

Supplementary material of “Constructive-destructive heuristics for the Safe Set Problem”

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The following tables report the detailed results obtained by the algorithm described in the above mentioned paper on the benchmark instances available at <https://homes.di.unimi.it/cordone/research/wssp.html>.

The solutions reported have been obtained in 10 seconds on an Intel Xeon E5-2620 with a 2.1 GHz CPU and 16 GB of RAM.

Each table refers to a benchmark or benchmark class, identified in the caption. The first three columns identify the graph corresponding to each row, reporting its number of vertices $|V|$ and edges $|E|$ and an identifier. The identifier is usually a progressive number, except for the real-world instances where it is the name of the graph and for the grid instances, where it is a description of its structure. Each graph yields two instances: a weighted one and an unweighted one. Correspondingly, the following two groups of three columns provide the value of the best known solution and the numbers of safe and unsafe components, for the weighted and the unweighted instance.

Tables 1, 2, 3 and 4 consider the 100 random graphs of benchmark H^+ , divided by density ($\delta = 0.1$, $\delta = 0.2$, $\delta = 0.3$ and $\delta = 0.4$). Table 5 considers

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the 9 real-world graphs of benchmark **GC**. Tables 6 and 7 consider the 50 small-world graphs of benchmark **SW**, divided by initial degree ($d = 6$ and $d = 10$). Tables 8 and 9 consider the 50 regular graphs of benchmark **Reg**, divided by degree ($d = 5$ and $d = 10$). Table 10 considers the 25 planar graphs of benchmark **P1a**. Table 11 considers the 5 two-dimensional and the 5 three-dimensional toroidal grid graphs of benchmark **Grid**.

$ V $	#	$ E $	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
100	1	508	212	1	2	43	1	2
100	2	494	201	1	2	42	1	2
100	3	489	187	1	2	42	1	3
100	4	541	200	1	6	43	1	3
100	5	520	213	1	3	43	1	4
150	1	1122	399	1	3	73	1	5
150	2	1161	407	1	6	72	1	5
150	3	1195	392	1	3	74	1	3
150	4	1144	388	1	3	73	1	3
150	5	1212	384	1	4	74	1	3
200	1	2027	543	1	4	100	1	1
200	2	2063	546	1	5	99	1	3
200	3	2115	568	1	2	99	1	2
200	4	2084	553	1	3	99	1	3
200	5	2064	529	1	4	99	1	3
250	1	3227	679	1	2	125	1	1
250	2	3259	673	1	3	125	1	1
250	3	3246	678	1	2	125	1	1
250	4	3183	646	1	3	125	1	1
250	5	3158	698	1	3	125	1	1
300	1	4634	835	1	2	150	1	1
300	2	4692	834	1	2	150	1	1
300	3	4607	858	1	2	150	1	1
300	4	4556	801	1	2	150	1	1
300	5	4486	794	1	2	150	1	1

Table 1: Results for the instances with $\delta = 0.1$ of benchmark \mathbb{H}^*

$ V $	#	$ E $	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
100	1	993	267	1	2	49	1	2
100	2	1014	259	1	3	49	1	2
100	3	956	242	1	2	49	1	3
100	4	1072	248	1	2	49	1	3
100	5	948	239	1	3	49	1	3
150	1	2243	415	1	3	75	1	1
150	2	2246	426	1	2	75	1	1
150	3	2263	418	1	2	75	1	1
150	4	2211	408	1	2	75	1	1
150	5	2346	397	1	2	75	1	1
200	1	4041	552	1	1	100	1	1
200	2	3968	563	1	1	100	1	1
200	3	4010	577	1	1	100	1	1
200	4	4027	563	1	1	100	1	1
200	5	3978	543	1	1	100	1	1
250	1	6265	684	1	1	125	1	1
250	2	6254	681	1	1	125	1	1
250	3	6275	683	1	1	125	1	1
250	4	6272	652	1	1	125	1	1
250	5	6285	706	1	1	125	1	1
300	1	8995	837	1	1	150	1	1
300	2	9078	839	1	1	150	1	1
300	3	9067	861	1	1	150	1	1
300	4	8985	805	1	1	150	1	1
300	5	9019	799	1	1	150	1	1

