

Image segmentation

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Image processing

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Segmentation by thresholding

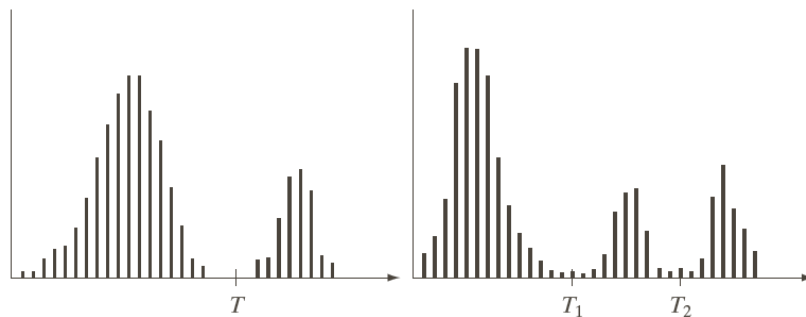
- ▶ Thresholding is the simplest segmentation method.
- ▶ The pixels are partitioned depending on their intensity value.
- ▶ Global thresholding, using an appropriate threshold T :

$$g(x, y) = \begin{cases} 1, & \text{if } f(x, y) > T \\ 0, & \text{if } f(x, y) \leq T \end{cases}$$

- ▶ Variable thresholding, if T can change over the image.
 - ▶ Local or regional thresholding, if T depends on a neighborhood of (x, y) .
 - ▶ adaptive thresholding, if T is a function of (x, y) .
- ▶ Multiple thresholding:

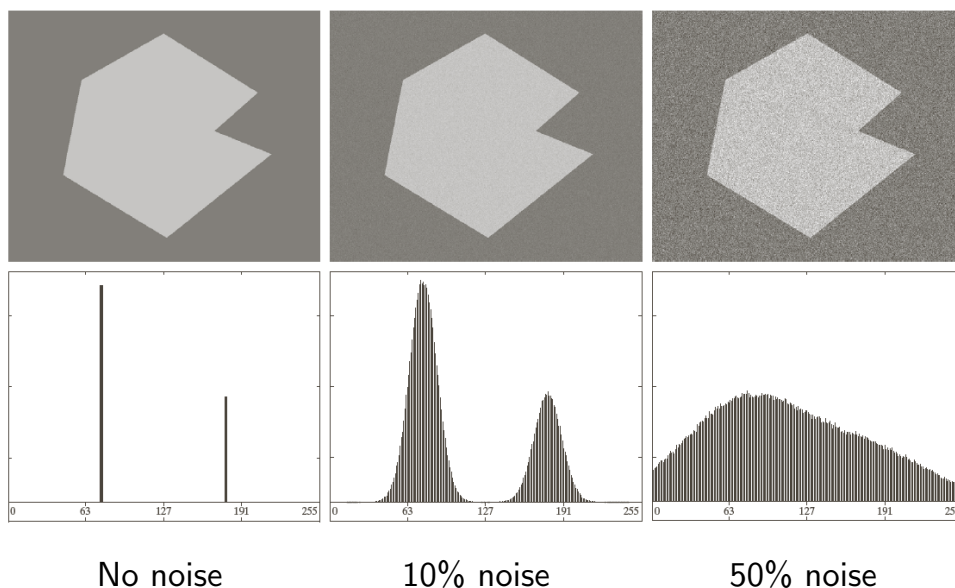
$$g(x, y) = \begin{cases} a, & \text{if } f(x, y) > T_2 \\ b, & \text{if } T_1 < f(x, y) \leq T_2 \\ c, & \text{if } f(x, y) \leq T_1 \end{cases}$$

Choosing the thresholds

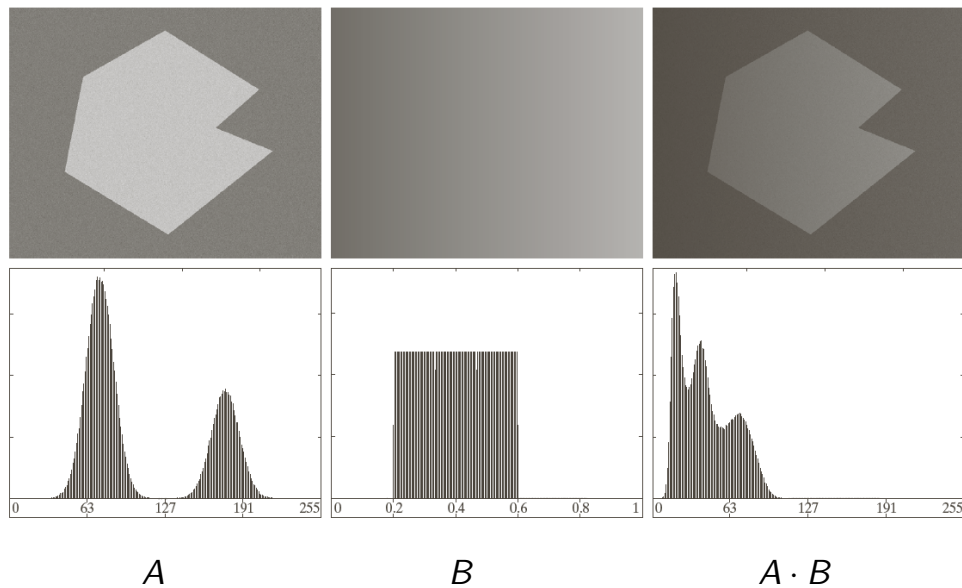


- ▶ Peaks and valleys of the image histogram can help in choosing the appropriate value for the threshold(s).
- ▶ Some factors affect the suitability of the histogram for guiding the choice of the threshold:
 - ▶ the separation between peaks;
 - ▶ the noise content in the image;
 - ▶ the relative size of objects and background;
 - ▶ the uniformity of the illumination;
 - ▶ the uniformity of the reflectance.

