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Sharpening through spatial filtering

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Elaborazione delle immagini
(Image processing I)

academic year 2011–2012

Sharpening

- The term *sharpening* is referred to the techniques suited for enhancing the intensity transitions.
- In images, the borders between objects are perceived because of the intensity change: more crisp the intensity transitions, more sharp the image.
- The intensity transitions between adjacent pixels are related to the derivatives of the image.
- Hence, operators (possibly expressed as linear filters) able to compute the derivatives of a digital image are very interesting.

First derivative of an image

- Since the image is a discrete function, the traditional definition of derivative cannot be applied.
- Hence, a suitable operator have to be defined such that it satisfies the main properties of the first derivative:
  1. it is equal to zero in the regions where the intensity is constant;
  2. it is different from zero for an intensity transition;
  3. it is constant on ramps where the intensity transition is constant.
- The natural derivative operator is the difference between the intensity of neighboring pixels (spatial differentiation).
- For simplicity, the monodimensional case can be considered:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- Since $\frac{\partial f}{\partial x}$ is defined using the next pixel:
  - it cannot be computed for the last pixel of each row (and column);
  - it is different from zero in the pixel before a step.

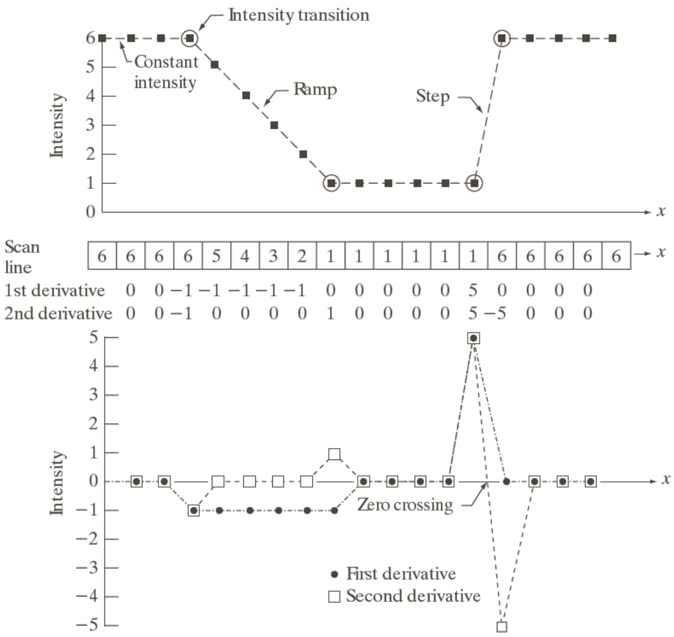
Second derivative of an image

- Similarly, the second derivative operator can be defined as:

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= f(x+1) - f(x) - (f(x) - f(x-1)) \\ &= f(x+1) - 2f(x) + f(x-1)\end{aligned}$$

- This operator satisfies the following properties:
  1. it is equal to zero where the intensity is constant;
  2. it is different from zero at the begin of a step (or a ramp) of the intensity;
  3. it is equal to zero on the constant slope ramps.
- Since $\frac{\partial^2 f}{\partial x^2}$ is defined using the previous and the next pixels:
  - it cannot be computed with respect to the first and the last pixels of each row (and column);
  - it is different from zero in the pixel that precedes and in the one that follows a step.

Derivatives of an image: an example



Note: at the step, the second derivative switch the sign (*zero crossing*).

Laplacian

- Usually the sharpening filters make use of the second order operators.
  - A second order operator is more sensitive to intensity variations than a first order operator.
- Besides, partial derivatives has to be considered for images.
  - The derivative in a point depends on the direction along which it is computed.
- Operators that are invariant to rotation are called *isotropic*.
  - Rotate and differentiate (or filtering) has the same effects of differentiate and rotate.
- The *Laplacian* is the simpler isotropic derivative operator (wrt. the principal directions):

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Laplacian filter

- In a digital image, the second derivatives wrt. x and y are computed as:

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= f(x+1, y) - 2f(x, y) + f(x-1, y)\\ \frac{\partial^2 f}{\partial y^2} &= f(x, y+1) - 2f(x, y) + f(x, y-1)\end{aligned}$$

- Hence, the Laplacian results:

$$\begin{aligned}\nabla^2 f(x, y) &= f(x+1, y) + f(x-1, y) + f(x, y+1)\\ &\quad +f(x, y-1) - 4f(x, y)\end{aligned}$$

