Supplementary material of "A combinatorial branch-and-bound for the Safe Set Problem"

Alberto Boggio Tomasaz

Dipartimento di Informatica, Università degli Studi di Milano

Roberto Cordone*

Dipartimento di Informatica, Università degli Studi di Milano

Pierre Hosteins

Université Gustave Eiffel, COSYS-ESTAS, Villeneuve d'Ascq, France

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The following tables report the detailed results obtained by the algorithm described in the above mentionsed paper on the benchmark instances available at `https://homes.di.unimi.it/cordone/research/wssp.html`.

The computational times reported refer to an Intel Xeon E5-2620 with a 2.1 GHz CPU and 16 GB or RAM with a time limit of one hour.

All tables have one row for each instance. The first column always reports the size of the graph, that is the number of vertices $|V|$. Tables 1, 2 and 3 refer to the 126 instances of the benchmark proposed in [2], divided by density ($\delta = 0.3$, $\delta = 0.5$ and $\delta = 0.7$). The following two groups of three columns refer to the weighted and unweighted instances, and provide the value of the optimum, the computational time (in seconds) and the number of branching nodes required to find it.

Tables 4, 5, 6 and 7 consider the 240 instances of the benchmark proposed in [1] and enlarged in the submitted paper with 40 instances of $|V| = 60$ vertices. Once again, they are divided by density ($\delta = 0.1$, $\delta = 0.2$, $\delta = 0.3$ and $\delta = 0.4$). Since the benchmark includes 5 instances with the same size

*Corresponding author

Email addresses: `alberto.boggio@unimi.it` (Alberto Boggio Tomasaz), `roberto.cordone@unimi.it` (Roberto Cordone), `pierre.hosteins@univ-eiffel.fr` (Pierre Hosteins)

and density, the second column of the each table reports the index of each instance. Then, two groups of four columns provide (respectively, for the weighted and unweighted instances) the value of upper and lower bound on the optimum, the computational time (in seconds) and the number of branching nodes required to find them. The label OM marks the instances in which the algorithm was terminated before the time limit for lack of memory. In 26 cases out of 280, the bounds (marked in italics) are different, and the instance is still open. In the other 254 cases, both bounds coincide with the optimum.

	Weighted			Unweighted		
$\|V\|$	z^*	T	BN	z^*	T	BN
10	249	0.000	93	4	0.000	103
11	214	0.000	63	4	0.000	149
12	197	0.001	123	4	0.000	117
13	116	0.000	113	5	0.001	171
14	235	0.001	205	5	0.001	273
15	251	0.001	315	6	0.004	807
16	216	0.001	253	6	0.004	711
17	257	0.003	487	6	0.006	1097
18	210	0.001	147	6	0.005	703
19	281	0.007	1241	7	0.017	2659
20	433	0.009	1385	8	0.037	5231
21	374	0.006	933	8	0.014	1827
22	421	0.028	3873	8	0.033	4319
23	297	0.009	1121	9	0.050	5937
24	427	0.040	4861	10	0.107	13335
25	425	0.046	5147	11	0.216	25075
26	510	0.131	13555	11	0.267	27463
27	587	0.127	12543	11	0.240	23143
28	444	0.100	9111	12	0.421	40675
29	595	0.416	36881	12	0.494	43319
30	562	0.328	27017	13	1.723	137661

Table 1: Results for the benchmarks instances with $\delta = 0.3$ proposed in [2]

	Weighted			Unweighted		
$\|V\|$	z^*	T	BN	z^*	T	BN
10	254	0.001	139	5	0.000	155
11	214	0.000	67	4	0.000	113
12	251	0.000	97	6	0.001	303
13	149	0.000	87	6	0.001	263
14	299	0.001	253	6	0.002	285
15	301	0.002	481	7	0.002	465
16	279	0.001	213	7	0.007	1289
17	336	0.004	553	8	0.004	677
18	326	0.002	253	8	0.003	409
19	378	0.003	411	9	0.019	2407
20	489	0.005	569	9	0.019	2199
21	545	0.031	3367	10	0.016	1729
22	524	0.032	3201	10	0.040	4139
23	410	0.012	1175	11	0.057	5415
24	536	0.022	1979	11	0.155	15115
25	539	0.046	3819	12	0.097	8397
26	638	0.109	8591	13	0.182	14079
27	725	0.226	16411	13	0.090	6981
28	572	0.084	5753	13	0.218	15275
29	735	0.469	31129	14	0.303	19961
30	710	0.456	27165	15	0.549	34145