Noise role in thresholding



Illumination and reflection role in thresholding



Global thresholding

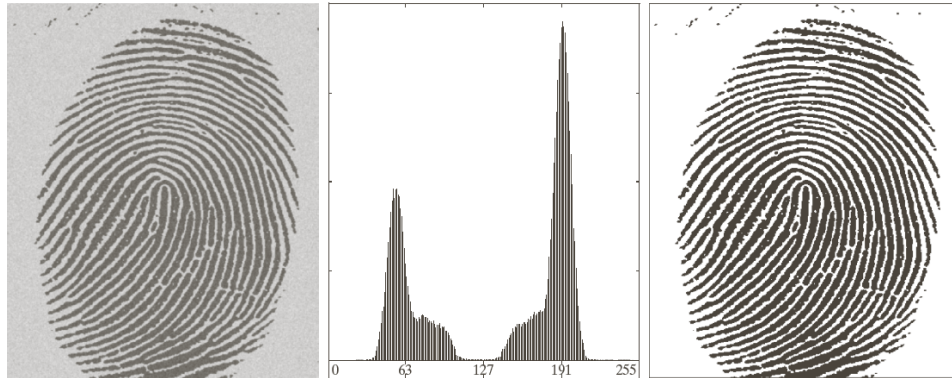
A simple algorithm:

1. Initial estimate of T
2. Segmentation using T :
 - ▶ G_1 , pixels brighter than T ;
 - ▶ G_2 , pixels darker than (or equal to) T .
3. Computation of the average intensities m_1 and m_2 of G_1 and G_2 .
4. New threshold value:

$$T_{\text{new}} = \frac{m_1 + m_2}{2}$$

5. If $|T - T_{\text{new}}| > \Delta T$, back to step 2, otherwise stop.

Global thresholding: an example



Otsu's method

- ▶ Otsu's method is aimed in finding the optimal value for the global threshold.
- ▶ It is based on the interclass variance maximization.
 - ▶ Well thresholded classes have well discriminated intensity values.
- ▶ $M \times N$ image histogram:
 - ▶ L intensity levels, $[0, \dots, L - 1]$;
 - ▶ n_i #pixels of intensity i :

$$MN = \sum_{i=0}^{L-1} n_i$$

- ▶ Normalized histogram:

$$p_i = \frac{n_i}{MN}$$

$$\sum_{i=0}^{L-1} p_i = 1, \quad p_i \geq 0$$

Otsu's method (2)

- ▶ Using k , $0 < k < L - 1$, as threshold, $T = k$:
 - ▶ two classes: C_1 (pixels in $[0, k]$) and C_2 (pixels in $[k + 1, L - 1]$)
 - ▶ $P_1 = P(C_1) = \sum_{i=0}^k p_i$, probability of the class C_1
 - ▶ $P_2 = P(C_2) = \sum_{i=k+1}^{L-1} p_i = 1 - P_1$, probability of the class C_2
 - ▶ m_1 , mean intensity of the pixels in C_1 :

$$\begin{aligned} m_1 &= \sum_{i=0}^k i \cdot P(i|C_1) \\ &= \sum_{i=0}^k i \frac{P(C_1|i)P(i)}{P(C_1)} \\ &= \frac{1}{P_1} \sum_{i=0}^k i \cdot p_i \end{aligned}$$

where $P(C_1|i) = 1$, $P(i) = p_i$ e $P(C_1) = P_1$.

Otsu's method (3)

- ▶ Similarly, m_2 , mean intensity of the pixels in C_2 :

$$m_2 = \frac{1}{P_2} \sum_{i=k+1}^{L-1} i \cdot p_i$$

- ▶ Mean global intensity, m_G :

$$m_G = \sum_{i=0}^{L-1} i \cdot p_i$$

- ▶ while the mean intensity up to the k level, m :

$$m = \sum_{i=0}^k i \cdot p_i$$

- ▶ Hence:

$$P_1 m_1 + P_2 m_2 = m_G$$

$$P_1 + P_2 = 1$$

Otsu's method (4)

- ▶ The global variance σ_G^2 :

$$\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 \cdot p_i$$

- ▶ The *between-class variance*, σ_B , can be defined as:

$$\begin{aligned}\sigma_B^2 &= P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 \\ &= P_1 P_2 (m_1 - m_2)^2 \\ &= \frac{(m_G P_1 - m)^2}{P_1(1 - P_1)}\end{aligned}$$

- ▶ The *goodness* of the choice $T = k$ can be estimated as the ratio η :

$$\eta = \frac{\sigma_B^2}{\sigma_G^2}$$

Otsu's method (5)

- ▶ The quantities required for the computation of η , can be obtained from the histogram:
- ▶ Hence, for each value of k , $\eta(k)$ can be computed:

$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2}$$

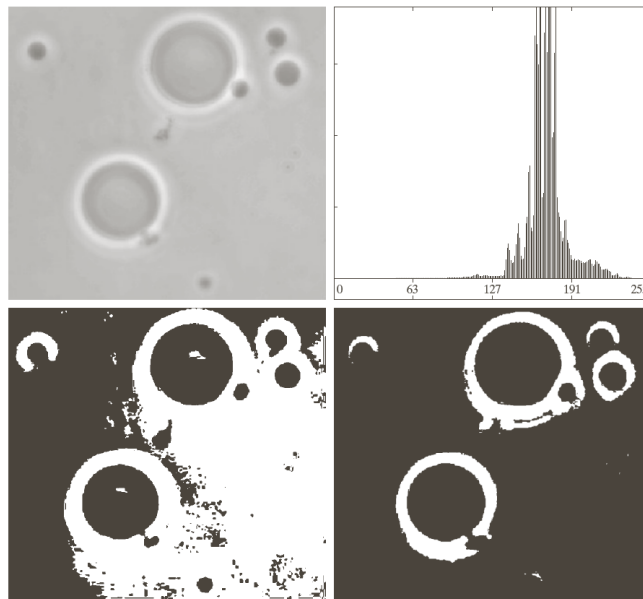
where

$$\sigma_B^2(k) = \frac{(m_G P_1(k) - m(k))^2}{P_1(k)(1 - P_1(k))}$$

- ▶ The optimal threshold value, k^* , satisfies:

$$\sigma_B^2(k^*) = \max_{0 < k < L-1} \sigma_B^2(k)$$

Otsu's method: an example



a	b
c	d

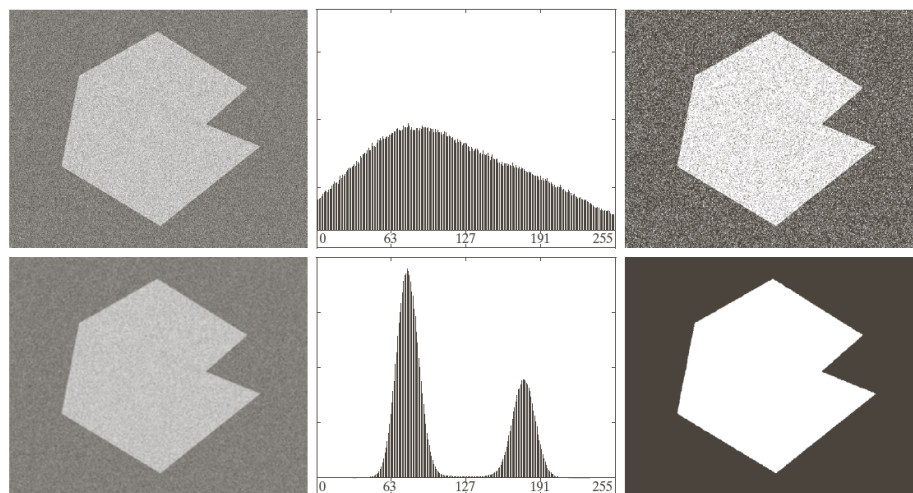
(a) original image;

(b) histogram of (a);

(c) global threshold:
 $T = 169$,
 $\eta = 0.467$;

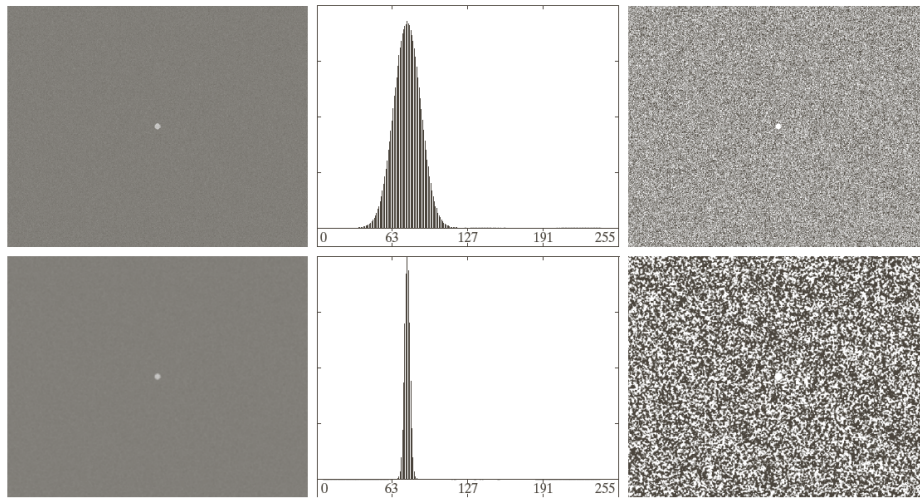
(d) Otsu's method:
 $T = 181$,
 $\eta = 0.944$.

Smoothing



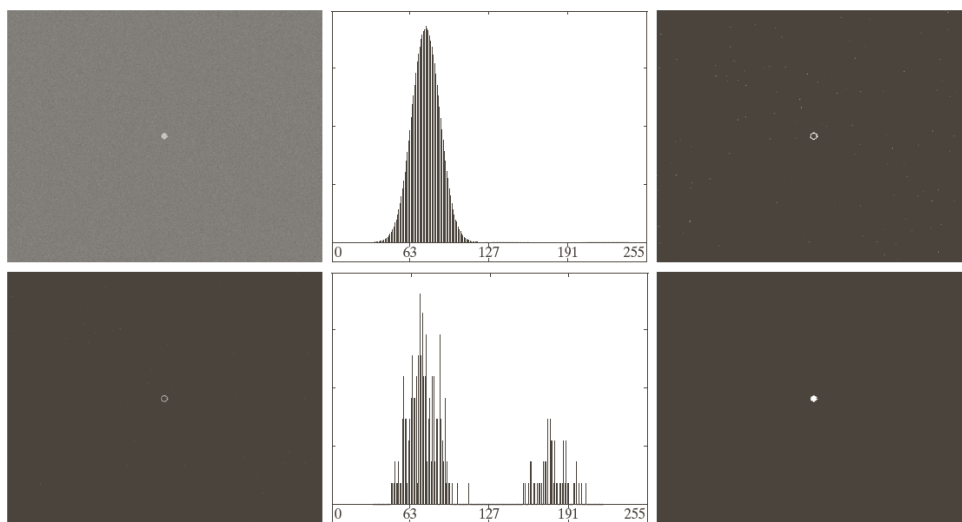
- ▶ Otsu's method may not work in presence of noise.
- ▶ Smoothing can produce a histogram with separated peaks.

Significance of the histogram



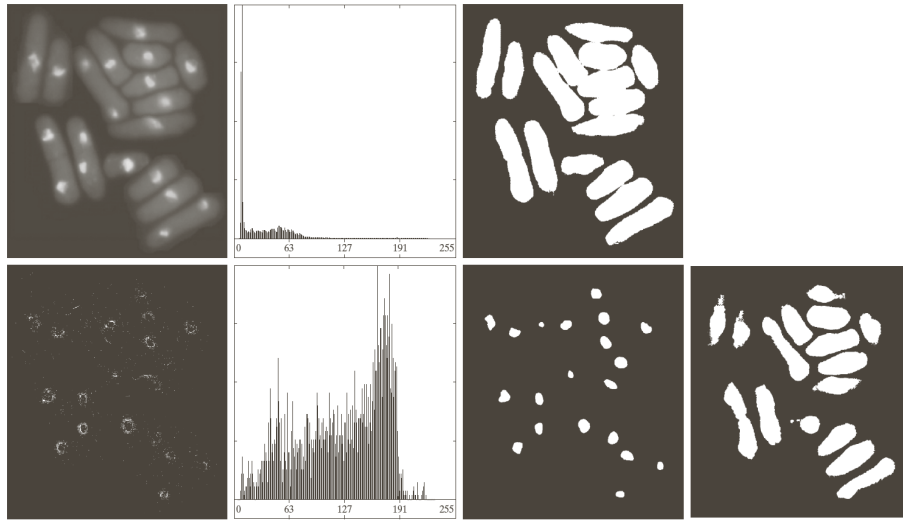
- ▶ If the distribution is not balanced, no information can be extracted from the histogram.
- ▶ Smoothing cannot help.