- Also the derivatives along to the diagonals can be considered:

$$\begin{aligned}\nabla^2 f(x, y) &+ f(x-1, y-1) + f(x+1, y+1)\\ &+ f(x-1, y+1) + f(x+1, y-1) - 4f(x, y)\end{aligned}$$

Laplacian filter (2)

0	1	0
1	−4	1
0	1	0

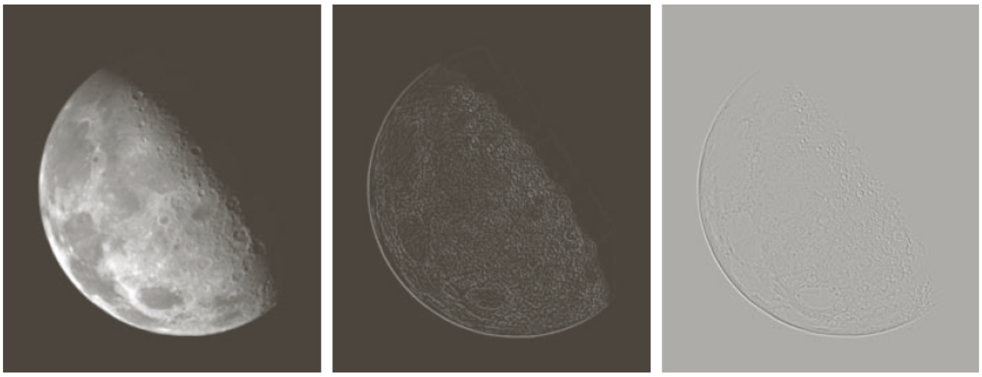
Laplacian filter invariant to 90° rotations

1	1	1
1	−8	1
1	1	1

Laplacian filter invariant to 45° rotations

Laplacian filter: example

- The Laplacian has often negative values.
- In order to be visualized, it must be properly scaled to the representation interval $[0, \ldots, L-1]$.



(a) (b) (c)

(a) Original image, (b) its Laplacian, (c) its Laplacian scaled such that zero is displayed as the intermediate gray level.

Laplacian filter: example (2)

- The Laplacian is positive at the onset of a step and negative at the end.
- Subtracting the Laplacian (or a fraction of it) from the image, the height of the step is increased.

(a) (b) (c)

(a) Original image, (b) Laplacian filtered, (c) Laplacian with diagonals filtered.

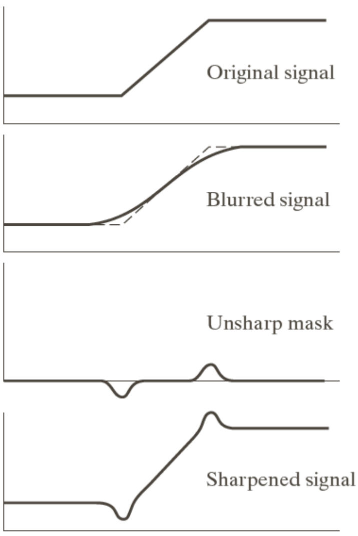
Unsharp masking

- The technique known as *unsharp masking* is a method of common use in graphics for making the images sharper.
- It consists of:
  1. defocusing the original image;
  2. obtaining the mask as the difference between the original image and its defocused copy;
  3. adding the mask to the original image.
- The process can be formalized as:

$$g = f + k \cdot (f - f * h)$$

  where f is the original image, h is the smoothing filter and k is a constant for tuning the mask contribution.
  - If $k > 1$, the process is called *highboost* filtering.

Unsharp masking (2)



original image

defocused image

unsharping mask

unsharped image

highboosted image

Gradient

- The *gradient* of a function is the vector formed by its partial derivatives.
- For a bidimensional function, $f(x, y)$:

$$\nabla f \equiv \mathrm{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

- The gradient vector points toward the direction of maximum variation.
- The gradient *magnitude*, $M(x, y)$ is:

$$M(x, y) = \mathrm{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

  - It is also called *gradient image*.
  - Often approximated as $M(x, y) \approx |g_x| + |g_y|$.

