.

# Edge detection

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**Elaborazione delle immagini**
**(Image processing I)**

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## Image segmentation

- Several image processing based applications require the detection and the recognition of the objects on the scene.
- Segmentation is a critical task for such applications.
- It consists in assigning each pixel to an object.
- Two main criteria are used for guiding the partitioning:
  - discontinuity;
  - similarity.

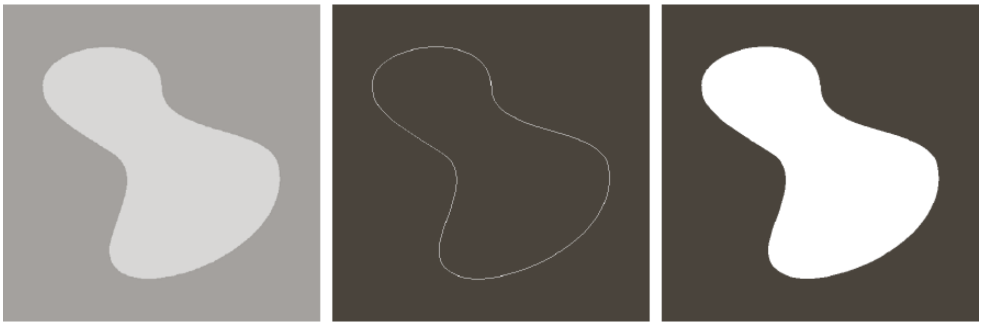
## Formal description of the segmentation

The *segmentation* of an image, $R$, can be defined as the partitioning of $R$ in $n$ subregions, $R_1, \ldots, R_n$, such that:

- $\bigcup_{i=1}^{n} R_i = R$
- $R_i$ is a connected set, $i = 1, \ldots, n$
- $R_i \cap R_j =, \forall i, j, i \neq j$
- $Q(R_i) = \text{TRUE}, i = 1, \ldots, n$
- $Q(R_i \cup R_j) = \text{FALSE}$, for each pair of adjacent regions $R_i$ e $R_j$

where $Q$ is a logical predicate defined on the points of the considered region.
$Q$ is used for characterizing the objects of the image.

## Edge based segmentation



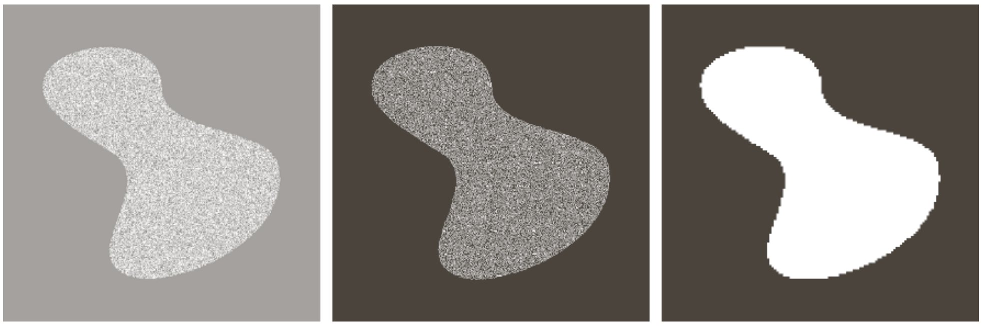
a | b | c

(a) An image is composed of object and background both of constant intensity.

(b) Discontinuities are easily detected and identify the boundary.

(c) $Q(R)$ can be defined as TRUE iff all the pixels of $R$ are on the same side of the boundary.

## Region based segmentation



a | b | c

(a) An image is composed of a textured object and a constant background.

(b) Discontinuities form an intricate pattern of small edges.

(c) The image in (a) can be subdivided in 4×4 regions and each region is marked as white if the standard deviation of the intensity of its pixel is positive (black, otherwise).

## Discontinuity types

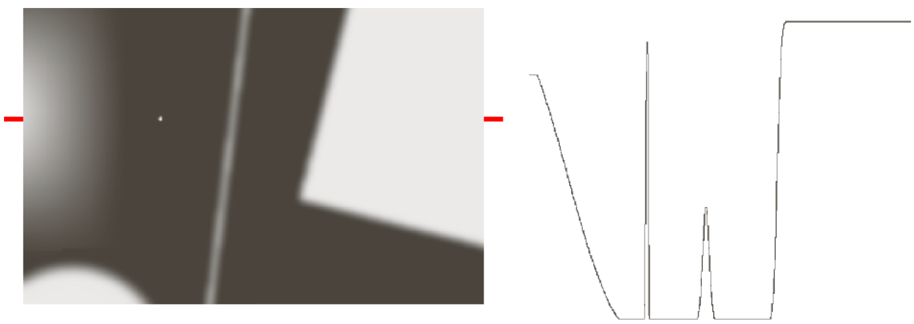
There are three main types of discontinuity that usually carry the information:

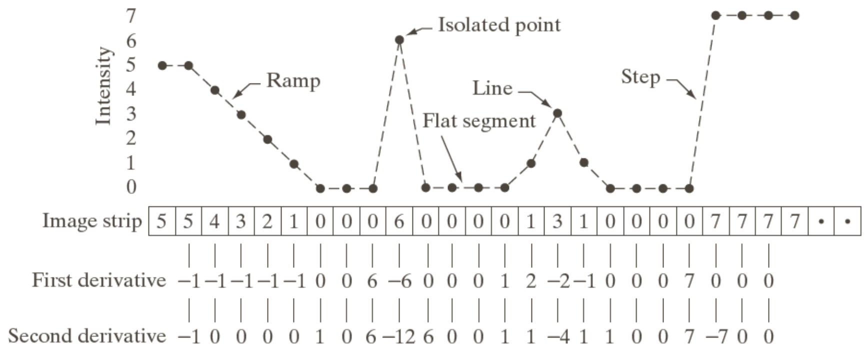
- isolated (edge) point
  - abrupt local change of intensity
- edge segment
  - connected set of discontinuity points
- line
  - thin region where the intensity changes on both the sides