Selection of the border region



- ▶ Edge extraction techniques (e.g., Laplacian), can be used for selecting the region that carry the valuable information:
 - ▶ Those pixels that belong to the objects and to the background with an equal probability.

Use of edge for global thresholding (2)



- ▶ Changing the threshold of the Laplacian, several segmentations are obtained.
 - ▶ It can be useful for nested classes.

Multiple thresholds Otsu's method

- ▶ The Otsu's method can be applied also for the multiple thresholds segmentation (generally, double threshold).
- ▶ Between-class variance:

$$\sigma_B^2(k_1, k_2) = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 + P_3(m_3 - m_G)^2$$

- ▶ The optimal thresholds k_1^* and k_2^* can be computed as:

$$\sigma_B^2(k_1^*, k_2^*) = \max_{0 < k_1 < k_2 < L-1} \sigma_B^2(k_1, k_2)$$

- ▶ The separability degree can be measured as:

$$\eta(k_1^*, k_2^*) = \frac{\sigma_B^2(k_1^*, k_2^*)}{\sigma_G^2}$$

Multiple thresholds Otsu's method: an example

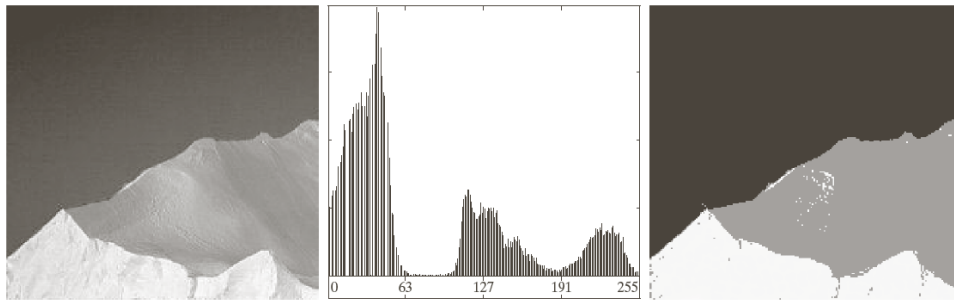
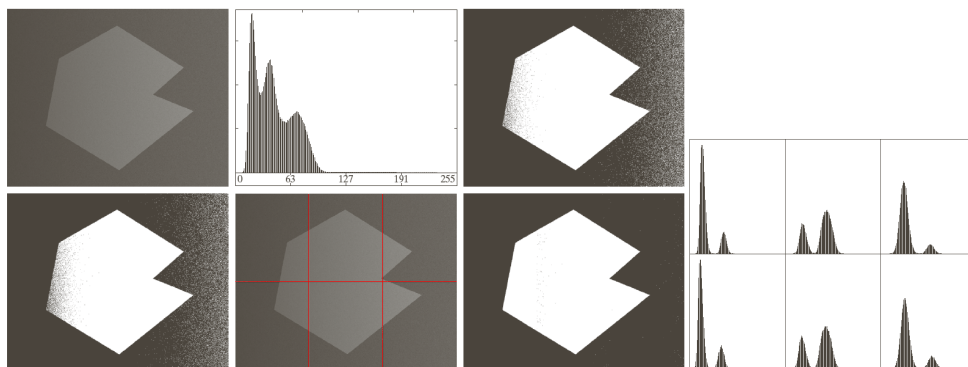


Image partitioning based thresholding

- ▶ In order to face non uniform illumination or reflectance, the image is partitioned and the thresholding is operated on each partition.
 - ▶ In each partition, the illuminance and reflectance is supposed uniform.
 - ▶ In each partition, objects and background have to be equally represented.



Local properties based thresholding

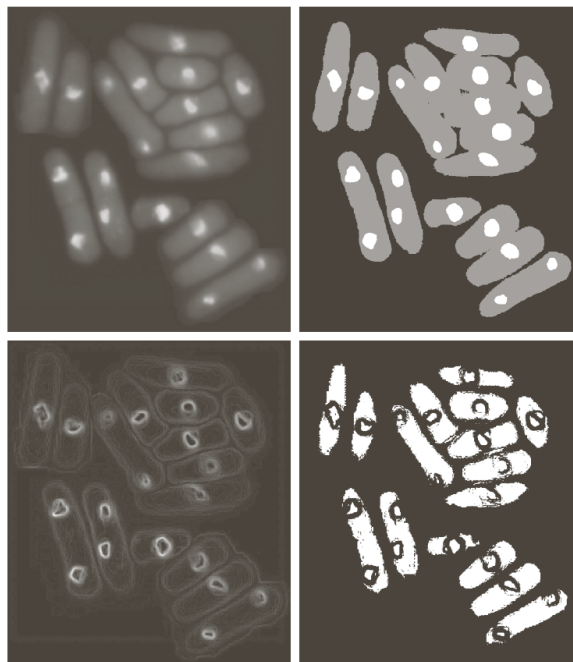
- ▶ Local properties (e.g., statistics) based criteria can be used for adapting the threshold.
- ▶ For example:
 - ▶ $T_{xy} = a\sigma_{xy} + bm_{xy}$
 - ▶ $T_{xy} = a\sigma_{xy} + bm_G$
- ▶ The segmentation is operated using a suitable predicate, Q_{xy} :

$$g(x, y) = \begin{cases} 1, & \text{if } Q_{xy} \\ 0, & \text{otherwise} \end{cases}$$

where Q_{xy} can be, for instance:

- ▶ $f(x, y) > T_{xy}$
- ▶ $f(x, y) > a\sigma_{xy}$ AND $f(x, y) > bm_{xy}$
- ▶ This technique can be easily generalized to multiple thresholds segmentation.

Local properties based thresholding: an example



a	b
c	d

- (b) segmentation of (a) with double threshold Otsu;
- (c) local (3×3) standard deviation;
- (d) segmentation with local thresholding.

Moving averages thresholding

- Pixels are visited following a zigzag path and the statistics are computed using only the last n visited pixels.



Growing based segmentation

- **Region growing** is a technique based on a controlled growing of some initial pixels (*seeds*).
- The selection of the *seeds* can be operated manually or using automatic procedures based on appropriate criteria.
 - A-priori knowledge can be included.
 - It is strictly application-dependent.
- The growing is controlled by the connectivity.
- The stop rule is another parameter of the algorithm.
 - It can depend on the a-priori knowledge on the problem.