Table 2: Results for the instances with $\delta = 0.2$ of benchmark \mathbb{H}^+

$ V $	#	$ E $	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
100	1	1510	271	1	2	50	1	1
100	2	1488	265	1	2	50	1	1
100	3	1438	254	1	2	50	1	1
100	4	1528	251	1	2	50	1	1
100	5	1453	247	1	2	50	1	1
150	1	3355	420	1	1	75	1	1
150	2	3321	431	1	1	75	1	1
150	3	3374	423	1	1	75	1	1
150	4	3354	413	1	1	75	1	1
150	5	3453	401	1	1	75	1	1
200	1	5976	552	1	1	100	1	1
200	2	6002	563	1	1	100	1	1
200	3	5998	577	1	1	100	1	1
200	4	6061	563	1	1	100	1	1
200	5	5934	543	1	1	100	1	1
250	1	9323	684	1	1	125	1	1
250	2	9428	681	1	1	125	1	1
250	3	9371	683	1	1	125	1	1
250	4	9358	652	1	1	125	1	1
250	5	9275	706	1	1	125	1	1
300	1	13464	837	1	1	150	1	1
300	2	13596	839	1	1	150	1	1
300	3	13405	861	1	1	150	1	1
300	4	13362	805	1	1	150	1	1
300	5	13522	799	1	1	150	1	1

Table 3: Results for the instances with $\delta = 0.3$ of benchmark \mathbb{H}^+

$ V $	#	$ E $	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
100	1	2014	276	1	1	50	1	1
100	2	1973	268	1	1	50	1	1
100	3	1936	256	1	2	50	1	1
100	4	2017	255	1	1	50	1	1
100	5	1958	252	1	1	50	1	1
150	1	4490	420	1	1	75	1	1
150	2	4439	431	1	1	75	1	1
150	3	4458	423	1	1	75	1	1
150	4	4469	413	1	1	75	1	1
150	5	4585	401	1	1	75	1	1
200	1	7964	552	1	1	100	1	1
200	2	7971	563	1	1	100	1	1
200	3	8044	577	1	1	100	1	1
200	4	8023	563	1	1	100	1	1
200	5	7944	543	1	1	100	1	1
250	1	12435	684	1	1	125	1	1
250	2	12558	681	1	1	125	1	1
250	3	12460	683	1	1	125	1	1
250	4	12537	652	1	1	125	1	1
250	5	12476	706	1	1	125	1	1
300	1	17927	837	1	1	150	1	1
300	2	18070	839	1	1	150	1	1
300	3	17999	861	1	1	150	1	1
300	4	17927	805	1	1	150	1	1
300	5	18108	799	1	1	150	1	1

Table 4: Results for the instances with $\delta = 0.4$ of benchmark \mathbb{H}^+

Name	$ V $	$ E $	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
adjnoun	112	425	151	1	18	31	1	18
celegans_metabolic	453	2025	239	2	45	47	1	44
celegansneural	297	2148	528	1	32	89	1	27
dolphins	62	159	83	1	9	14	1	9
football	115	613	212	1	2	40	1	2
jazz	198	2742	380	1	6	85	1	6
karate	34	78	27	1	14	6	1	15
lesmis	77	254	57	1	21	11	1	17
polbooks	105	441	119	1	8	24	1	9

Table 5: Results for the instances of benchmark GC

$ V $	#	$ E $	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
100	1	299	112	1	4	25	1	4
100	2	298	113	1	4	25	1	4
100	3	300	94	1	5	25	1	3
100	4	297	125	1	4	24	1	4
100	5	300	126	1	5	24	1	4
150	1	450	150	1	6	38	1	3
150	2	449	188	1	4	35	1	4
150	3	449	141	1	6	37	1	4
150	4	449	160	1	5	37	1	4
150	5	449	188	1	5	35	1	5
200	1	600	203	1	5	45	1	5
200	2	600	193	1	6	43	2	8
200	3	597	220	1	6	52	1	3
200	4	600	206	1	6	52	1	4
200	5	600	262	1	6	50	1	5
250	1	750	298	1	7	63	1	4
250	2	749	267	1	7	61	2	6
250	3	749	269	1	6	61	1	4
250	4	748	312	1	5	61	1	4
250	5	749	325	1	4	63	1	5
300	1	899	287	1	6	67	1	4
300	2	899	312	1	6	68	1	4
300	3	899	355	1	6	76	2	7
300	4	898	414	1	4	79	1	5
300	5	899	319	1	6	74	1	4

Table 6: Results for the instances with $d = 6$ of benchmark SW

$ V $	#	$ E $	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
100	1	498	141	1	3	27	1	3
100	2	498	116	1	4	27	1	3
100	3	496	121	1	3	28	1	3
100	4	498	163	1	3	27	1	3
100	5	498	138	1	4	28	1	3
150	1	748	185	1	4	41	1	3
150	2	750	177	1	4	40	1	3
150	3	745	194	1	4	39	1	3
150	4	749	181	1	4	40	1	3
150	5	748	220	1	4	47	1	3
200	1	997	269	1	3	51	1	3
200	2	998	258	1	4	51	1	3
200	3	999	250	1	4	56	1	3
200	4	998	258	1	4	51	1	3
200	5	999	287	1	3	65	1	3
250	1	1248	345	1	3	81	1	3
250	2	1248	317	1	4	63	1	3
250	3	1249	335	1	4	69	1	3
250	4	1246	359	1	4	77	1	4
250	5	1249	376	1	3	66	1	3
300	1	1497	425	1	3	93	1	4
300	2	1497	379	1	5	85	1	3
300	3	1500	421	1	3	92	1	3
300	4	1500	481	1	3	88	1	3
300	5	1496	395	1	5	87	1	3

