Spatial filtering

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Methods for Image Processing

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Filtering

- The term *filtering* refers to a technique developed for the frequency domain.
- Rapid variations of intensity can be associated to high frequency components, while slow changes can be associated to low frequency components.
- The effects of the filtering process is the attenuation or the enhancement of the components in a target interval of frequency.

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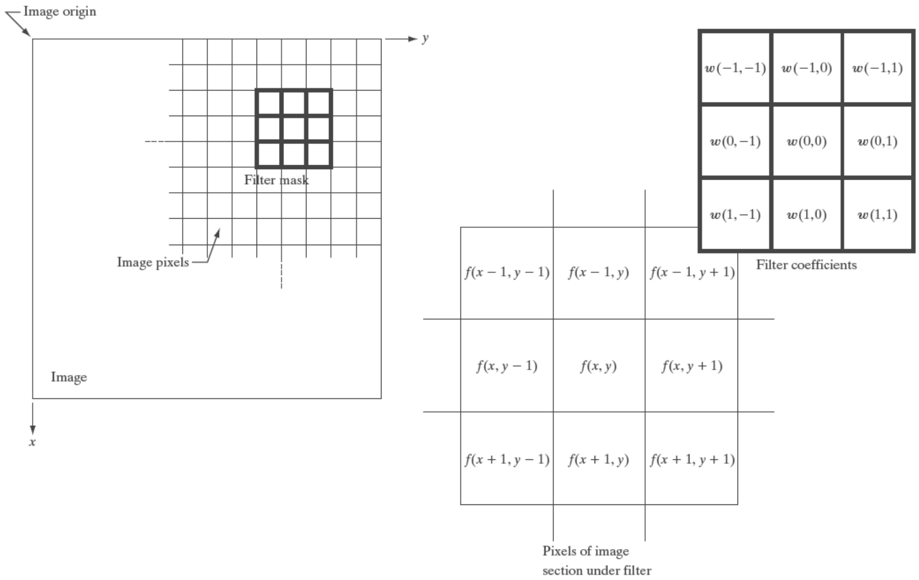
Spatial filtering

- Similar effects can be obtained also in the spatial domain: techniques of this kind are defined as *spatial filtering*.
- In general, the spatial filtering techniques operate on an image taking into consideration the intensity values in an suitable neighborhood of each pixel.
  - For each pixel of the original image, the intensity of the corresponding pixel in the "filtered" image is computed.
  - The transformation rule is often described by a matrix, called *filter*, with the same size of the considered neighborhood.
  - If the transforming rule is a linear function of the intensities in the considered neighborhood, the technique is called *spatial linear filtering* (*non-linear*, otherwise).

Linear filtering



Linear filtering (2)

- The pixel (x, y) in the filtered image, g, is obtained as the weighted average of the pixels in the original image, f, of a suitable neighborhood of (x, y):

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)\, f(x + s, y + t)$$

- The weight matrix, w, is called *filter*.
  - Or *mask*, *template*, *window*.
  - For convenience, often a matrix with an odd number of rows $2a + 1$, and columns, $2b + 1$, is used.

Correlation and convolution

- Correlation

$$g(x, y) = w(x, y) \star f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)\, f(x+s, y+t)$$

- Convolution

$$g(x, y) = w(x, y) * f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)\, f(x-s, y-t)$$

- Vectorial representation

$$R = w_1 z_1 + \cdots + w_{mn} z_{mn} = \sum_{k=1}^{mn} w_k z_k = \mathbf{w}^T \mathbf{z}$$

Correlation

f									w				
0	0	0	1	0	0	0	0		1	2	3	2	8

					↓							
f					0	0	0	1	0	0	0	0
w	1	2	3	2	8							
					↑	starting position alignment						

zero padding

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w	1	2	3	2	8											

Correlation (2)

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w	1	2	3	2	8											
$w \star f$					0											

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w		1	2	3	2	8										
$w \star f$					0	0										

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w			1	2	3	2	8									
$w \star f$					0	0	0									

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w				1	2	3	2	8								
$w \star f$					0	0	0	8								

Correlation (3)

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w				1	2	3	2	8								
$w \star f$					0	0	0	8								

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w					1	2	3	2	8							
$w \star f$					0	0	0	8	2							

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w						1	2	3	2	8						
$w \star f$					0	0	0	8	2	3						

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w							1	2	3	2	8					
$w \star f$					0	0	0	8	2	3	2					

Correlation (4)

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w								1	2	3	2	8				
$w \star f$					0	0	0	8	2	3	2	1				