## Discontinuity detection

- Derivative are the most suited tools for detecting changes
  - but they are defined on continuous domains.
- Since the digital images are discrete, approximations have to be used (using Taylor series):
  - first derivative:
  $$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$
  - second derivative:
  $$\frac{\partial^2 f}{\partial x^2} = f(x+1) - 2f(x) + f(x-1)$$
- Hence, the value of the derivative estimated in a pixel will be considered a measure of discontinuity of the intensity in that point.

## Discontinuity detection (2)





## Isolated points detection

- The *Laplacian* is the simplest derivative operator that is isotropic with respect to the four principal directions:
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$
- Partial derivatives:
$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= f(x+1, y) - 2f(x, y) + f(x-1, y)\\ \frac{\partial^2 f}{\partial y^2} &= f(x, y+1) - 2f(x, y) + f(x, y-1)\end{aligned}$$
- Hence, the Laplacian can be computed as:
$$\begin{aligned}\nabla^2 f(x, y) &= f(x+1, y) + f(x-1, y) + f(x, y+1)\\ &\quad +f(x, y-1) - 4f(x, y)\end{aligned}$$
- Optionally, the diagonals contribution can be added:
$$\begin{aligned}\nabla^2 f(x, y) &+ f(x-1, y-1) + f(x+1, y+1) + f(x-1, y+1)\\ &+ f(x+1, y-1) - 4f(x, y)\end{aligned}$$

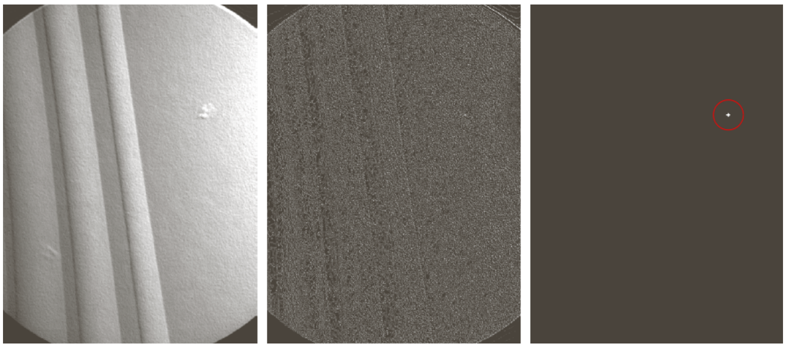
## Thresholding

- Thresholding is the simplest technique for obtaining the pixels where a substantial discontinuity takes place.
- Hence, the image $g$ is considered:
$$g(x, y) = \begin{cases} 1, & \text{if } |\nabla^2 f(x, y)| \geq T \\ 0, & \text{otherwise} \end{cases}$$

for a suitable threshold $T$.

## Thresholding (2)

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | −8 | 1 |
| 1 | 1 | 1 |



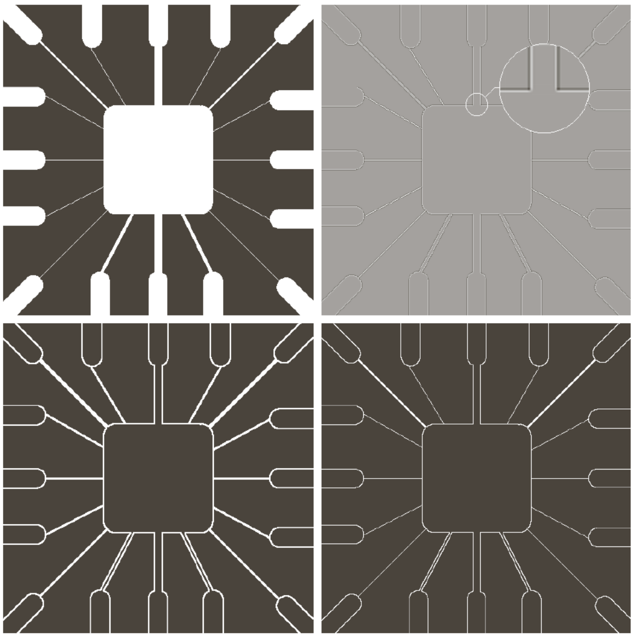
$f$ $\nabla^2 f$ $g$

- The X-ray image, $f$, of a turbine blade shows some porosity.
- The Laplacian of $f$ is computed for detecting isolated points.
- The porosity is detected in the image $g$, obtained thresholding $\nabla^2 f$ with a threshold equal to the 90% of the highest value of the Laplacian.

## Lines detection

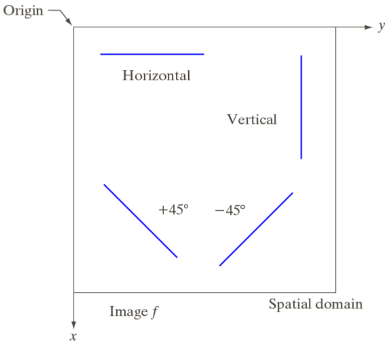
- A line is a group of discontinuity pixels structured in linear arrangement.
- Hence, the detection of a line can be realized using isolated point detection techniques, such as the Laplacian.
- However, problems such as the double response of the Laplacian have to be handled.
- Besides, the thickness of the line have to be defined.
  - Thick lines can be better detected by edge detectors.

