Topological structure of images

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Image processing

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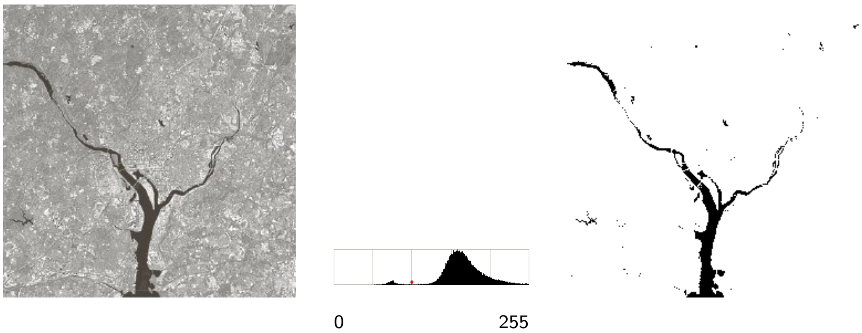
Use of simple relationships between pixels

- The structure of a digital image allows to state some basic relationships between pixels that can be useful in some practical cases.
  - The pixels are organized in a regular structure and can have a limited number of values.
- Some operations consider groups of pixels that share the same features or are related by some peculiar characteristics.

Example: estimation of the area covered by the river



- identify the pixels which have the color of the river;
  - several shades for the river;
- select among these the connected pixels;
  - other “water” regions are not of interest;
- consider the regions that are “naturally” aligned;
  - sections of the rivers parted by bridges, bottlenecks, or covered.

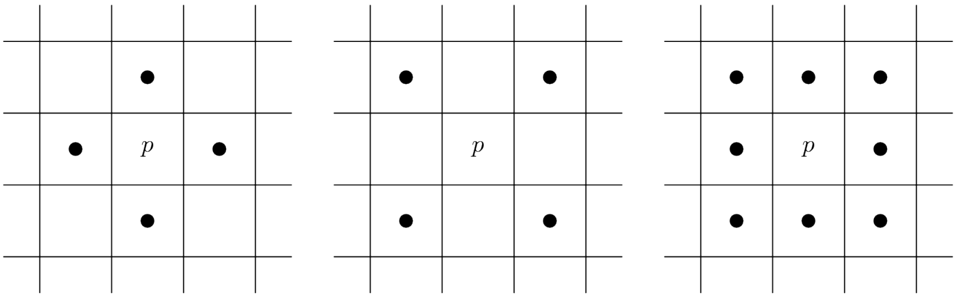
Relationships between pixels

- Some tools can be very useful:
  - a neighborhood relationship for pixels;
  - an adjacency relationship for pixels;
  - a connectivity relationship for pixels;
  - a distance measure on the image domain.
- The same concepts can be easily extended to subset of pixels.

Neighborhood

The pixel p, positioned in (x, y) has the following *neighbors*:

- N_4: $(x + 1, y)$, $(x - 1, y)$, $(x, y + 1)$, $(x, y - 1)$;
- N_D: $(x + 1, y + 1)$, $(x - 1, y + 1)$, $(x + 1, y - 1)$, $(x - 1, y - 1)$;
- N_8: $N_4 \cup N_D$.

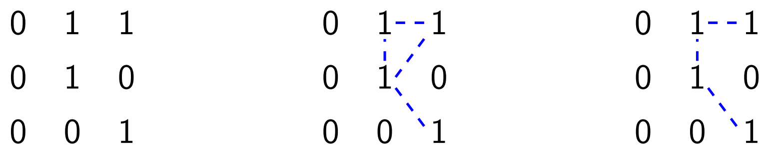


$N_4(p)$ $\quad$ $N_D(p)$ $\quad$ $N_8(p)$

Adjacency

- The *adjacency* relationship is defined considering only "similar" pixels.
- In this context, the pixel intensity is used for defining the similarity relationship: the set V is composed by the intensities that (arbitrarily) have to be considered in the definition of the adjacency.
- Three types of adjacency are used: *4-adjacency*, *8-adjacency*, and *m-adjacency* (*mixed*).
- The pixels p and q having intensity values in V are:
  - 4-adjacent if $q \in N_4(p)$;
  - 8-adjacent if $q \in N_8(p)$;
  - *m*-adjacent if:
    - $q \in N_4(p)$, or
    - $q \in N_D(p)$ and none of the pixels in $N_4(p) \cap N_4(q)$ have values in V.
- *m*-adjacency is used for avoiding the ambiguities that arise using the 8-adjacency.

Adjacency (2)



(a) (b) (c)

(a) A set of pixels. (b) 8-adjacency. (c) m-adjacency.

Path

- The *path* from pixel p to pixel q having coordinates (x_p, y_p) and (x_q, y_q) is a sequence of $n+1$ pixels having coordinates:

  $$(x_0, y_0)\,(x_1, y_1)\; \ldots, (x_n, y_n)$$

  where:
  - $(x_0, y_0) = (x_p, y_p)$ and $(x_n, y_n) = (x_q, y_q)$
  - (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent $(1 \leq i \leq n)$
- The number n is the path *length*.
- If $(x_0, y_0) = (x_n, y_n)$, the path is *closed*.
- The path definition depends from the adjacency definition:
  - 4-paths, 8-paths, or m-paths.

