \bullet b)^c = f^c \circ \hat{b}$
- $(f \circ b)^c = f^c \bullet \hat{b}$

Opening and closing (2)

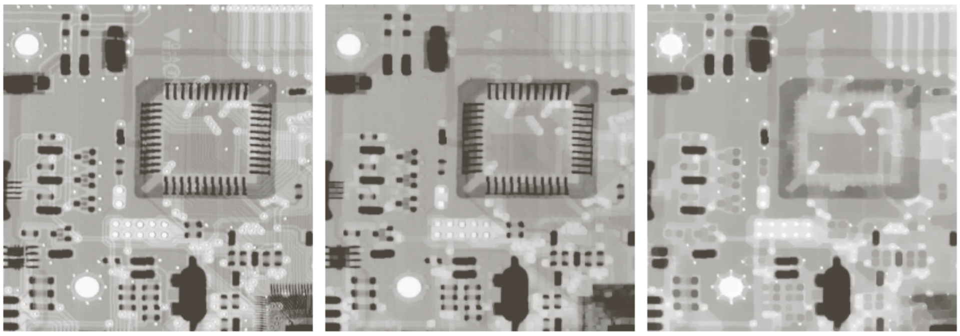
- The opening shaves the peaks.
- The closing fills the valleys.
- The following properties hold:
  - $f \circ b \leq f$
  - if $f_1 \leq f_2$, then $f_1 \circ b \leq f_2 \circ b$
  - $(f \circ b) \circ b = f \circ b$

  and
  - $f \leq f \bullet b$
  - if $f_1 \leq f_2$, then $f_1 \bullet b \leq f_2 \bullet b$
  - $(f \bullet b) \bullet b = f \bullet b$

  where $f \leq g$ if the domain of f is a subset of the domain of g and if $f(x, y) \leq g(x, y)$ for every points of the domain of f.



Opening and closing (3)

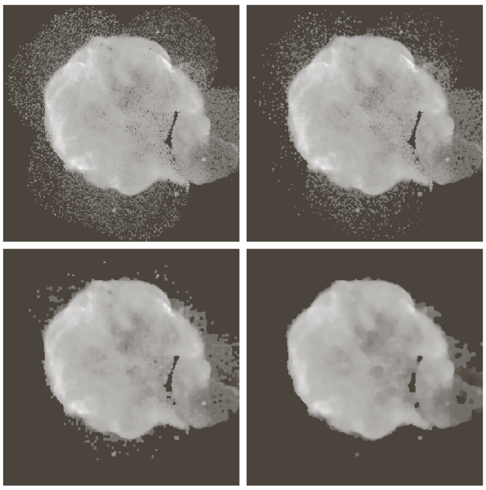


f $\quad f \circ b_1 \quad f \bullet b_2$

- f, 448 × 425 pixels;
- b_1, flat disk of radius 3;
- b_2, flat disk of radius 5.

Morphological smoothing

- opening + closing = smoothing

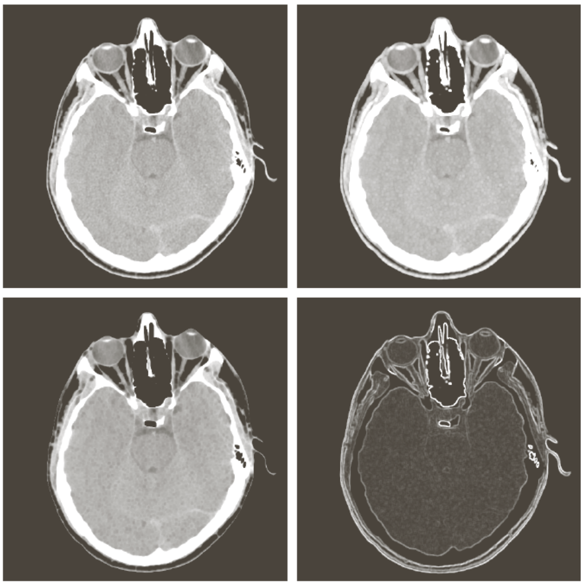


Morphological smoothing with disks of 1, 3, and 5 pixels of radius.

Alternatively, smoothing with the same SE can be iterated.