## Lines detection (2)



- The Laplacian have a double response to the edges.
- Considering the absolute value, the thickness of the detected lines results doubled.
- Lines can be handled considering only the strictly positive values.

## Line detection masks




- Often, the direction of the lines is an important feature.
- The four masks act in the four specified directions.
- Each pixel can be associated to the direction for which the response is the highest.

Horizontal:

| | | |
|---|---|---|
| −1 | −1 | −1 |
| 2 | 2 | 2 |
| −1 | −1 | −1 |

+45°:

| | | |
|---|---|---|
| 2 | −1 | −1 |
| −1 | 2 | −1 |
| −1 | −1 | 2 |

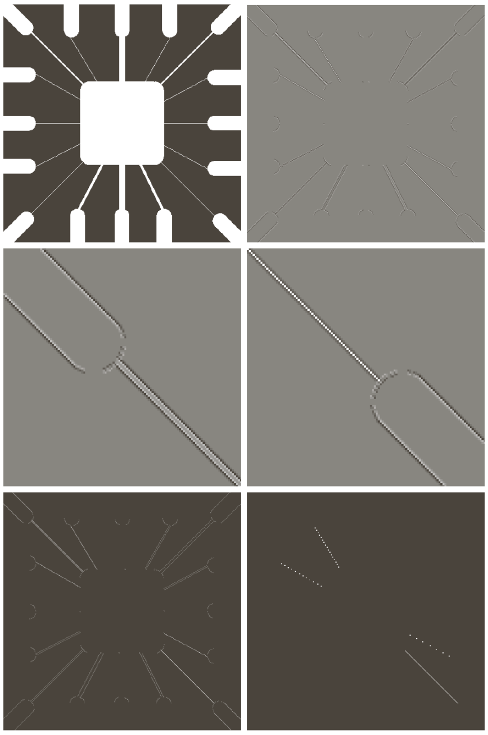
Vertical:

| | | |
|---|---|---|
| −1 | 2 | −1 |
| −1 | 2 | −1 |
| −1 | 2 | −1 |

−45°:

| | | |
|---|---|---|
| −1 | −1 | 2 |
| −1 | 2 | −1 |
| 2 | −1 | −1 |

## Specific directions detection

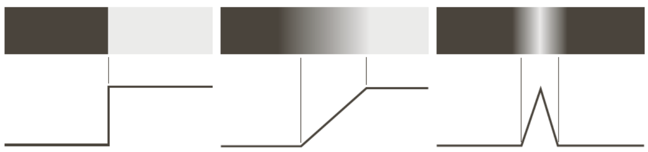


| a | b |
|---|---|
| c | d |
| e | f |

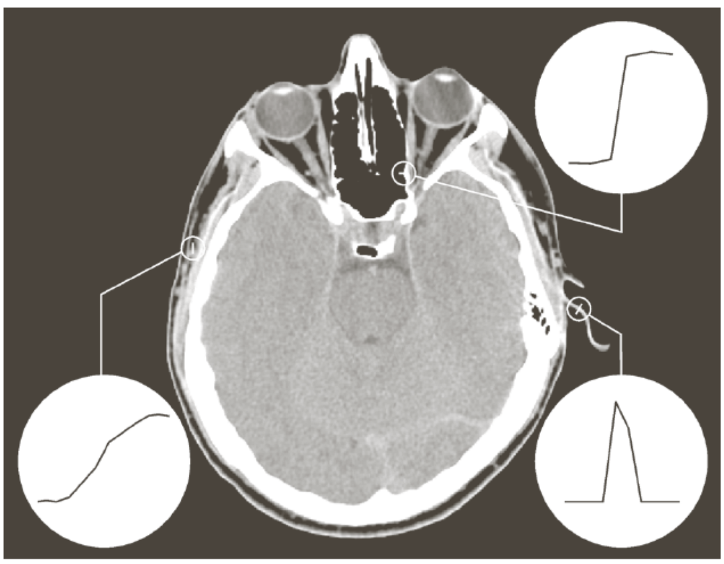
- The image (a) is processed with the 45° mask producing the image (b).
- (c) and (d) shows the details in the upper-left and bottom-right corners of (b) respectively. The response in (d) is stronger then in (c), due to the different lines thickness.
- In (e), only positive values are shown.
- In (f), (e) is thresholded using the its maximum value as threshold.

