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Histogram equalization

Stefano Ferrari

Università degli Studi di Milano
stefano.ferrari@unimi.it

Elaborazione delle immagini
(Image processing I)

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Histogram

- The *histogram* of an L-valued image is a discrete function:

$$h(k) = n_k, \quad k \in [0, \ldots, L-1]$$

  where n_k is the number of pixels with intensity k.
- Often it is preferable to consider the histogram normalized with respect to the number of pixels, $M \times N$:

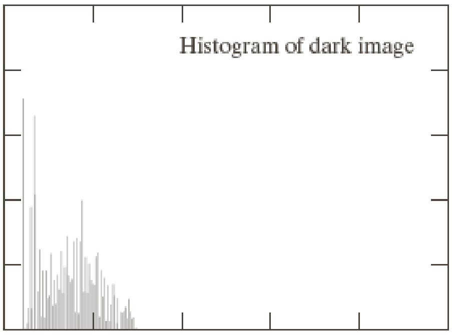
$$p(k) = \frac{n_k}{MN}$$

  - M and N are the number of rows and columns of the image.
- The function $p(k)$ estimates the probability density of k;
  - the sum $\sum_k p(k)$ is equal to 1.

Histogram based trasformations

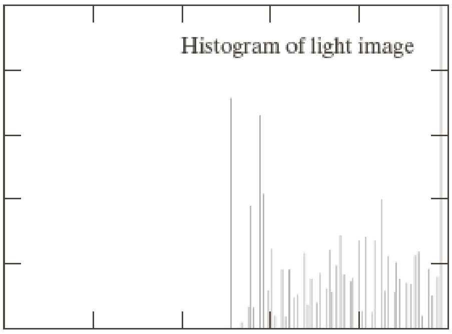
- The histogram provides an intuitive (visual) tool for evaluating some statistical properties of the image.
- Histogram based transformations are numerous:
  - enhancement,
  - compression,
  - segmentation;
- and can be easily implemented:
  - cheap;
  - dedicated hardware.

Dark image



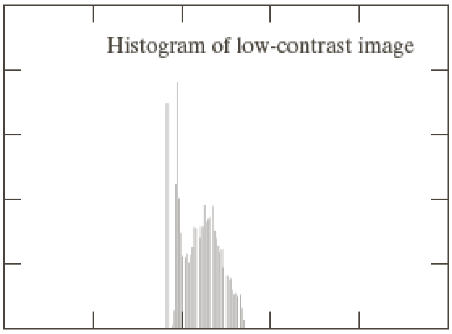
- The histogram components are localized to low intensity values.

Bright image



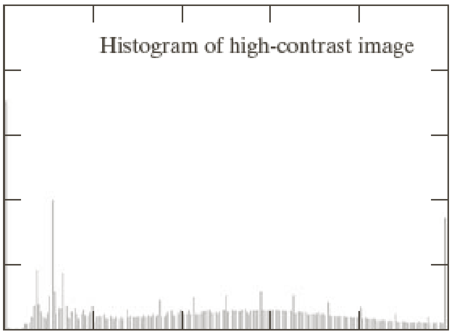
- The histogram components are localized to high intensity values.

Low contrast image



- The histogram components are localized in a narrow region of the intensity values.

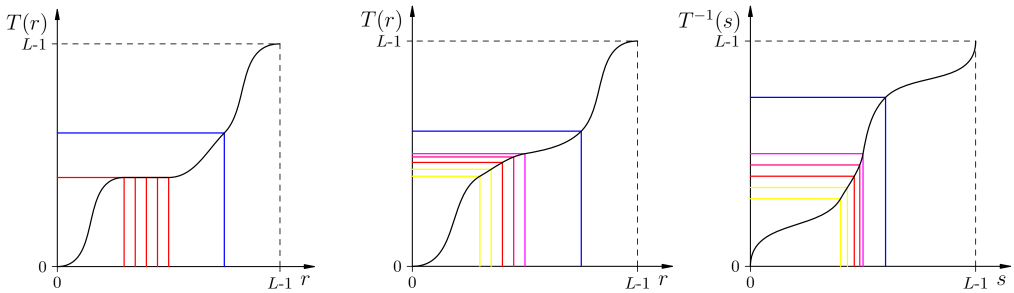
High contrast image



- The histogram components are distributed over all the intensity range.
- The distribution is almost uniform, with few peaks.
- If the distribution is uniform, the image tends to have a high dynamic range and the details are more easily perceived.
- This is the effect pursued by the histogram based transformations.