Region growing: the basic algorithm

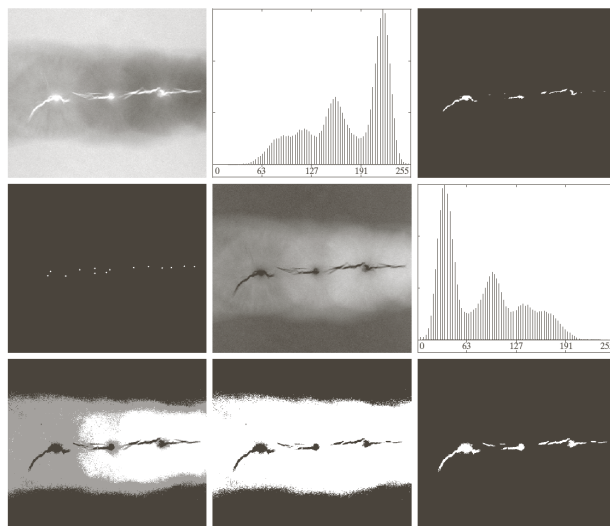
Given:

- ▶ $f(x, y)$, the image to be segmented;
- ▶ $S(x, y)$, binary image with the seeds (it is 1 only where the seeds are located);
- ▶ Q , predicate to be tested for each location (x, y) .

A simple region growing algorithm (based on 8-connectivity) is the following:

1. Erode all the connected components of S until they are only one pixel wide.
2. Generate the binary image f_Q such that $f_Q(x, y) = 1$ if $Q(x, y)$ is true.
3. Create the binary image g where $g(x, y) = 1$ if $f_Q(x, y) = 1$ and (x, y) is 8-connected to a seed in S .
4. The resulting connected components in g are the segmented regions.

Region growing: an example



- (a) f
- (c) $S(x, y) := f(x, y) > 254$
- (d) erosion of S
- (e) $|f - S|$
- (h) $|f - S| > 68$
- (i) segmentation by region growing with $Q := |f - S| \leq 68$

a	b	c
d	e	f
g	h	i

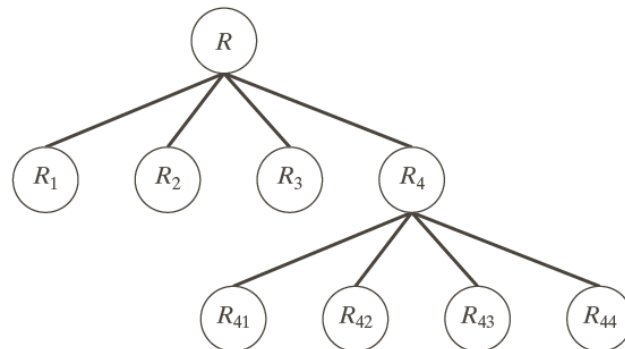
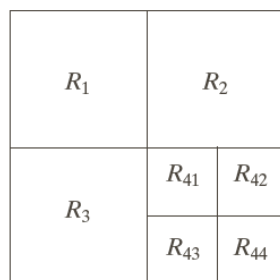
Region splitting and merging

- ▶ Iterative subdivision of the image in homogeneous regions (*splitting*).
- ▶ Joining of the adjacent homogeneous regions (*merging*).

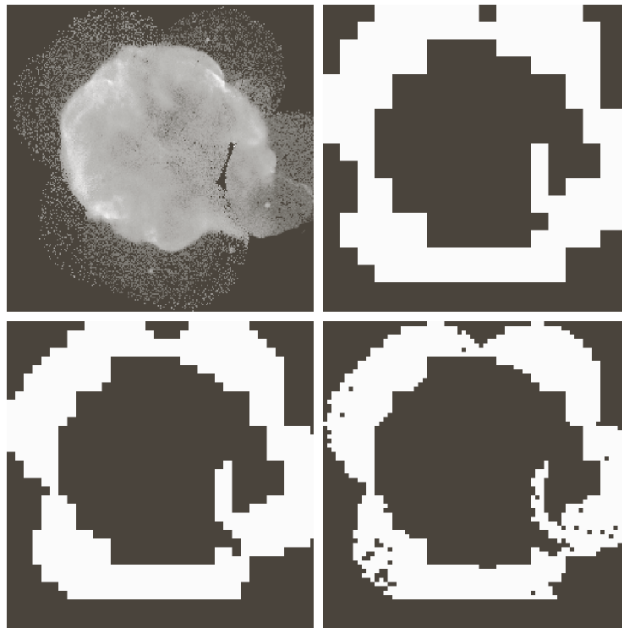
Given an image f and a predicate Q , the basic algorithm is:

1. $R_0 = f$
2. Iterative subdivision in quadrants of each region R_i for which $Q(R_i) = \text{FALSE}$.
3. If $Q(R_i) = \text{TRUE}$ for every regions, merge those adjacent regions R_i and R_j such that $Q(R_i \cup R_j) = \text{TRUE}$; otherwise, repeat step 2.
4. Repeat the step 3 until no merging is possible.

Quadtree based partitioning



Splitting and merging: an example

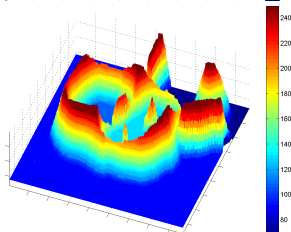
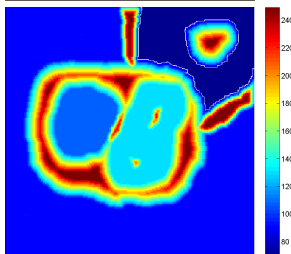
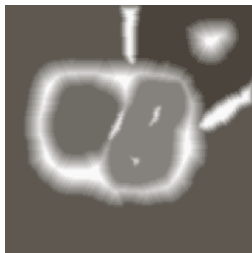