## Edge models

Ideal edges



Real edges



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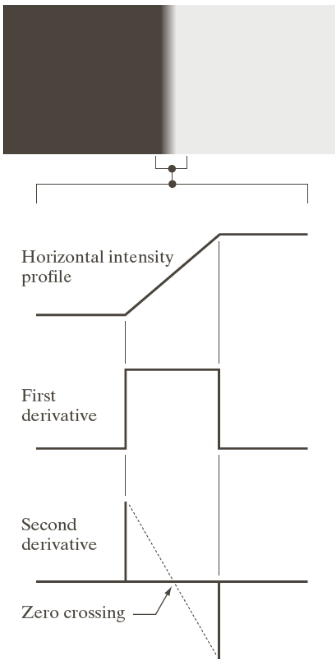
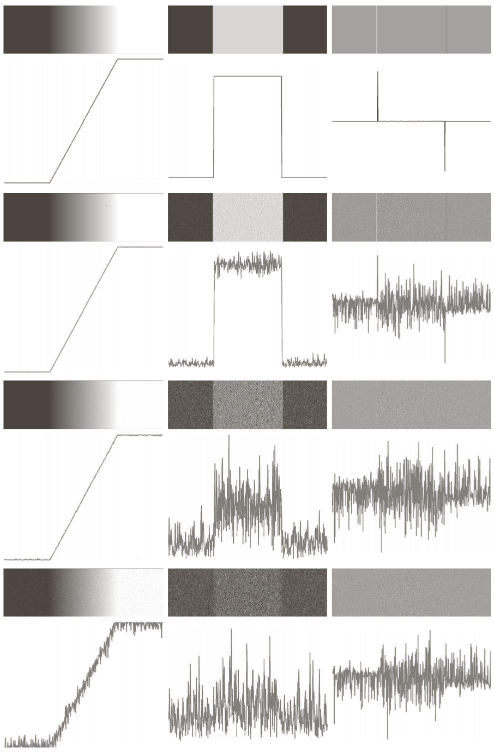
## Ramp edges




- In real images, ramp edges are the most common (blur).
- The first derivative magnitude can be used to detect an edge.
- The sign of the second derivative tells if the pixel is on the dark or light side of the edge.
- The second derivative:
  - produces a double response (undesirable);
  - allows the localization through zero crossing.
- These consideration hold for edges of every orientation, considering the section of the image operated perpendicularly to the edge.

## Edge and noise



Noise free ramp.
The gray levels range in [0, 255];

Gaussian noise, zero mean, standard deviation of 0.1 (0.0392%).

Gaussian noise, zero mean, standard deviation of 1.0 (0.392%).

Gaussian noise, zero mean, standard deviation of 10.0 (3.92%).

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## Edge and noise (2)

- Although noise is almost invisible in the images, even a tiny amount of noise strongly affects the second derivative and a modest amount of noise can ruin the first derivative.
- Since derivative operators are the principal tool for edge detection, their effectiveness in real image can be questioned.
- Three fundamental steps have to be performed in edge detection:
  - Smoothing for noise reduction
  - Detection of edge points
  - Edge localization

## Gradient based edge detection

- The *gradient* allows to find edge strength and direction.
- The gradient $\nabla f$ of a bidimensional function, $f(x, y)$ is:

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

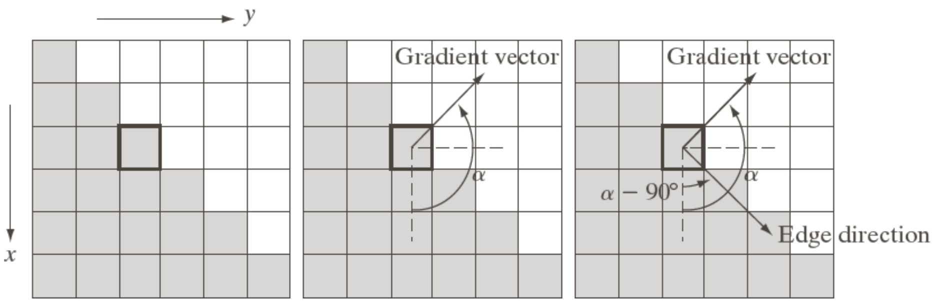
- The *magnitude* of the gradient, $M(x, y)$ is:

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

  - The approximation $M(x, y) \approx |g_x| + |g_y|$ is often used.
- The gradient vector points in the directions of the the greatest rate of change at the considered location:

$$\alpha(x, y) = \tan^{-1} \left[ \frac{g_y}{g_x} \right]$$

## Edge and gradient




- The direction of the edge is orthogonal to the direction of the gradient.
  - Gradient is also called *edge normal*.