Monotonic transformations

- In order to study the histogram transformations, it is useful to consider the (continuous) monotonic transforms on $[0, L-1]^2$:
  - $s = T(r), \qquad 0 \leq r \leq L-1$
  - $T(r_2) \geq T(r_1), \qquad r_2 > r_i$
  - $0 \leq T(r) \leq L-1, \qquad 0 \leq r \leq L-1$
- If T is strictly monotonically increasing, there is T^{-1}:
  - $r = T^{-1}(s), \qquad 0 \leq s \leq L-1$



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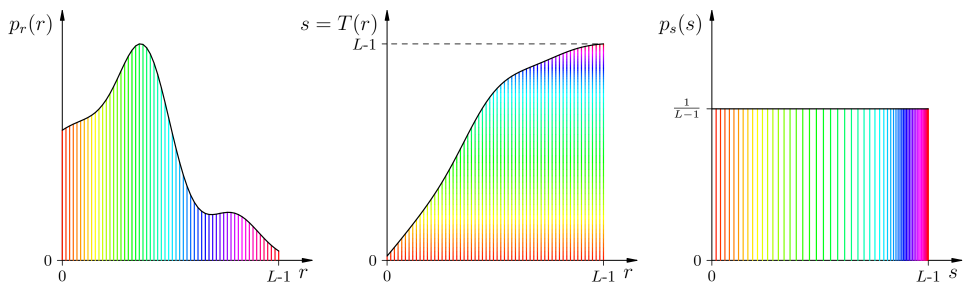
Intensities as random variables

- The (continuous) intensities can be intended as random variables in $[0, L-1]$.
- If $s = T(r)$ and $T(r)$ is continuous and differentiable:
  - $p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$
- In particular, the following transformation is interesting:
  - $s = T(r) = (L-1) \int_0^r p_r(w) dw$
- Then:
  - $\frac{ds}{dr} = \frac{T(r)}{dr} = (L-1) \frac{d}{dr} \left[\int_0^r p_r(w) dw \right] = (L-1) p_r(r)$
- Hence:
  - $p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| = p_r(r) \left| \frac{1}{(L-1) p_r(r)} \right|$
    $= \frac{1}{L-1}, \qquad 0 \leq s \leq L-1$
- That is: s is uniform, independently of p_r.

Equalization



- The equalization transformation, $T(r)$, is steeper where r is more probable.
- It results in mapping intervals of r values with low probability into narrow intervals of $s = T(r)$.
- On the contrary, intervals of r values with high probability are mapped into large intervals of s.

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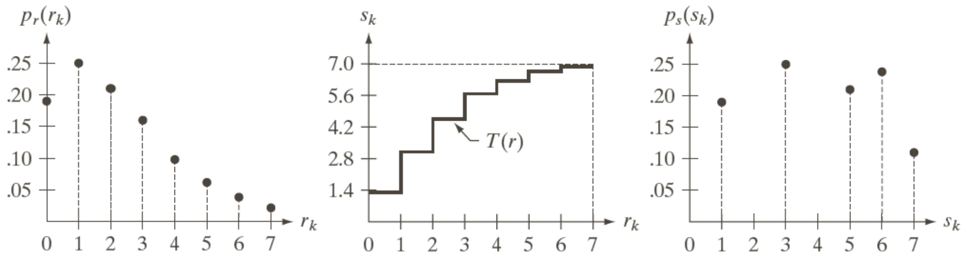
Equalization of a discrete random variable