Derivative operators

- Basic definitions:

$$g_x(x, y) = f(x+1, y) - f(x, y)$$

$$g_y(x, y) = f(x, y+1) - f(x, y)$$

$$g_x: \begin{array}{|c|c|} \hline -1 & 1 \\ \hline \end{array} \qquad g_y: \begin{array}{|c|} \hline -1 \\ \hline 1 \\ \hline \end{array}$$

- Roberts operators:

$$g_x(x, y) = f(x+1, y+1) - f(x, y)$$

$$g_y(x, y) = f(x, y+1) - f(x-1, y)$$

$$g_x: \begin{array}{|c|c|} \hline -1 & 0 \\ \hline 0 & 1 \\ \hline \end{array} \qquad g_y: \begin{array}{|c|c|} \hline 0 & -1 \\ \hline 1 & 0 \\ \hline \end{array}$$

Derivative operators (2)

- Sobel operators:

$$
\begin{aligned}
g_x(x, y) = &-f(x-1, y-1) - 2f(x-1, y) \\
&- f(x-1, y+1) + f(x+1, y-1) \\
&+ 2f(x+1, y) + f(x+1, y+1)
\end{aligned}
$$

$$
\begin{aligned}
g_y(x, y) = &-f(x-1, y-1) - 2f(x, y-1) \\
&- f(x+1, y-1) + f(x-1, y+1) \\
&+ 2f(x, y+1) + f(x+1, y+1)
\end{aligned}
$$

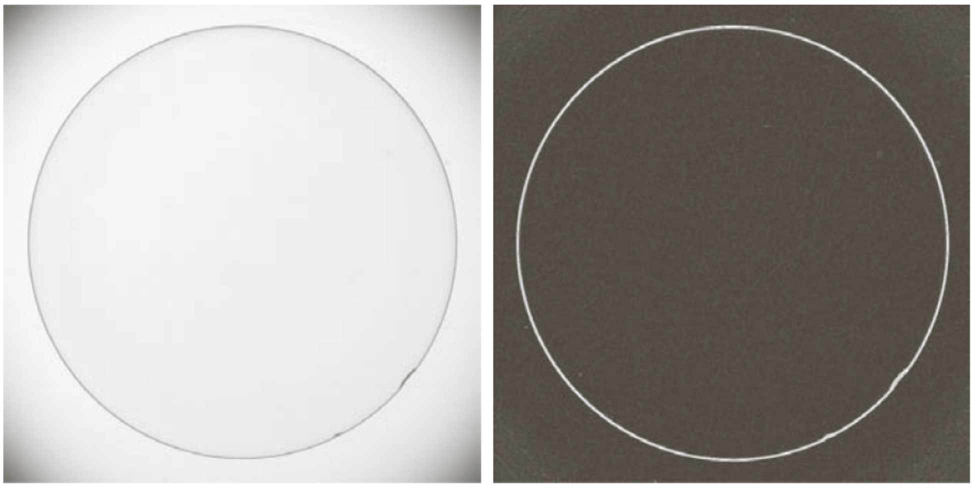
g_x:

-1	-2	-1
0	0	0
1	2	1

g_y:

-1	0	1
-2	0	2
-1	0	1

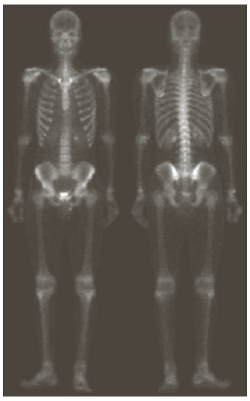
Example of gradient based application



- The Sobel filtering reduces the visibility of those regions in which the intensity changes slowly, allowing to highlight the defects (and making defects detection easier for automatic processing).

Combined methods

- Often, a single technique is not sufficient for obtaining the desired results.
- For example, the image (a) is affected by noise and has a narrow dynamic range.



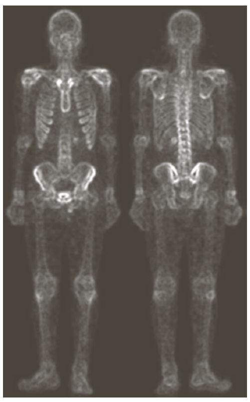
(a)

(b)

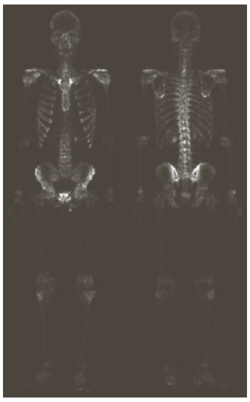
- Laplacian filtering (b) enhance the details, but also the noise.
- The gradient is less sensitive to the noise than the Laplacian (which is a second order operator).