$$\begin{array}{c|c} a & b \\ \hline c & d \end{array}$$

$$Q := \sigma > a \text{ AND } 0 < m < b$$

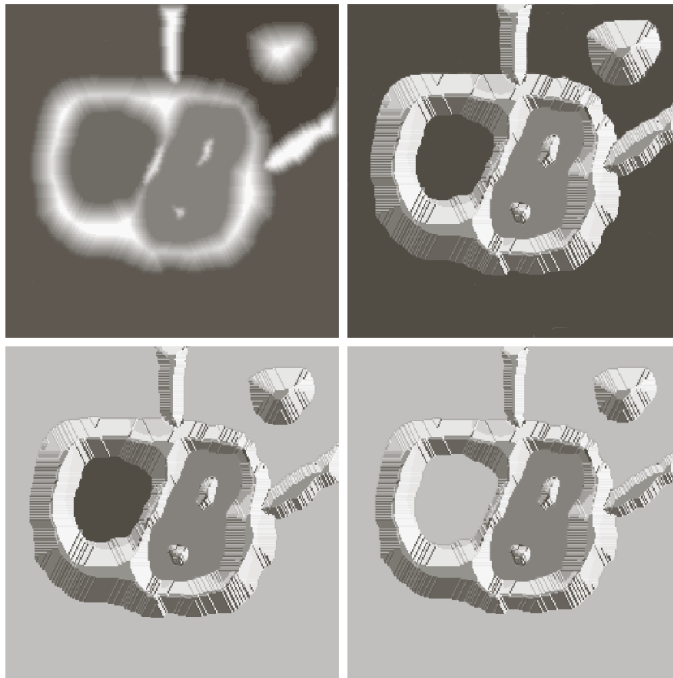
- ▶ (b) 32×32
- ▶ (c) 16×16
- ▶ (b) 8×8

Watershed

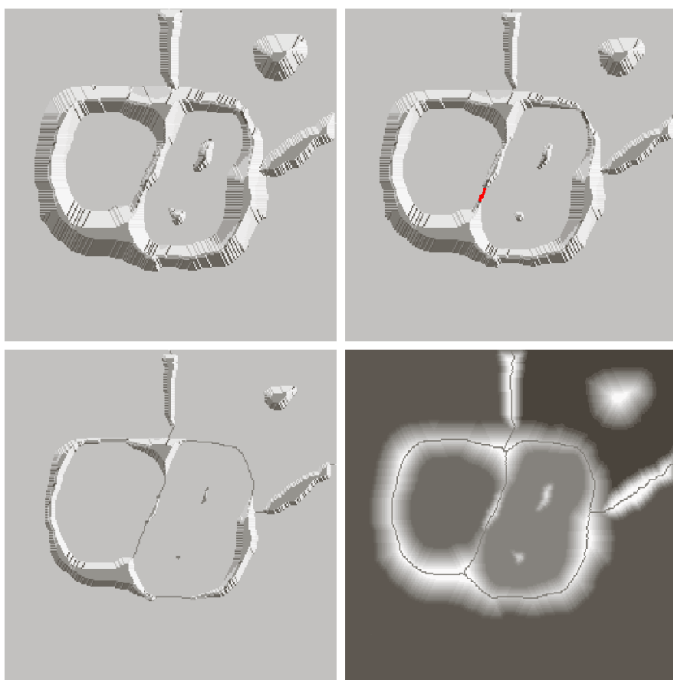


- ▶ The watershed technique is based on a topological interpretation of the image.
 - ▶ The intensity levels represent the height of the terrain that describe mountains and basins.
- ▶ For each basin, a hole in its minimum is supposed to be realized, from which, the rising underground water spills and fills the basins.
- ▶ As the water rises, the level reach the border of the basin and two or more adjacent basins tend to merge together.
- ▶ Dams are required for maintaining a separation between basins.
- ▶ These dams are the borders of the regions of the segmentation.

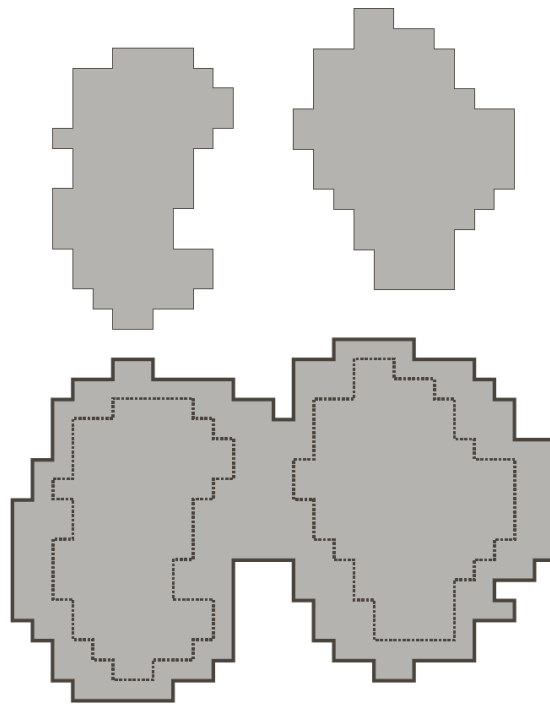
Watershed (2)



Watershed (3)