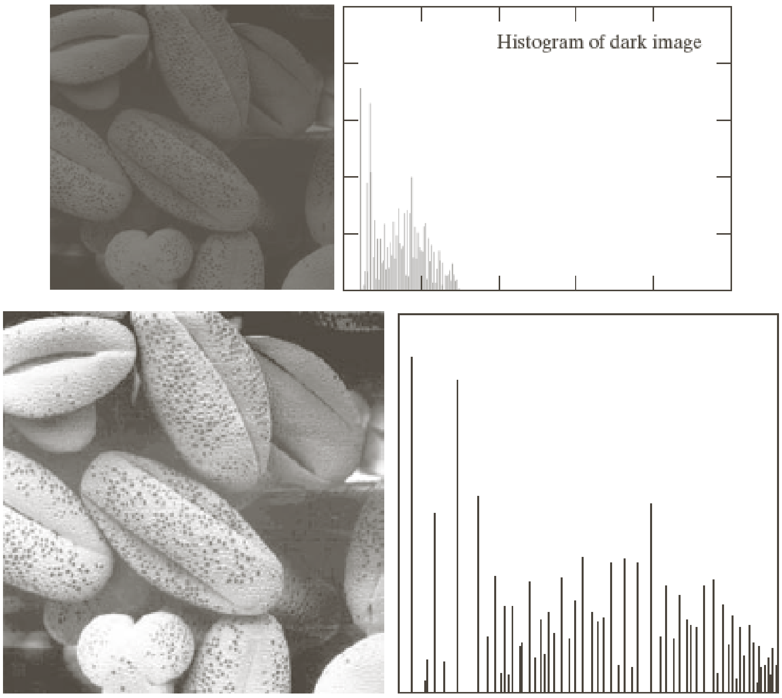
- r_k is the intensity level in $0, \ldots, L-1$
  - $p_r(r_k) = \frac{n_k}{MN}, \qquad k = 0, 1, \ldots, L-1$
- p_r can be equalized by assigning the intensity s_k to those pixels having intensity r_k:
  - $s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$
    $= \frac{L-1}{MN}\sum_{j=0}^{k} n_j, \qquad k = 0, 1, \ldots, L-1$

Equalization of a discrete random variable (2)

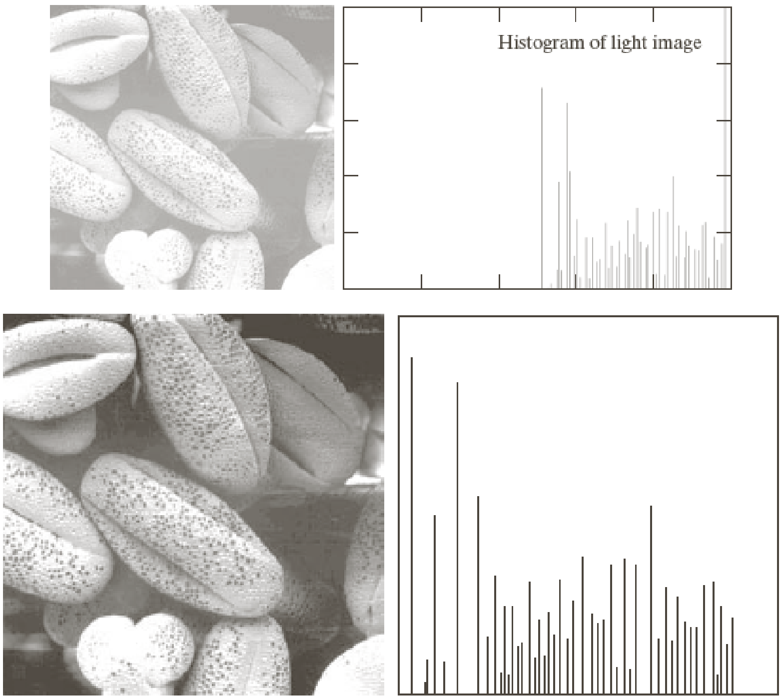
r_k	n_k	$p_r(r_k)$	$T(r_k)$	s_k	$p_s(s_k)$
$r_0 = 0$	790	0.19	1.33	1	0.19
$r_1 = 1$	1023	0.25	3.08	3	0.25
$r_2 = 2$	850	0.21	4.55	5	0.21
$r_3 = 3$	656	0.16	5.67	6	0.24
$r_4 = 4$	329	0.08	6.23	6	
$r_5 = 5$	245	0.06	6.65	7	0.11
$r_6 = 6$	122	0.03	6.86	7	
$r_7 = 7$	81	0.02	7.00	7	