Combined methods (2)

- The gradient, smoothed in order to avoid the noise, can be used for weighting the contribution of the Laplacian.
- The image (c), the Sobel filtering gradient smoothed using a 5 × 5 averaging filter is reported.



(c)

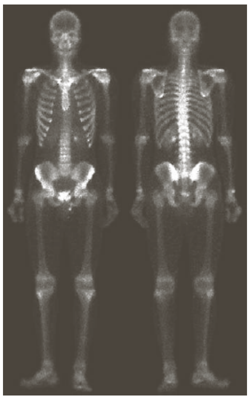


(d)

- This mask multiplied by the Laplacian results in the image in (d).
- The intensity changes are preserved, while the noise has been attenuated.

Combined methods (3)

- The image (d) can be added to the original image, which results in the image (e).
- The dynamical range can be enlarged applying a power transformation (e).



(e)

(f)

- The intensity transformation make the noise more visible, but also enhance other details, such as the tissues around the skeleton.

Bilateral Filtering *

Giving f an image, g is the image after bilateral filtering:

$$g(p) = \frac{1}{W_p} \sum_{q \in N_p} \exp\left(-\frac{||q-p||^2}{\sigma_s^2}\right) \exp\left(-\frac{||f(q)-f(p)||^2}{\sigma_i^2}\right) f(q)$$

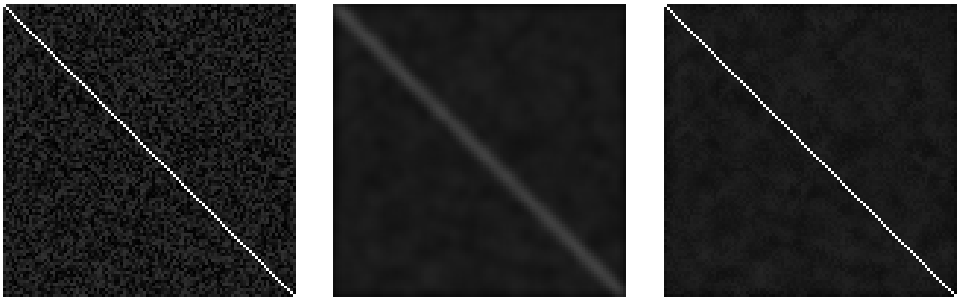
where W_p is the normalization factor:

$$W_p = \sum_{q \in N_p} \exp\left(-\frac{||q-p||^2}{\sigma_s^2}\right) \exp\left(-\frac{||f(q)-f(p)||^2}{\sigma_i^2}\right)$$

and N_p is a suitable neighborhood of p.

- What is the effect produced by the filter?
- Notes:
  - when σ_i grows, the filter tends to an averaging filter;
  - the filter is not linear.

Bilateral Filtering *



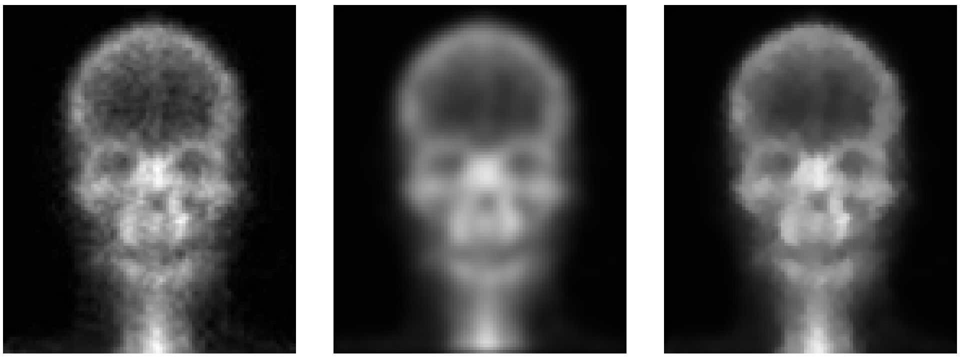
(a) (b) (c)

(a) original image (100×100, 256 gray levels)

(b) after filtering with Gaussian filter (7×7, $\sigma_s = 3$)

(c) after filtering with bilateral filter (7×7, $\sigma_s = 3$, $\sigma_i = 50$)

Bilateral Filtering *



(a) (b) (c)

(a) Original image (107×90, 256 gray levels)

(b) after filtering with Gaussian filter (7×7, $\sigma_s = 3$)

(c) after filtering with bilateral filter (7×7, $\sigma_s = 3$, $\sigma_i = 30$)