Connected components

- Let S a subset of pixels in an image.
- Two pixel, p and q, in S are said to be *connected in S* if there is a *path* p–q composed of pixels in S.
- For any pixel in S, the set of the pixels of S connected to that pixel constitutes a *connected component* of S.
- If there is only one connected component in S, then S is called *connected set*.
- In an image, any subset of pixel, R, that is a connected set is called a *region* of the considered image.

0	1	1	0	0	0	0	0	3	3	3	0	0	0	0	0
0	0	1	1	0	0	2	0	0	3	3	3	3	3	3	3
1	1	1	0	0	0	2	0	0	0	3	3	3	3	0	0
0	0	1	1	1	0	0	2	2	2	0	0	0	3	3	3
0	0	1	1	1	1	1	2	2	2	2	0	3	3	3	0
0	0	0	1	1	0	0	2	2	2	0	0	3	3	3	0

using 4-adjacency

note: two regions for 3

Relationship between regions

- The *adjacency* relationship can be extended to regions.
- Two regions R_i and R_j are *adjacent* if $R_i \cup R_j$ is a connected set.
- Regions that are not adjacent are called *disjoint*.
- The adjacency of regions depends on the neighborhood relationship adopted.
- The union of all the regions of an image is called *foreground*.
  - The terms *subject(s)* or *object(s)* of the image are also used.
- The complement of the foreground (i.e., the set of the pixels that do not belong to any regions) is called *background*.
- Since it is practical that the foreground objects are surrounded by the background, if required a frame of background pixels can be added to the image (*padding*).
  - Hence, one row at the top and one at the bottom and one line at the right and one at the left are added to the original image.

Boundary of a region

- The *boundary* (or *border* or *contour*) of a region R is the set of pixels of R adjacent to the complement of R.
  - In other words, the border of a region is composed of pixels that have at least one of their neighbors in the background.
- The boundary definition depends on the adjacency.

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	(1)	1	0
0	1	1	1	0
0	0	0	0	0

The circled pixel belongs to the boundary only if the 4-adjacency is used.

Boundary of a region (2)

- The boundary of a region is also called *inner border*, to be distinguished by the *outer border*, which is composed of background pixels that are adjacent to the inner border.
- The outer border is always a closed path.
  - The region can be an open path, and so its inner border.
  - The inner border can be identified as those foreground pixels that are adjacent to the outer border.

0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0

- The *edge* is an important concept that is similar to the border. It will be introduced later in the course.
  - The edge is often a subsequence of the border, but there are more complex cases.

Distance

- The *distance function* or *metric* is a real function defined on pairs of pixels such that:
  1. $D(p, q) \geq 0, \qquad (D(p, q) = 0 \text{ iff } p = q)$
  2. $D(p, q) = D(q, p)$
  3. $D(p, q) \leq D(p, z) + D(z, q)$
- *Euclidean distance*, D_e, (L_2 norm):

$$D_e(p, q) = \left[(x_p - x_q)^2 + (y_p - y_q)^2\right]^{\frac{1}{2}}$$

- D_4 *distance*, (L_1 norm):

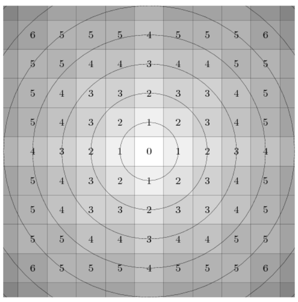
$$D_4(p, q) = \|x_p - x_q\| + \|y_p - y_q\|$$

  - aka *Manhattan*, or *taxi-cab* or *city-block* distance.

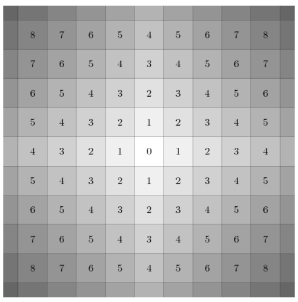
Distance (2)

- D_8 *distance*, or *chessboard distance* (L_∞ norm):

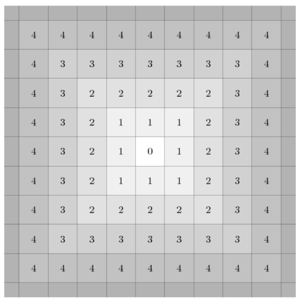
$$D_8(p, q) = \max(\|x_p - x_q\|, \|y_p - y_q\|)$$



D_e



D_4



D_8

- The pixels, q, are labeled with the integer radius, $r \in \mathbb{N}$, of the disk they belong to: $r - 1 < D(p, q) \leq r$, where p is the central pixel.

Distance (3)

- The distance can be evaluated also using the *m*-adjacency.
- The *m*-distance, D_m, differs from D_e, D_4, and D_8 since it depends on the image content.

(a)

0	1	1	1
1	1	0	0
1	0	0	0
0	1	0	0
0	0	1	1

(b)

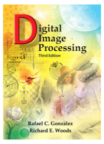
-	2	2	3
1	1	-	-
0	-	-	-
-	1	-	-
-	-	2	3

(c)

-	3	4	5
1	2	-	-
0	-	-	-
-	1	-	-
-	-	2	3

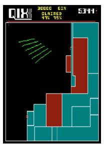
(a) Pixel arrangement. (b) D_8 distance (only for 1-valued pixels). (c) The corresponding D_m distance.

Homeworks and suggested readings



DIP, Section 2.5

- pp. 68–71



Flood fill, seed fill

- `http://en.wikipedia.org/wiki/Flood_fill`

GIMP

- Tools
  - Paint Tools
    - Bucket Fill