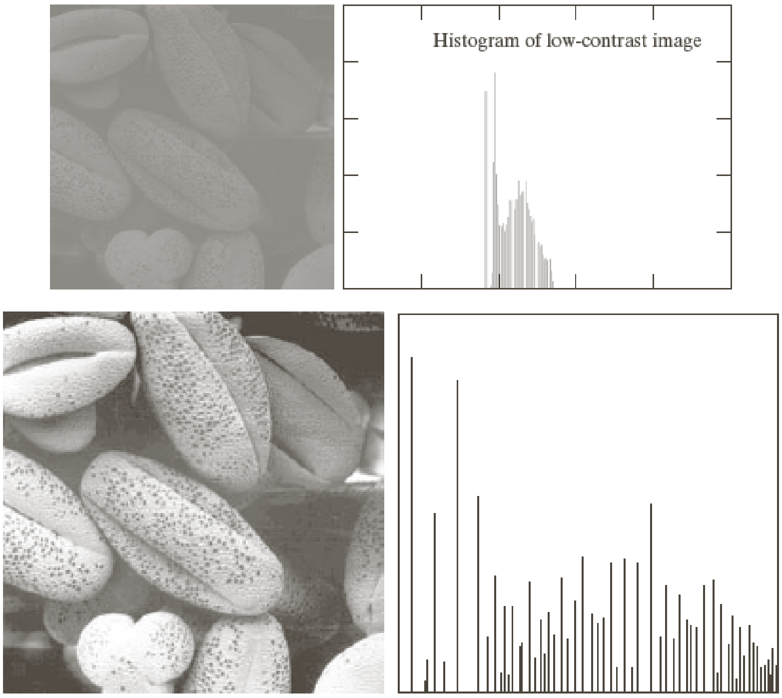
Examples



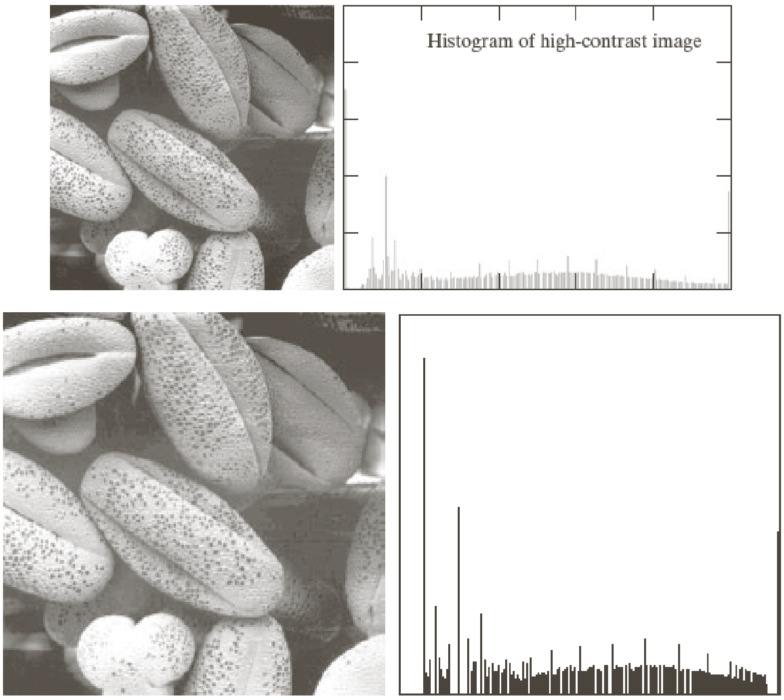
Examples (2)