## Derivative operators

Partial derivatives

| |
|---|
| −1 |
| 1 |

| | |
|---|---|
| −1 | 1 |

Derivative operators

| | |
|---|---|
| −1 | 0 |
| 0 | 1 |

| | |
|---|---|
| 0 | −1 |
| 1 | 0 |

Roberts

| | | |
|---|---|---|
| −1 | −1 | −1 |
| 0 | 0 | 0 |
| 1 | 1 | 1 |

| | | |
|---|---|---|
| −1 | 0 | 1 |
| −1 | 0 | 1 |
| −1 | 0 | 1 |

Prewitt

| | | |
|---|---|---|
| −1 | −2 | −1 |
| 0 | 0 | 0 |
| 1 | 2 | 1 |

| | | |
|---|---|---|
| −1 | 0 | 1 |
| −2 | 0 | 2 |
| −1 | 0 | 1 |

Sobel

Diagonal edge masks

| | | |
|---|---|---|
| 0 | 1 | 1 |
| −1 | 0 | 1 |
| −1 | −1 | 0 |

| | | |
|---|---|---|
| −1 | −1 | 0 |
| −1 | 0 | 1 |
| 0 | 1 | 1 |

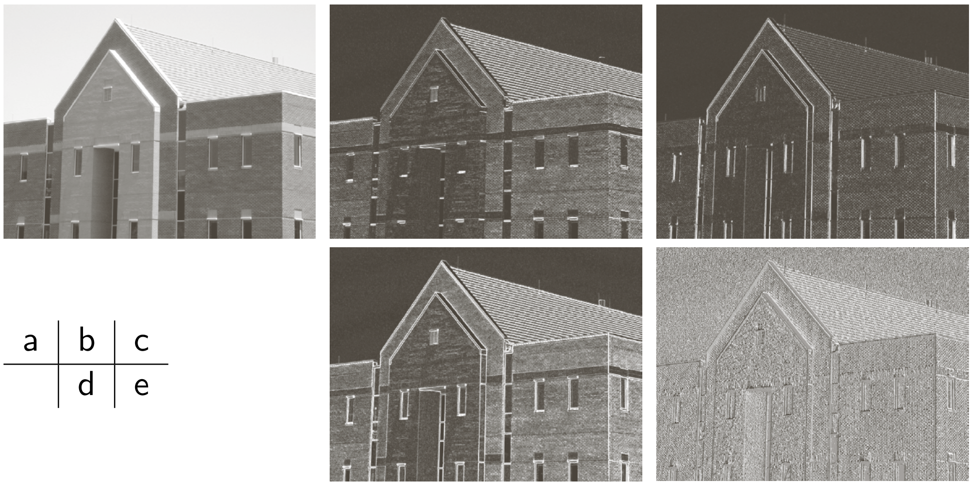
Prewitt

| | | |
|---|---|---|
| 0 | 1 | 2 |
| −1 | 0 | 1 |
| −2 | −1 | 0 |

| | | |
|---|---|---|
| −2 | −1 | 0 |
| −1 | 0 | 1 |
| 0 | 1 | 2 |

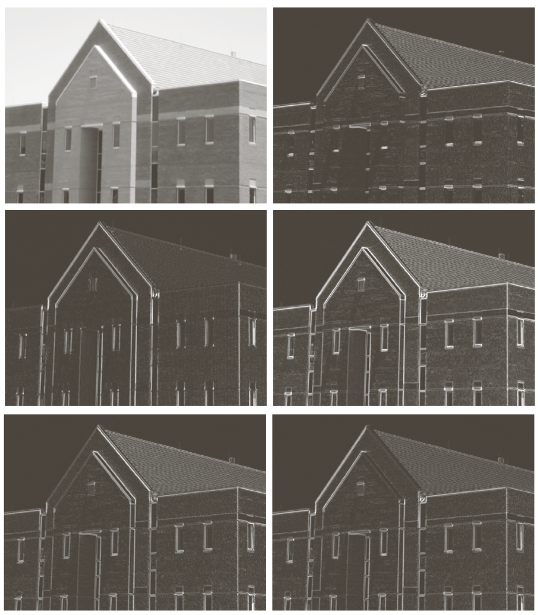
Sobel

## Example of use of the gradient




- The images (b) and (c) are respectively the magnitude of the (Sobel) horizontal and vertical gradients of (a), while (d) and (e) are the magnitude and the angle of the gradient.

## Example of use of the gradient (2)



- As well as noise, too small details can crowd the gradient image and make difficult the edge detection.
- Smoothing can handle this problem.

(a) Original image (834 × 1114) are smoothed with a 5 × 5 averaging filter.

(b-c) Horizontal and vertical gradients (Sobel) of (a).

(d) Gradient magnitude.

(e-f) Diagonal Sobel gradients.

## Example of use of the gradient (3)

a | b

- Thresholding can help in simplifying the edge detection.

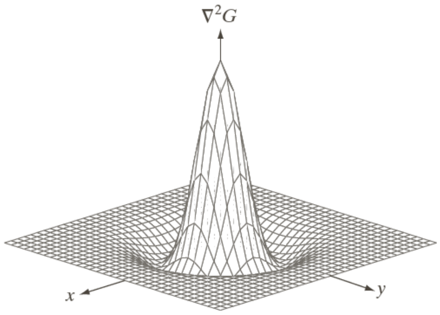
(a) Thresholding at the 33% of the maximum value of the gradient.

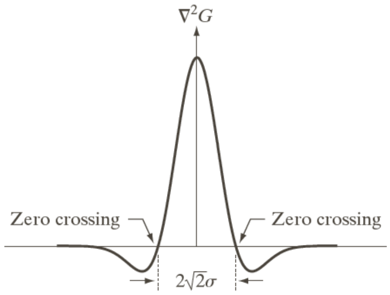
(b) Thresholding the gradient of the smoothed image.

## Marr-Hildreth edge detector

- The Marr-Hildreth edge detector is based on two observations:
  - intensity changes are not independent of image scale;
  - the derivatives of the function carry useful information on changes of intensity.
- Hence, operators that act at different scales can be used.
- The Laplacian of the Gaussian satisfies both the requirements.
- The Gaussian is smooth both in the spatial and in the frequency domains:
  - result is less affected by artifacts.
- The Laplacian is isotropic:
  - no need of multiple masks for covering different directions.