$\cdots$

f	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
w												1	2	3	2	8
$w \star f$					0	0	0	8	2	3	2	1	0	0	0	0

$w \star f$	0	0	0	8	2	3	2	1	0	0	0	0
Cropping	___	___	0	8	2	3	2	1	0	0	___	___
f			0	0	0	1	0	0	0	0		

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Convolution

	f								w rotated 180°				
0	0	0	1	0	0	0	0		8	2	3	2	1

					↓							
f					0	0	0	1	0	0	0	0
w	8	2	3	2	1							
					↑	starting position alignment						

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$w * f$	0	0	0	1	2	3	2	8	0	0	0	0
Cropping	——		0	1	2	3	2	8	0	0	——	
f			0	0	0	1	0	0	0	0		

Correlation and convolution on images (1)

↙ Origin $f(x, y)$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$w(x, y)$

1	2	3
4	5	6
7	8	9

Padded f

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

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Correlation and convolution on images (2)

Initial position for w

1	2	3	0	0	0	0	0	0
4	5	6	0	0	0	0	0	0
7	8	9	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Full correlation result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	9	8	7	0	0	0
0	0	0	6	5	4	0	0	0
0	0	0	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Cropped correlation result

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

Rotated w

9	8	7	0	0	0	0	0	0
6	5	4	0	0	0	0	0	0
3	2	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Full convolution result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	2	3	0	0	0
0	0	0	4	5	6	0	0	0
0	0	0	7	8	9	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Cropped convolution result

0	0	0	0	0
0	1	2	3	0
0	4	5	6	0
0	7	8	9	0
0	0	0	0	0

Vectorial representation

- When relative position of the coefficients is not important, it is more convenient to represent the effect of the filter using a vectorial representation.
  - A conventional indexing have to be specified.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$R = w_1 z_1 + \cdots + w_{mn} z_{mn} = \sum_{k=1}^{mn} w_k z_k = \mathbf{w}^T \mathbf{z}$$

Filter specification

- The filter coefficients have to be specified.
  - Their values depend on the desired effect.
- Direct specification:

$$R = \frac{1}{9}\sum_{i=1}^{9} z_i \qquad \Rightarrow \qquad w_i = \frac{1}{9}, \quad \forall i$$

- Function based specification:

$$h(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \qquad \Rightarrow \qquad w(s, t) = h(s, t)$$

- Algorithmic specification
  - Non linear filters
  - e.g., “max” filter

Smoothing filters

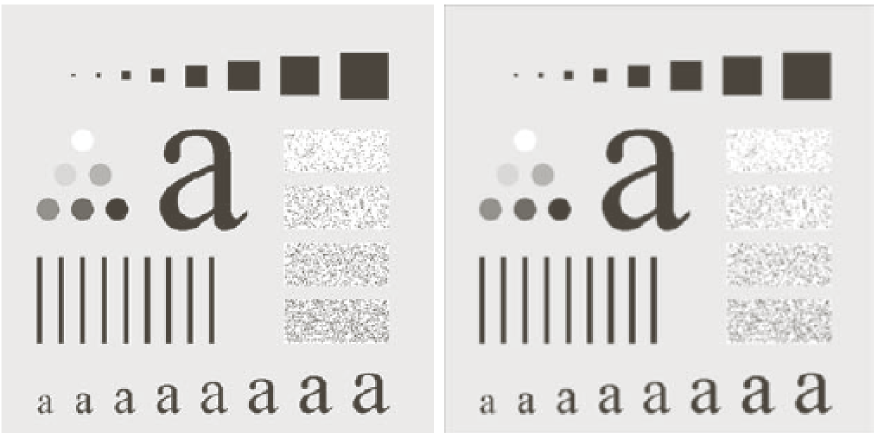
- Results in defocused images (*blur*).
- Small details (smaller that the size of the filter) removal:
  - more attention on large objects;
  - small gaps bridging;
  - noise reduction.
- Depending on the type of noise that affects the image, the filter can be linear or non-linear.

Linear smoothing filters

- Averaging filters (or *low pass*)
- Arithmetic average
- Weighted average

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Smoothing effects (1)



Original image and image filtered using a 3×3 averaging filter.