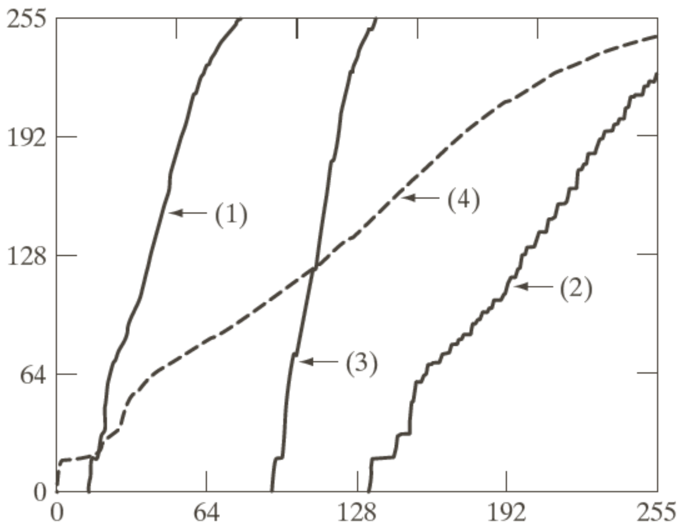
Examples (3)



Examples (4)



Examples (5)



- The transformation of each image maps values from the range of the original images to the whole range of intensity levels.
- The transformation for (4) is close to the identity.

Histogram specification

- The histogram equalization is a basic procedure that allow to obtain a processed image with a specified intensity distribution.
- Sometimes, the distribution of the intensities of a scene is known to be not uniform.
- The possibility of obtaining a processed image with a given distribution is appreciable:
  - *Histogram matching*
- The problem can be formalized as follows:
  - given an input image, whose pixels are distributed with probability density p_r,
  - given the desired intensity distribution, p_z,
  - find the transformation F, such that $z = F(r)$.

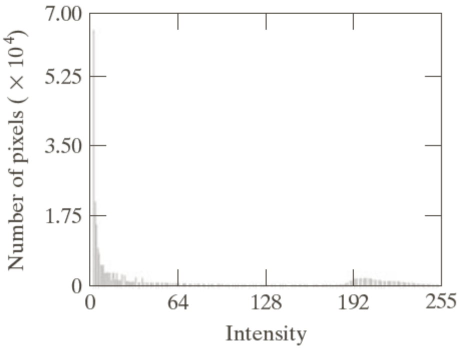
Histogram specification (2)

- Let s be a random variable such that:
  - $s = T(r) = (L-1)\int_0^r p_r(w)\mathrm{d}w$
- Define a random variable z that satisfies:
  - $G(z) = (L-1)\int_0^z p_z(t)\mathrm{d}t = s$
- Hence: $G(z) = s = T(r)$
- The desired mapping F, such that $z = F(r)$ can be obtained as:
  - $z = G^{-1}(T(r))$, i.e., $F = T \circ G^{-1}$

Histogram specification (3)

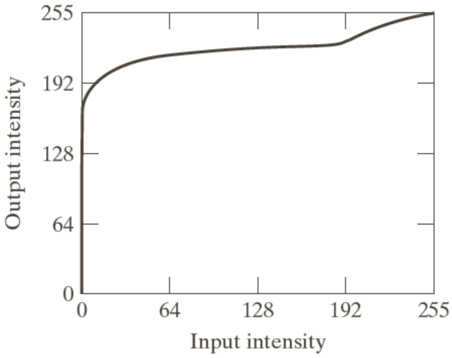
- When discrete random variables are considered, p_z can be specified by its histogram.
- The histogram matching procedure can be realized:
  1. obtain p_r from the input image;
  2. obtain the mapping T using the equalization relation;
  3. obtain the mapping G from the specified p_z;
  4. build F by scanning T and finding the matching value in G;
  5. apply the transformation F to the original image.
- In order to be invertible, G have to be strictly monotonic.
- In pratical cases, this property is rarely satisfied.
- Some approximations should be allowed
  - e.g., the first matching value can be accepted.