## Marr-Hildreth edge detector (2)




| | | | | |
|---|---|---|---|---|
| 0 | 0 | −1 | 0 | 0 |
| 0 | −1 | −2 | −1 | 0 |
| −1 | −2 | 16 | −2 | −1 |
| 0 | −1 | −2 | −1 | 0 |
| 0 | 0 | −1 | 0 | 0 |

## Marr-Hildreth edge detector (3)

- In order to achieve a good approximation of the Gaussian, $n$ should be chosen at least as the smallest odd integer greater or equal to $6\sigma$.
- $\nabla^2 G$ can be obtained sampling the Gaussian function and then computing its Laplacian through the above illustrated operators.
- Due to the linearity of the operations:

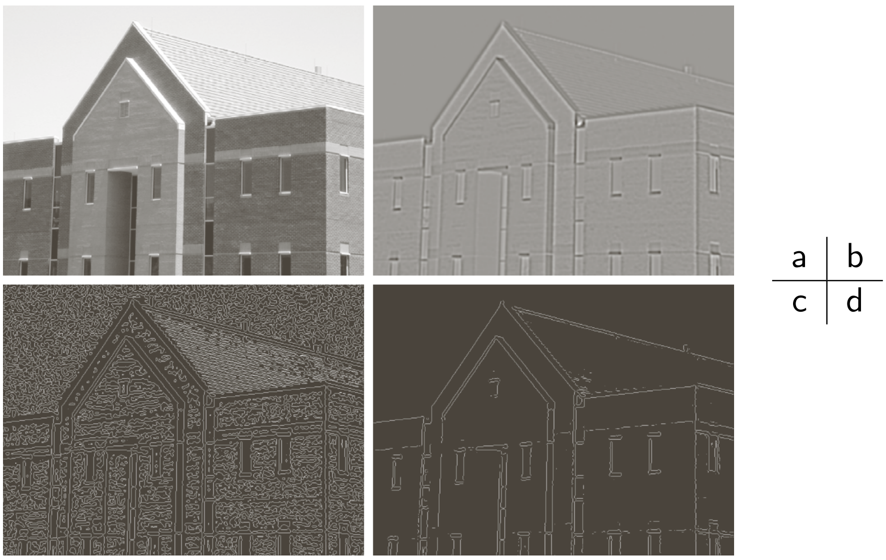
$$[\nabla^2 G(x, y)] \star f(x, y) = \nabla^2 [G(x, y) \star f(x, y)]$$

- The Marr-Hildreth algorithm is composed of three steps:
  - Filter the image with a $n \times n$ Gaussian filter with an appropriate scale, $\sigma$.
  - Compute the Laplacian (e.g., with a $3 \times 3$ mask).
  - Find the zero-crossings.

## Marr-Hildreth edge detector (4)

- A simple algorithm for finding zero crossing can be used:
  - if at least two opposite pixels have different sign, a zero crossing happens;
  - a more robust variant can be obtained adding a condition on the absolute value of the difference of the opposite pixels.

## Marr-Hildreth edge detector (5)



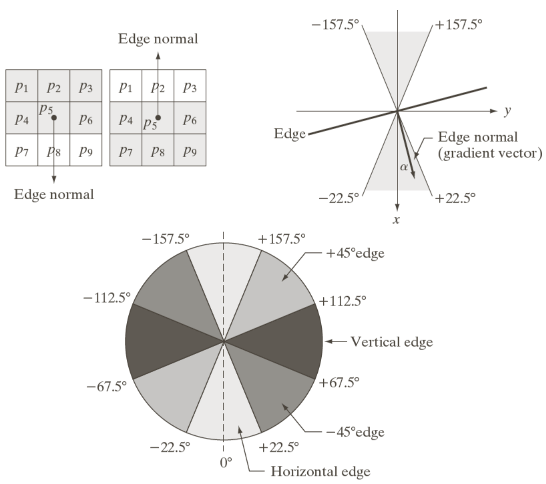

- A 834 $\times$ 1114 image (2) is processed with a Laplacian of Gaussian filter ($\sigma = 4$, $n = 25$) (b).
- Zero crossing with threshold 0 (c) and 4% of the maximum value in (b).

## Canny edge detector

- The Canny algorithm has three goals:
  - Low error rate
    - no missing edge, no false edge
  - Well localized edge points
  - Single edge point response
- Canny proved that a good approximation to the optimal step detector in 1-D is the first derivative of the Gaussian.
- Generalizing this result to 2-D would require to process the image for all the directions.
  - Convolving the image with a Gaussian and then computing the gradient provides a good approximation.
- The gradient is used for estimating strength and direction of the edge in every point.

## Canny edge detector (2)

- Robust estimation of the direction using (*non-maxima suppression*) criterion:




- approximation of the direction to one of the four principal directions (horizontal, vertical and diagonal)
- if at least one of the neighboring along the edge direction has a gradient magnitude larger, the corresponding pixel in the edge image, $g_N$, is set to 0, otherwise it is set to the gradient value.

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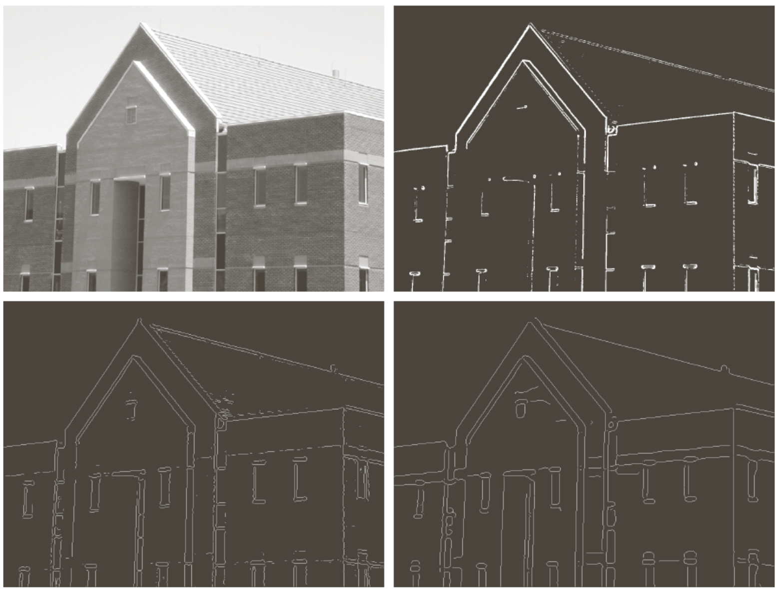
## Canny edge detector (3)