Morphological gradient

- $g = (f \oplus b) - (f \ominus b)$



f	$(f \oplus b)$
$(f \ominus b)$	g

Top-hat and *bottom-hat* transformations

- *Top-hat* transformation:

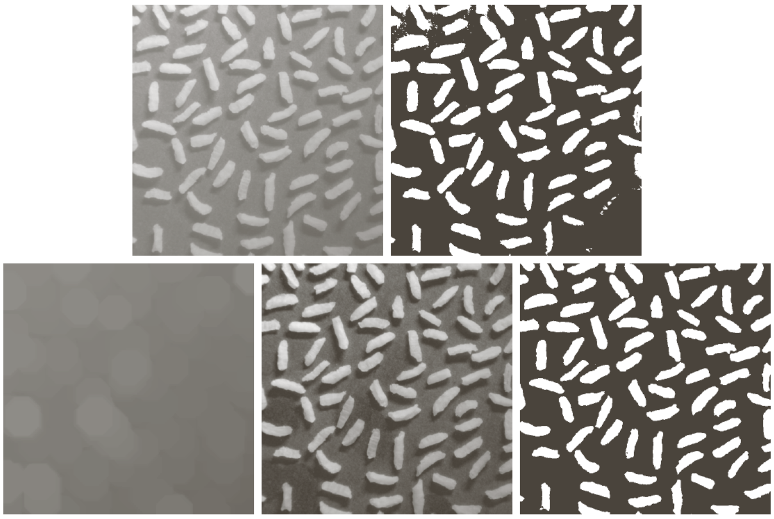
$$T_{\text{hat}} = f - (f \circ b)$$

- *Bottom-hat* transformation:

$$B_{\text{hat}} = (f \bullet b) - f$$

- These transformations preserve the information removed by the opening and closing operations, respectively.
- They are often cited as *white top-hat* and *black top-hat*.

Top-hat transformation for granulometry



A non uniform illumination can prevent thresholding effectiveness. Top-hat with a 40 pixels radius disk (on a 600 $\times$ 600 pixels image) solves the problem.

Advanced morphological operations on gray levels images

- Geodesic operations and morphological reconstruction can be defined also for grayscale images:

  geodesic dilation of size 1
  $$D_g^{(1)}(f) = (f \oplus b) \wedge g$$
  geodesic erosion of size 1
  $$E_g^{(1)}(f) = (f \ominus b) \vee g$$
  morphological reconstruction by dilation/erosion

  geodesic operations at stability

  where $\wedge$ and $\vee$ are the pointwise minimum and maximum operators.
- Opening and closing by reconstruction can be obtained similarly as in the binary case:

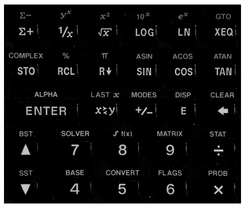
$$O_R^{(n)}(f) = R_f^D(f \ominus n\,b) \quad \text{and} \quad C_R^{(n)}(f) = R_f^E(f \oplus n\,b)$$

Top-hat by reconstruction

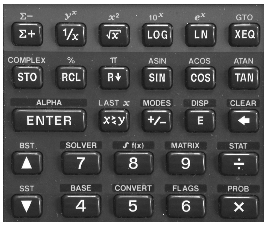
- Top-hat by reconstruction makes use of opening by reconstruction instead of just opening:

$$T_{\text{hat}R}(f) = f - (R_f^D\,(f \ominus b))$$

$R_f^D\,(f \ominus b)$

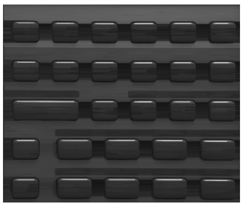


$T_{\text{hat}R}(f)$



f : 1360 × 1134

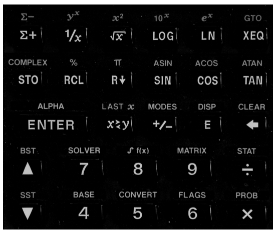
b : 1 × 71



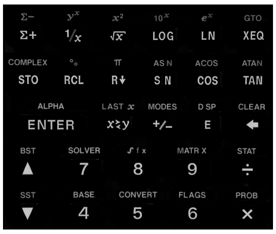
$f \circ b$

$T_{\text{hat}}(f)$

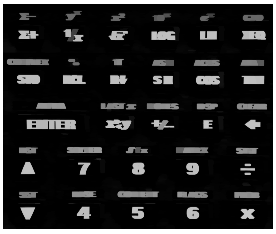
Top-hat by reconstruction (2)



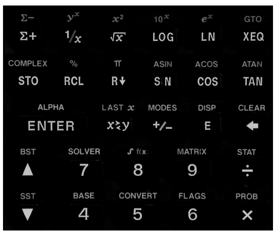
$T_{\text{hat}R}(f)$



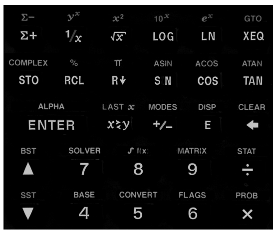
$O_R^1(T_{\text{hat}R}(f))$
b_2 : 1 × 11



$g = O_R^1(T_{\text{hat}R}(f) \oplus b_3$
b_3 : 1 × 21



$h = T_{\text{hat}R}(f) \wedge g$



$R_h^D(g)$

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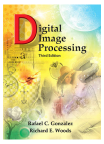
Homeworks and suggested readings



DIP, Sections 9.5.9–9.6

- pp. 656–679