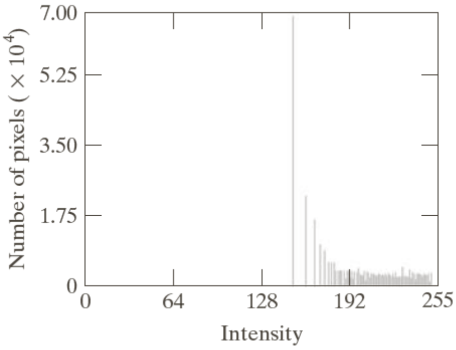
Example

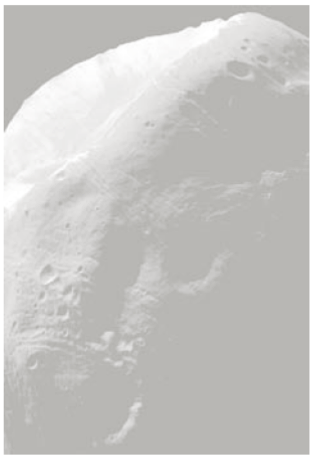


- Large concentration of pixels in the dark region of the histogram.

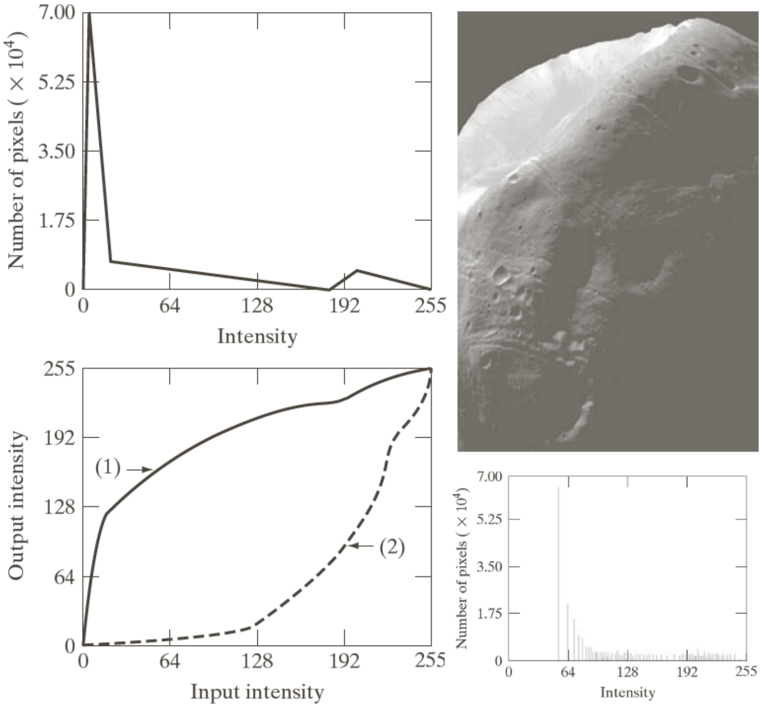
Example (2)