Table 7: Results for the instances with $d = 10$ of benchmark **SW**

$ V $	#	$ E $	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
100	1	250	158	1	4	42	1	3
100	2	250	167	1	5	41	1	3
100	3	250	128	1	4	41	1	2
100	4	250	181	1	5	41	1	2
100	5	250	180	1	6	42	1	2
150	1	375	225	1	6	65	1	7
150	2	375	248	1	4	65	1	6
150	3	375	215	1	4	64	1	3
150	4	375	258	1	3	65	1	5
150	5	375	286	1	4	62	1	2
200	1	500	306	1	5	88	1	11
200	2	500	306	1	6	88	1	8
200	3	500	306	1	6	86	1	6
200	4	500	332	1	6	88	1	9
200	5	500	356	1	8	87	2	16
250	1	625	402	1	8	109	2	15
250	2	625	416	1	16	110	3	19
250	3	625	423	1	7	110	2	13
250	4	625	452	1	5	109	2	13
250	5	625	461	1	8	110	2	16
300	1	750	504	1	10	133	2	16
300	2	750	510	1	11	133	2	19
300	3	750	516	1	10	134	2	16
300	4	750	550	1	9	134	2	16
300	5	750	532	1	16	134	2	16

Table 8: Results for the instances with $d = 5$ of benchmark **Reg**

$ V $	#	$ E $	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
100	1	500	215	1	2	47	1	3
100	2	500	228	1	2	47	1	3
100	3	500	178	1	2	46	1	2
100	4	500	249	1	6	46	1	2
100	5	500	254	1	3	45	1	2
150	1	750	338	1	3	70	1	2
150	2	750	360	1	2	72	1	4
150	3	750	329	1	2	70	1	2
150	4	750	384	1	5	70	1	2
150	5	750	382	1	2	72	1	4
200	1	1000	464	1	2	94	1	3
200	2	1000	488	1	10	94	1	3
200	3	1000	468	1	5	94	1	3
200	4	1000	515	1	7	94	1	3
200	5	1000	517	1	5	94	1	2
250	1	1250	613	1	9	117	2	15
250	2	1250	615	1	12	118	2	14
250	3	1250	607	2	18	117	2	14
250	4	1250	665	1	11	119	1	3
250	5	1250	665	1	12	118	2	17
300	1	1500	747	1	6	140	2	16
300	2	1500	746	1	7	140	2	18
300	3	1500	740	1	12	139	2	18
300	4	1500	786	1	17	140	2	17
300	5	1500	788	2	15	140	2	17

Table 9: Results for the instances with $d = 10$ of benchmark **Reg**

$ V $	#	$ E $	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
100	1	285	85	1	5	21	1	4
100	2	281	85	1	6	21	1	4
100	3	278	81	1	7	19	1	6
100	4	280	90	1	6	21	1	6
100	5	284	101	1	6	21	1	4
150	1	435	137	1	5	31	1	5
150	2	428	134	1	6	31	1	6
150	3	438	128	1	7	32	1	4
150	4	420	128	1	9	29	1	10
150	5	419	130	1	8	29	1	7
200	1	577	172	1	8	41	1	5
200	2	572	194	1	8	40	1	6
200	3	578	145	1	8	38	1	7
200	4	570	182	1	7	36	1	8
200	5	578	186	1	7	37	1	6
250	1	716	223	1	8	51	1	6
250	2	715	216	1	9	49	1	8
250	3	728	207	1	9	44	1	5
250	4	715	218	1	9	47	1	6
250	5	716	209	1	10	47	1	5
300	1	868	279	1	7	57	1	9
300	2	870	266	1	9	55	1	6
300	3	865	268	1	13	61	1	9
300	4	852	284	1	10	57	1	8
300	5	859	259	1	11	56	1	9

Table 10: Results for the instances of benchmark P1a

V	Descr.	E	Weighted			Unweighted		
			UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$	UB	$\mathcal{C}_G(S)$	$\mathcal{C}_G(V \setminus S)$
100	10x10	200	149	1	5	32	1	3
150	10x15	300	208	1	4	41	1	3
200	10x20	400	273	1	4	54	1	3
250	10x25	500	336	1	4	68	1	4
300	15x20	600	384	1	7	80	1	4
100	4x5x5	300	178	1	5	40	1	2
150	5x5x6	450	275	1	3	56	1	2
200	5x5x8	600	373	1	5	67	1	2
250	5x5x10	750	426	1	3	86	1	3
300	5x6x10	900	535	1	8	108	1	6

Table 11: Results for the instances of benchmark **Grid**