- Robust detection and linking of the edge points:
  - Hysteresis thresholding:
    - two thresholds, $T_H$ and $T_L$, such that: $T_L < T_H$
    - $g_{NL}(x, y) = T_L \leq g_N(x, y) \leq T_H$
    - $g_{NH}(x, y) = g_N(x, y) \geq T_H$
    - $g_{NH}$ and $g_{NL}$ contain respectively the strong and the weak edge points
  - After thresholding, the edges in $g_{NH}$ are usually broken, and in $g_{NL}$ there are some valid points.
  - The valid points in $g_{NL}$ can be selected:
    - for every point in $g_{NH}$, mark as valid all the points in $g_{NL}$ that are 8-connected to the considered points;
    - at the end, all the valid points in $g_{NL}$ are added to $g_{NH}$.

## Marr vs. Canny comparison



Marr Canny

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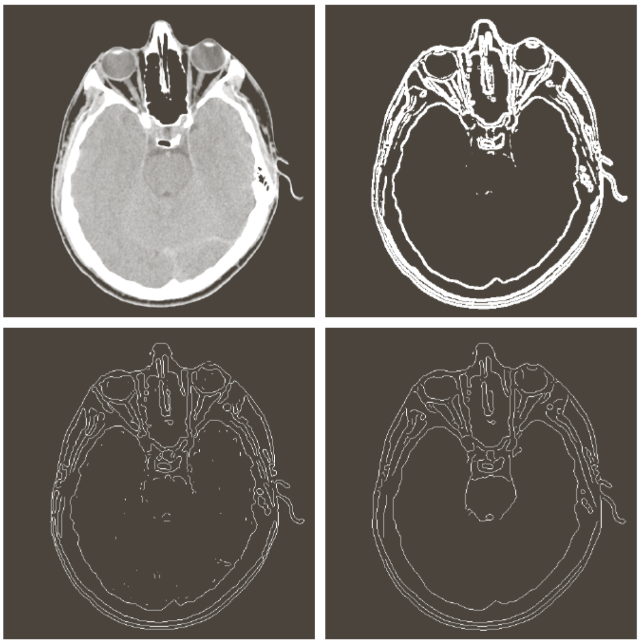
## Marr vs. Canny comparison (2)



Marr Canny

## Edge linking

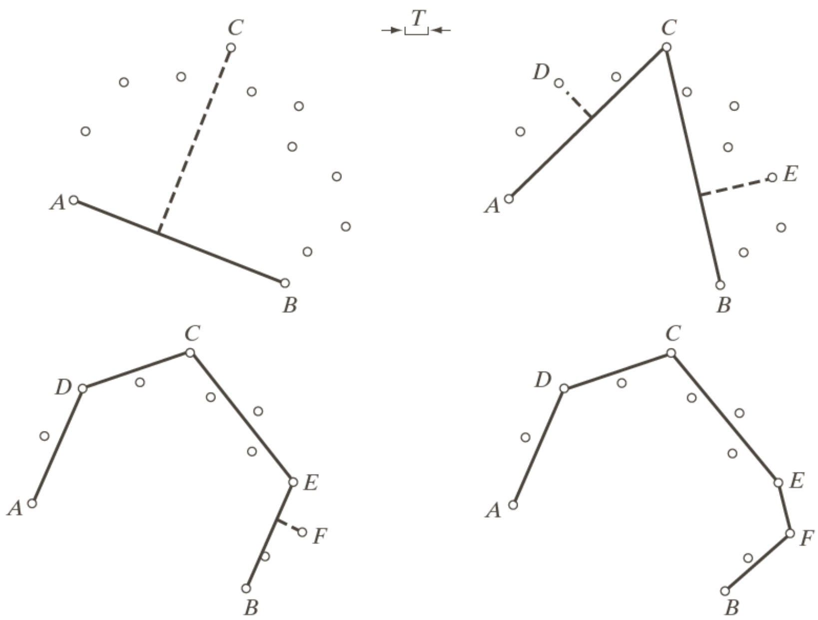
Since in real images the edge detection does not produce a clean set of edge points, some processing for obtaining the structure of the edges is required.

- Local processing:
  - each point is linked to the adjacent if magnitude and direction of the gradient are similar.
- Polygonal approximation:
  - starting from a set of edge points, segments of a polygonal line are iteratively added until all the points are close enough to a segment.
- Hough transform:
  - finding of the most probable lines.