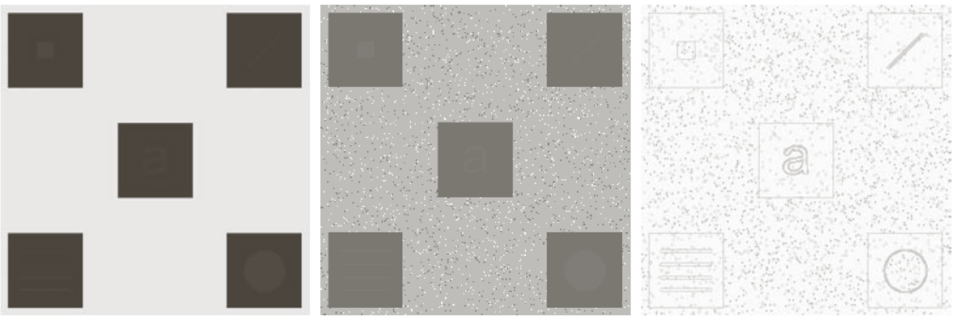
Example (3)



Local histogram processing

- Histogram equalization is a global approach.
- Local histogram equalization is realized selecting, for each pixel, a suitable neighborhood on which the histogram equalization (or matching) is computed.
  - More computational intensive, but neighboring pixels shares most of the neighborhood.
- Non overlapping regions may produce “blocky” effect.

Example



a | b | c

(a) original image
(b) equalized image
(c) locally equalized image (3×3 neighborhood)

Histogram statistics

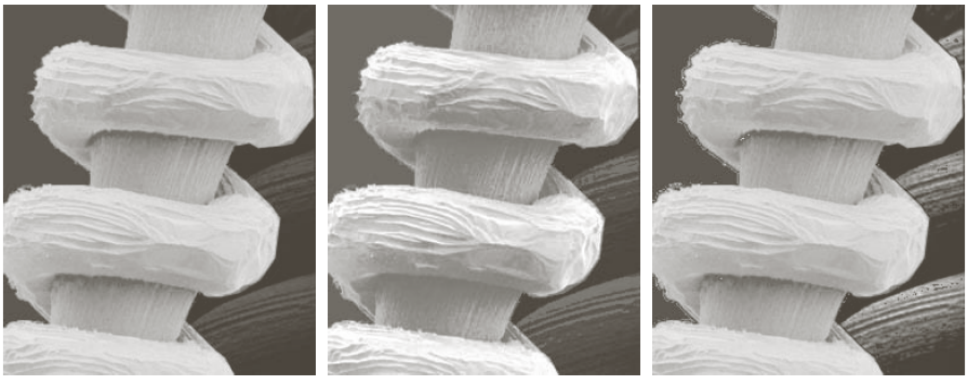
Some statistical indices can be easily computed from the histogram:

- Mean (average):
  - $m = \sum_{i=0}^{L-1} r_i p(r_i)$
- Variance:
  - $\sigma^2 = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$
  - Standard deviation: $\sigma = \sqrt{\sigma^2}$
- n-th moment:
  - $\mu_n = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$

Local statistical indices can be computed by bounding the histogram to a given neighborhood, S_{xy}:

- $m_{S_{xy}} = \sum_{i=0}^{L-1} r_i p_{S_{xy}}(r_i)$
- $\sigma^2_{S_{xy}} = \sum_{i=0}^{L-1} (r_i - m_{S_{xy}})^2 p_{S_{xy}}(r_i)$

Example



a | b | c

(a) original image
(b) equalized image
(c) local statistics enhanced image (3×3 neighborhood)

Example (2)

- Only dark regions need to be enhanced
  - $m_{S_{xy}} \leq k_0 m_G$
- Uniform region have to be preserved
  - $\sigma_{S_{xy}} \geq k_1 \sigma_G$
- Low contrasted regions have to be enhanced
  - $\sigma_{S_{xy}} \leq k_2 \sigma_G$

$$g(x,y) = \begin{cases} E \cdot f(x,y) & \text{if } m_{S_{xy}} \leq k_0 m_G \\ & \quad \text{AND } k_1 \sigma_G \leq \sigma_{S_{xy}} \leq k_2 \sigma_G \\ f(x,y) & \text{otherwise} \end{cases}$$

$E = 4,\ k_0 = 0.4,\ k_1 = 0.02,\ k_2 = 0.4.$