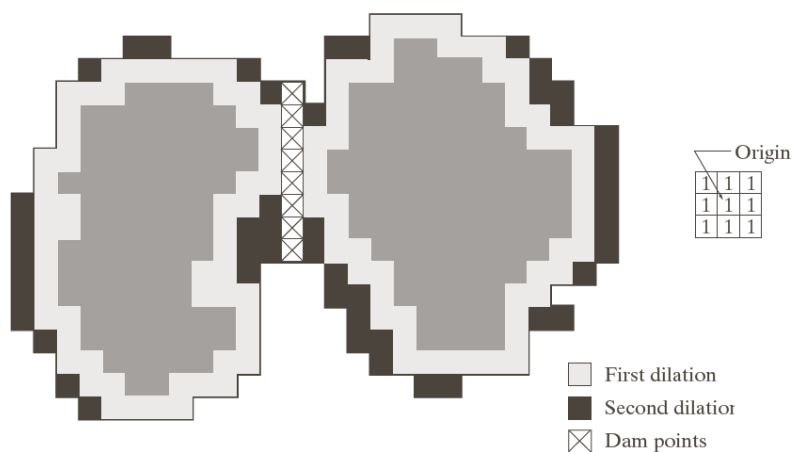


Dams construction



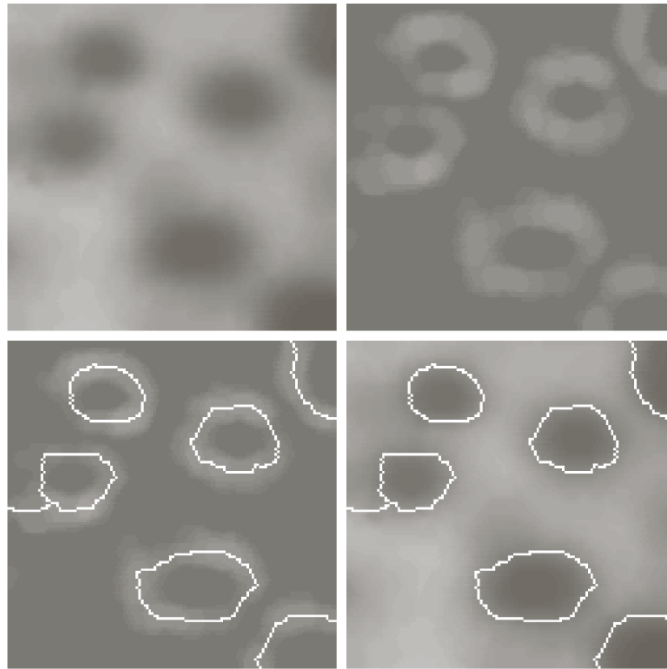
- ▶ The dams can be build using morphological dilation.
- ▶ Starting from the last step before merging, dilation can be performed until the two disjoint components become connected.

Dams construction (2)



- ▶ The dams can be realized setting the pixels at L (where the levels of intensity are in $[0, L - 1]$).
- ▶ Generally, the watershed algorithm is applied to the gradient of the image to be segmented.

Watershed: an example



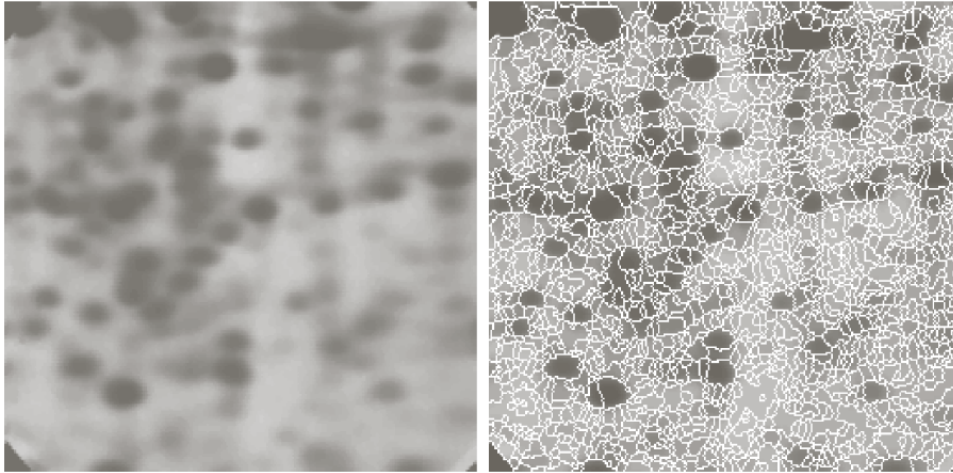
a	b
c	d

- ▶ (b) the gradient of (a)
- ▶ (c) resulting watershed segmentation
- ▶ (b) dams overimposed to (a)

Watershed with marker

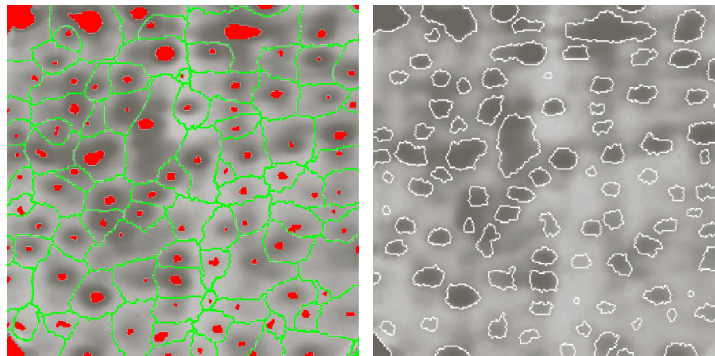
- ▶ Noise as well as irrelevant details make difficult the application of the watershed technique in real images.
 - ▶ *Oversegmentation* can be produced.
- ▶ These problems can be handled limiting the flooding through markers:
 - ▶ *internal*, associated to the object of interest;
 - ▶ *external*, associated to the background (border of the objects).
- ▶ The watershed algorithm can then be applied considering the marker as the only minimum points from which starting the procedure.
- ▶ The criteria used for defining the markers incorporate the a-priori knowledge on the problem.

Watershed with markers: an example



- Oversegmentation obtained applying watershed to the image gradient.

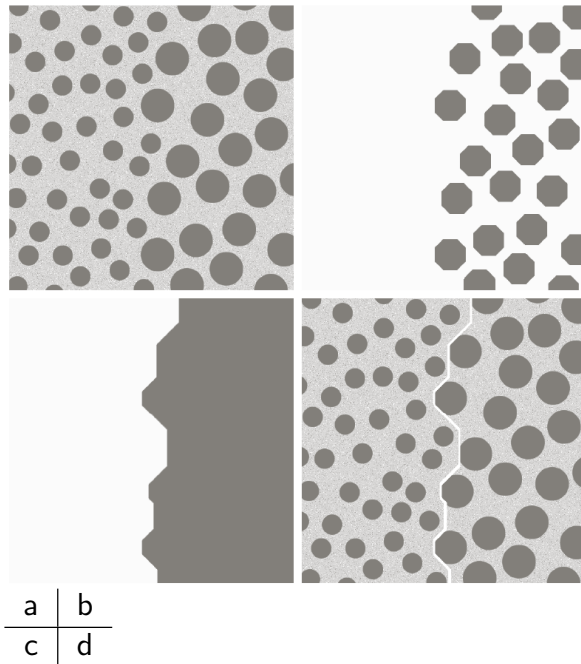
Watershed with markers: an example (2)