Table 2: Results for the benchmarks instances with $\delta = 0.5$ proposed in [2]

	Weighted			Unweighted		
$\|V\|$	z^*	T	BN	z^*	T	BN
10	312	0.000	71	5	0.000	67
11	260	0.000	71	5	0.000	79
12	300	0.001	109	6	0.000	95
13	173	0.001	99	6	0.001	189
14	320	0.001	205	7	0.001	137
15	348	0.003	509	7	0.003	399
16	323	0.002	285	8	0.002	343
17	377	0.003	337	8	0.002	315
18	380	0.003	357	9	0.002	267
19	420	0.008	881	9	0.015	1683
20	533	0.011	1079	10	0.004	389
21	606	0.036	3171	10	0.002	215
22	571	0.028	2267	11	0.007	587
23	410	0.008	607	11	0.018	1417
24	624	0.050	3615	12	0.012	843
25	583	0.029	1883	12	0.079	5437
26	681	0.041	2457	13	0.017	1069
27	756	0.075	4285	13	0.186	11979
28	636	0.051	2819	14	0.036	1983
29	775	0.241	11703	15	0.516	27415
30	759	0.423	19361	15	0.053	2483

Table 3: Results for the benchmarks instances with $\delta = 0.7$ proposed in [2]

		Weighted				Unweighted			
$\|V\|$	#	UB	LB	T	BN	UB	LB	T	BN
20	1	43	43	0.003	507	7	7	0.030	6789
20	2	40	40	0.001	339	7	7	0.020	6201
20	3	46	46	0.001	467	7	7	0.024	8059
20	4	43	43	0.001	555	7	7	0.012	4309
20	5	41	41	0.001	555	7	7	0.014	4813
25	1	54	54	0.019	5541	8	8	0.039	10185
25	2	43	43	0.007	1751	7	7	0.017	3865
25	3	43	43	0.006	1651	7	7	0.036	8549
25	4	33	33	0.006	1757	7	7	0.019	4473
25	5	48	48	0.005	1567	8	8	0.100	24557
30	1	51	51	0.040	7165	8	8	0.092	14759
30	2	43	43	0.085	15709	8	8	0.139	23935
30	3	40	40	0.017	3545	7	7	0.052	8679
30	4	49	49	0.033	6303	8	8	0.149	25739
30	5	38	38	0.056	11145	9	9	0.511	91117
35	1	45	45	0.225	32903	9	9	1.780	247973
35	2	54	54	0.177	27905	9	9	1.548	194447
35	3	45	45	0.039	6683	11	11	7.507	997005
35	4	39	39	0.073	11627	9	9	1.194	161133
35	5	46	46	0.223	40845	11	11	7.688	1155675
40	1	64	64	14.262	1676227	13	13	143.283	18952173
40	2	54	54	2.200	274715	12	12	96.266	10757845
40	3	61	61	1.196	152773	11	11	26.345	2720511
40	4	52	52	1.748	232327	13	13	144.700	16456887
40	5	53	53	2.123	257553	13	13	127.125	14323929
50	1	73	73	89.058	7765789	*18*	*14*	2030.481	OM
50	2	79	79	290.193	23077925	*22*	*14*	1916.588	OM
50	3	59	59	78.384	6769767	*23*	*13*	903.281	OM
50	4	87	87	537.576	44435869	*23*	*13*	1732.622	OM
50	5	82	82	191.621	15178881	*21*	*14*	2000.994	OM
60	1	77	77	3474.908	214559949	*31*	*13*	1405.774	OM
60	2	79	79	637.409	36033589	*34*	*14*	1741.854	OM
60	3	83	83	1130.850	74132535	*30*	*13*	842.090	OM
60	4	*103*	*84*	2641.847	OM	*33*	*13*	844.763	OM
60	5	86	86	3486.326	208196017	*32*	*13*	1455.969	OM