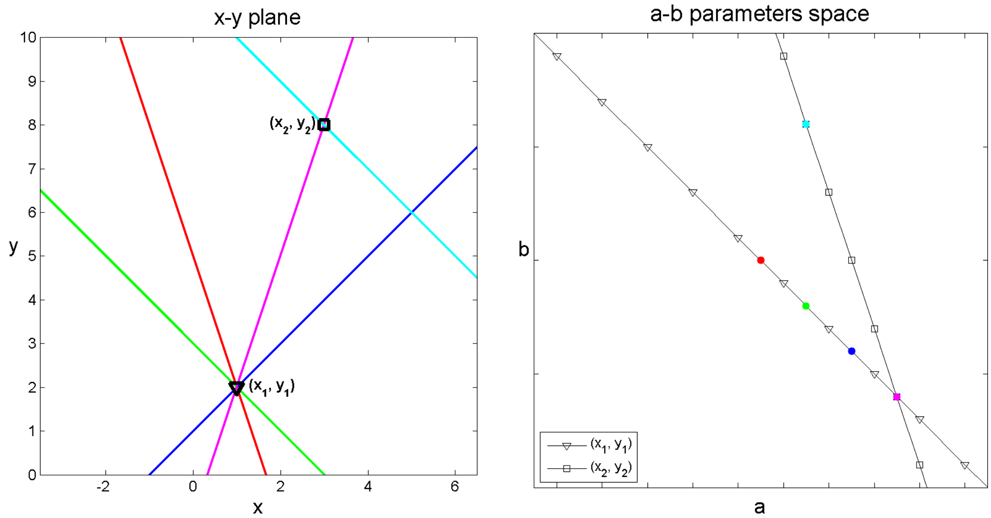
## Local edge linking

Gradient, horizontal edges, vertical edges, their union, final result after morphological thinning.

## Polygonal edge linking



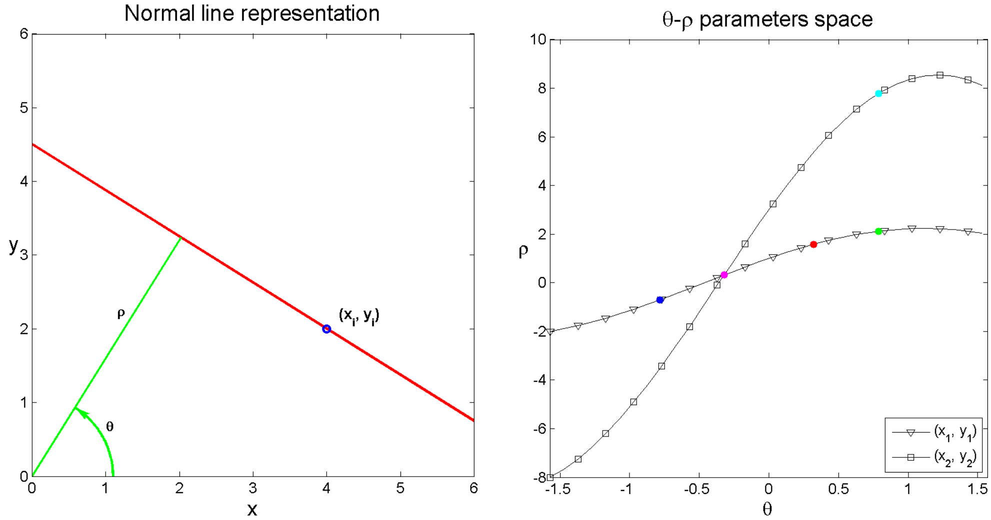
## Hough transform




$$y = a\,x + b$$

- Vertical lines cannot be represented.

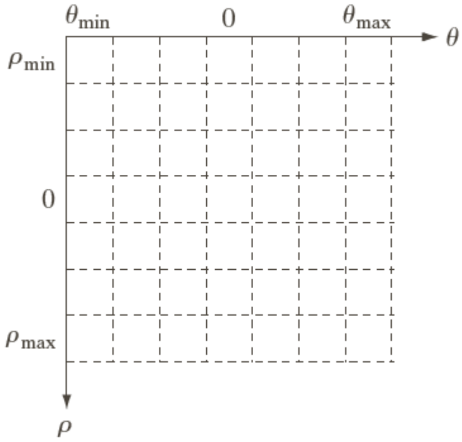
## Hough transform (2)




$$x\,\cos\theta + y\,\sin\theta = \rho$$

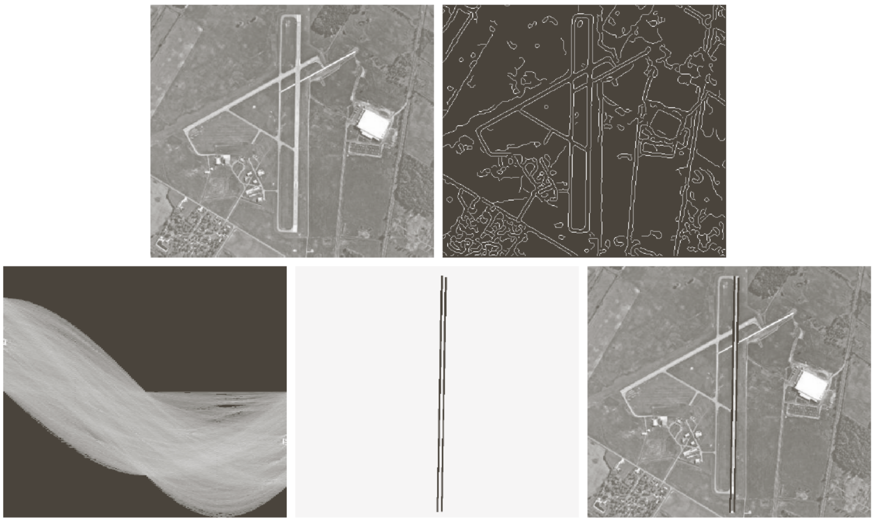
- Normal representation

## Hough transform (3)




- Quantization of the parameter space $\theta$-$\rho$.
- Count of the possible lines.
- Use of the most probable ones.
  - Optionally, some constraints can be added (e.g., direction).

## Hough transform: example



- Selection of the edge points through the Canny algorithm.
- Hough transform.
- Selection of the (nearly) vertical lines.