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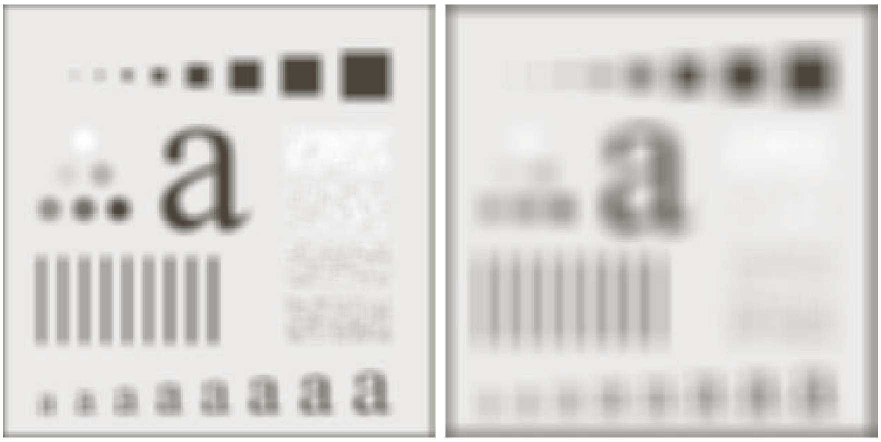
Smoothing effects (2)

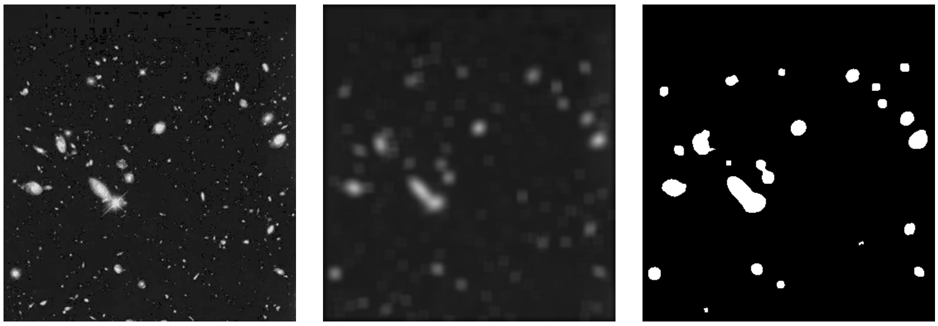
Filtering with averaging filters (5×5 and 9×9).

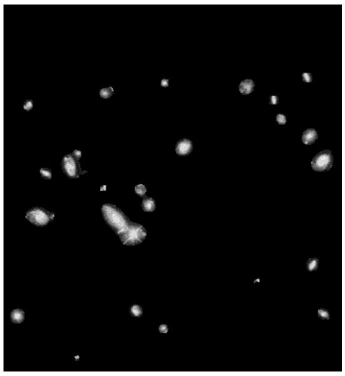
Smoothing effects (3)



Filtering with averaging filters (15×15 and 35×35).

Example: details removal





a	b	c
d		

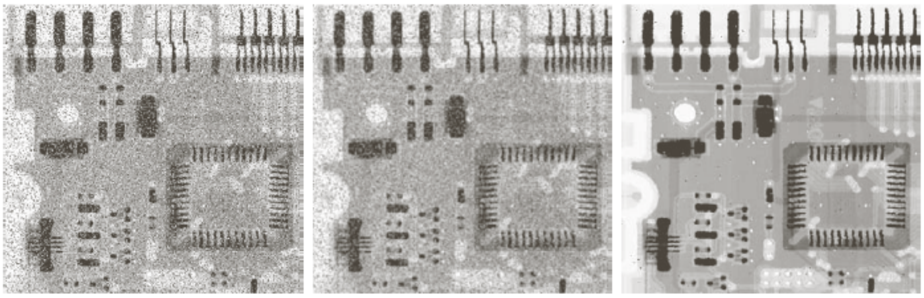
- (a) image 353 × 382
- (b) image filtered with 13 × 13 smoothing filter
- (c) thresholded image
- (d) masked image

Non-linear filters

- Statistics-based filters are the most used non-linear filters.
- Ranking (which requires sorting) is more computationally expensive than linear operations.
- In particular:
  - Median filter
  - Max and Min filters
  - Percentiles based filters

Example: median filter

Salt-and-pepper noise removal



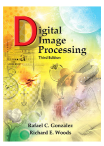
a | b | c

(a) Original image, corrupted by salt-and-pepper noise.

(b) Image filtered with averaging filter 3×3.

(c) Image filtered with median filter 3×3.

Homeworks and suggested readings



DIP, Sections 3.4, 3.5

- pp. 144–156

GIMP

- Filters
  - Blur
  - Enhance
    - Despeckle
  - Generic
    - Convolution matrix