Table 4: Results for the benchmarks instances with $\delta = 0.1$ proposed in [1]

		Weighted				Unweighted			
$\|V\|$	#	UB	LB	T	BN	UB	LB	T	BN
20	1	26	26	0.011	1999	7	7	0.067	12437
20	2	33	33	0.011	2197	7	7	0.022	4461
20	3	23	23	0.006	1023	7	7	0.017	3993
20	4	26	26	0.010	1741	7	7	0.025	5183
20	5	31	31	0.005	889	7	7	0.039	6837
25	1	39	39	0.090	12381	8	8	0.283	36561
25	2	44	44	0.146	19673	9	9	0.232	31837
25	3	35	35	0.088	12985	9	9	0.607	81427
25	4	53	53	0.242	34467	9	9	0.460	60773
25	5	38	38	0.123	16491	9	9	0.589	76775
30	1	54	54	0.574	60951	10	10	0.766	73663
30	2	52	52	0.373	40809	11	11	3.291	334735
30	3	50	50	0.251	27115	11	11	0.918	103341
30	4	51	51	2.275	239267	11	11	1.678	196109
30	5	50	50	0.543	54359	11	11	2.352	239987
35	1	61	61	2.799	224721	14	14	18.546	1539771
35	2	73	73	5.608	474225	14	14	40.049	3656217
35	3	64	64	10.780	862205	13	13	3.765	331355
35	4	51	51	3.994	317221	13	13	11.947	1015895
35	5	73	73	7.455	598641	14	14	26.090	2254417
40	1	82	82	47.942	3064887	16	16	55.998	3869533
40	2	76	76	19.076	1241443	15	15	26.785	1783551
40	3	68	68	7.488	488615	16	16	71.242	4839509
40	4	62	62	10.491	673553	16	16	33.020	2299495
40	5	75	75	24.398	1617767	16	16	91.396	6085663
50	1	104	104	262.538	12056497	22	22	2670.754	133206761
50	2	126	126	1516.176	70921773	21	21	2468.393	115567309
50	3	102	102	555.008	24748309	21	21	2076.367	104467103
50	4	114	114	698.322	32267773	22	22	2415.351	118672561
50	5	130	130	2329.665	107396009	22	22	2873.221	148760933
60	1	*124*	*115*	3600.000	114519193	*30*	*24*	3600.000	124741717
60	2	111	111	818.154	26933287	*30*	*25*	3600.000	122956349
60	3	*145*	*127*	3600.000	118277525	*32*	*24*	1864.234	OM
60	4	*133*	*130*	3600.000	115488031	*30*	*25*	3600.000	119412243
60	5	*138*	*125*	3600.000	110968265	*30*	*25*	3600.000	121037137

Table 5: Results for the benchmarks instances with $\delta = 0.2$ proposed in [1]

		Weighted				Unweighted			
$\|V\|$	#	UB	LB	T	BN	UB	LB	T	BN
20	1	33	33	0.007	1127	8	8	0.028	4349
20	2	39	39	0.011	1695	7	7	0.015	2093
20	3	36	36	0.012	1779	7	7	0.009	1517
20	4	31	31	0.009	1399	7	7	0.014	2085
20	5	40	40	0.024	3735	7	7	0.013	1847
25	1	48	48	0.034	3971	10	10	0.096	11073
25	2	57	57	0.151	16381	11	11	0.433	52421
25	3	62	62	0.125	14079	10	10	0.146	15665
25	4	53	53	0.147	15941	10	10	0.079	8689
25	5	48	48	0.113	12707	10	10	0.213	24319
30	1	64	64	0.771	63793	13	13	1.753	154635
30	2	59	59	0.367	30017	13	13	1.193	98815
30	3	70	70	0.626	50827	13	13	1.078	87217
30	4	45	45	0.054	4587	13	13	0.472	40913
30	5	68	68	0.588	47047	13	13	0.850	68665
35	1	86	86	3.311	214223	16	16	11.524	844865
35	2	88	88	6.472	417821	15	15	3.598	248141
35	3	83	83	4.425	294179	16	16	8.657	598511
35	4	78	78	0.479	32321	16	16	7.119	494713
35	5	66	66	0.977	62515	15	15	3.913	254381
40	1	95	95	3.323	173395	18	18	33.469	1872129
40	2	108	108	19.158	1006897	18	18	32.012	1699269
40	3	94	94	10.956	573439	18	18	33.107	1809069
40	4	84	84	4.048	215217	18	18	24.390	1334825
40	5	105	105	16.619	911487	18	18	25.638	1346543
50	1	115	115	140.265	4965171	23	23	454.234	17448179
50	2	117	117	99.621	3458577	23	23	443.934	16557027
50	3	119	119	63.087	2267147	24	24	1522.297	59870131
50	4	110	110	55.138	1977397	23	23	467.829	17517217
50	5	110	110	92.972	3310181	24	24	1479.911	58918873
60	1	129	129	849.539	20954933	*30*	*27*	3600.000	89251817
60	2	124	124	500.606	12054917	*30*	*27*	3600.000	90466873
60	3	157	157	3002.098	72243237	*30*	*27*	3600.000	90122711
60	4	149	149	979.856	22742691	*29*	*28*	3600.000	88076127
60	5	144	144	1480.667	36611413	*30*	*27*	3600.000	90393441