a | b

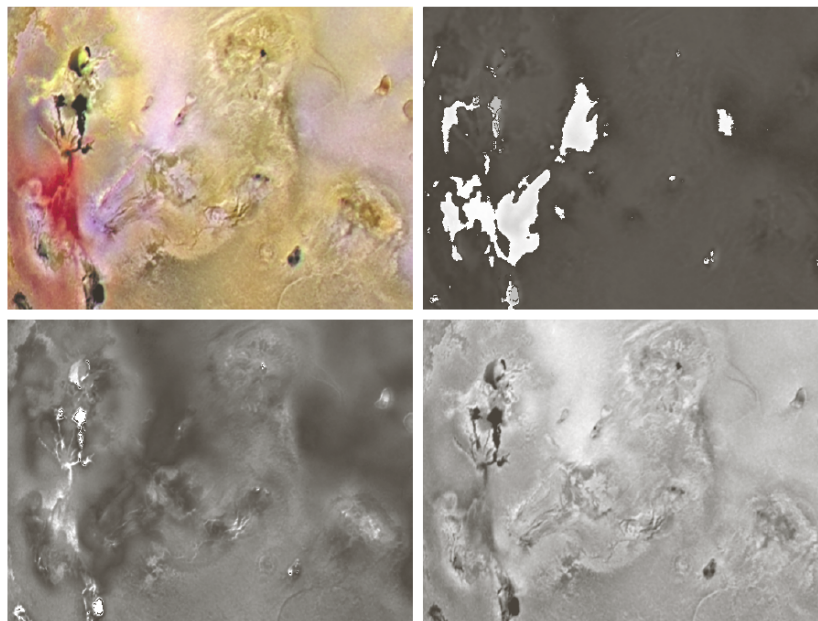
- (a) Smoothing of the original image.
 - Internal markers are defined as minimum points that forms connected components (in red).
 - The application of the watershed starting from internal markers generates the dams (in green), that can be used as external markers.
- (b) Segmentation obtained applying watershed in each region of (a).
 - Other segmentation algorithms can be applied to the single regions, as well.

Texture based segmentation



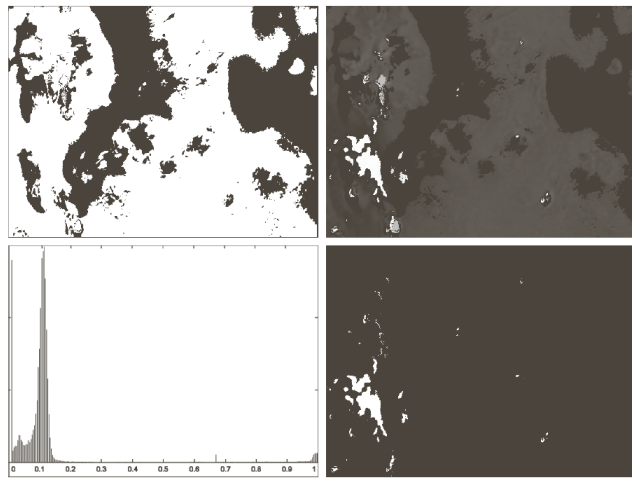
- (a) 600×600 pixels image
- (b) Closing of (a) using a disk of 30 pixels of radius.
- (c) Opening using a disk of 60 pixels of radius.
- (d) Segmentation boundary obtained as morphological gradient.

Color based segmentation



► HSI space

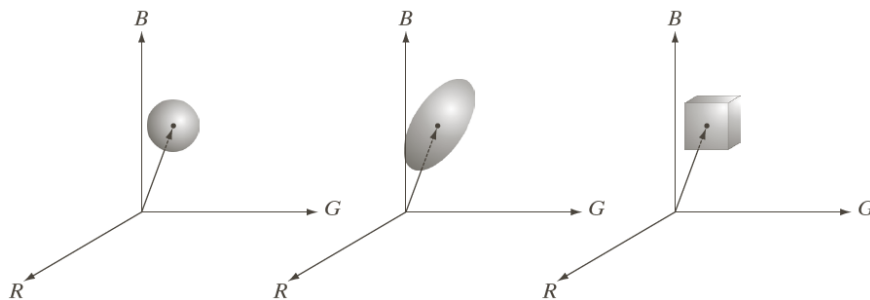
Color based segmentation (2)



a	b
c	d

- (a) Binary saturation mask (threshold at 90%);
- (b) product of the mask by the hue;
- (d) segmentation of (b) based on its histogram (c).

Color based segmentation (3)

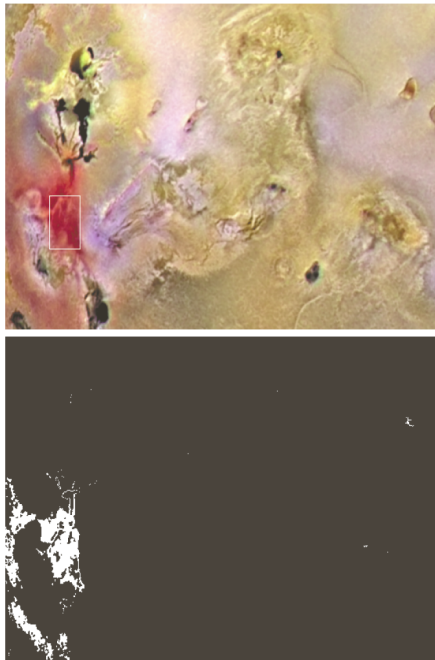


- RGB space: selection of colors similar to a
 - Criterion: $D(z, a) < D_0$

$$D(z, a) = \|z - a\| = \left((z - a)^T (z - a) \right)^{\frac{1}{2}}$$

$$D(z, a) = \left((z - a)^T C^{-1} (z - a) \right)^{\frac{1}{2}}$$

Color based segmentation (4)



$$\frac{a}{b}$$

(a) Manual selection of the color of interest.

- ▶ Average color computation,
 $a = [a_R \ a_G \ a_B]$
- ▶ Standard deviation of the color of the selected pixels computation,
 $\sigma = [\sigma_R \ \sigma_G \ \sigma_B]$.

(b) Segmentation of the pixels that have a red channel value in the interval $[a_R - \sigma_R, a_R + \sigma_R]$.

Homeworks and suggested readings



DIP, Sections 10.3–10.5

- ▶ pp. 738–778
- Sections 6.7.1–6.7.2
- ▶ pp. 443–447
- Section 9.6.3
- ▶ pp. 675–676



GIMP

- ▶ Tools
 - ▶ Selection Tools
 - ▶ Foreground Select
 - ▶ By Color Select