Table 6: Results for the benchmarks instances with $\delta = 0.3$ proposed in [1]

		Weighted				Unweighted			
$\|V\|$	#	UB	LB	T	BN	UB	LB	T	BN
20	1	43	43	0.016	2165	9	9	0.025	3455
20	2	49	49	0.015	2031	9	9	0.023	2913
20	3	42	42	0.006	747	9	9	0.010	1439
20	4	50	50	0.030	3957	9	9	0.009	1221
20	5	48	48	0.013	1697	9	9	0.033	4245
25	1	57	57	0.081	7685	12	12	0.144	14891
25	2	71	71	0.158	15027	12	12	0.239	25243
25	3	56	56	0.070	6647	11	11	0.125	12269
25	4	57	57	0.085	8283	11	11	0.093	8763
25	5	54	54	0.047	4379	12	12	0.227	23829
30	1	71	71	0.353	24869	14	14	0.882	65851
30	2	72	72	0.559	38975	14	14	0.733	53979
30	3	76	76	0.400	29315	14	14	0.773	58033
30	4	68	68	0.285	20053	14	14	0.966	68799
30	5	73	73	0.728	50725	14	14	0.745	55681
35	1	93	93	2.301	128843	17	17	5.491	333551
35	2	93	93	1.974	112131	16	16	8.094	497719
35	3	96	96	3.150	177521	17	17	5.288	325563
35	4	85	85	1.961	104751	16	16	2.356	136469
35	5	95	95	1.369	74845	17	17	6.014	364143
40	1	98	98	6.250	266341	19	19	27.390	1287939
40	2	96	96	7.649	322289	19	19	15.596	691835
40	3	105	105	2.866	126703	19	19	12.245	545577
40	4	108	108	2.773	122315	19	19	14.695	669075
40	5	93	93	1.655	72105	19	19	16.470	747527
50	1	127	127	52.420	1423333	24	24	272.972	7766813
50	2	142	142	83.399	2312399	24	24	168.931	4868125
50	3	122	122	26.058	708093	24	24	242.142	7094047
50	4	123	123	38.493	1080615	24	24	194.666	5791291
50	5	116	116	53.370	1503081	24	24	192.525	5674063
60	1	137	137	169.997	3122629	29	29	2178.099	40799191
60	2	130	130	236.432	4292569	29	29	3097.001	60593849
60	3	163	163	968.583	17609435	29	29	3199.095	63802285
60	4	154	154	294.560	5287225	29	29	1597.201	30996299
60	5	155	155	640.122	11745113	*30*	*29*	3600.000	71328489

Table 7: Results for the benchmarks instances with $\delta = 0.4$ proposed in [1]

References

[1] Hosteins, P. A compact mixed integer linear formulation for safe set problems. *Optimization Letters*, 14, 2127-–2148, 2020.

[2] Macambira, A.F.U., Simonetti, L., Barbalho, H., Silva, P.H.G., Maculan, N. A new formulation for the safe set problem on graphs. *Computers and Operations Research*, 111, 346–356, 2019.