Decision Methods and Models

Master's Degree in Computer Science/Mathematics

Roberto Cordone

DI - Università degli Studi di Milano

Schedule:	Thursday 16.30 - 18.30 in the Aula magna of the CS department
	Friday 12.30 - 14.30 in classroom 100
Office hours:	on appointment
E-mail:	roberto.cordone@unimi.it
Web page:	https://homes.di.unimi.it/cordone/courses/2023-mmd/2023-mmd.html
Ariel site:	https://rcordonedmm.ariel.ctu.unimi.it

Aims of the course

A decision problem intrinsically implies two fundamental conditions:

1. freedom, that is the availability of different choices
   (*otherwise, there is no decision*)
2. rationality, that is the existence of preference criteria
   (*otherwise, the choice cannot be motivated*)

The focus is on practical decisions where

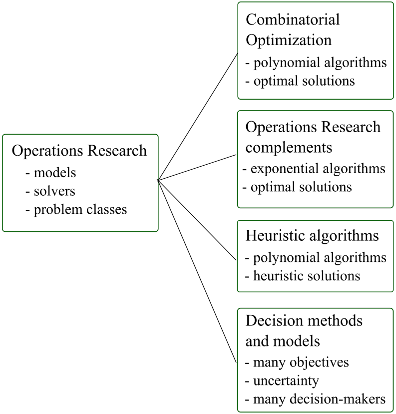
- a large amount of data must be taken into account
- there are many possible choices
- the costs of a wrong choice are high

This course aims to

1. discuss some of the factors that make a decision complicated
2. present the mathematical models to describe complicated situations
3. present the mathematical methods to deal with such situations
4. discuss limits and errors of such models and methods

The *Analytics and Optimization* track

This teaching belongs to the *Analytics and Optimization* track
of the Computer Science course



This gives a specific slant to the presentation of the subject

but the students do not need an Operations Research background

(check the list of prerequisites!)

What about other students?

Students not specifically interested in optimisation could be interested

Decision models are a guide for action in many application fields

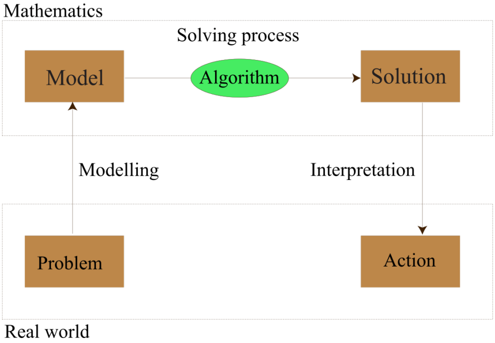
- finance, to define investment strategies
- marketing, to define advertising campaigns or pricing strategies
- natural resource management, to define harvesting strategies
- videogames, to define game strategies
- public or private works, to define the alternative to implement

This is not a technical course on optimisation models and algorithms

- it focuses on the factors that complicate in principle the possibility of building a model
- it presents some impossibility results to build satisfactory models
- it helps to use models as a support in decision-making
  - on the one hand recognising their utility
  - on the other hand using their results with critical sensibility

The modelling approach

The correct strategy is first make a model, then compute, finally decide



This is a classical concept, but nowadays it is strongly enhanced by

- Big Data: huge amounts of precise, structured and cheap data, from which to extract information
- Cloud Computing: pervasive capacity to access and process data
- Business Analytics: a business culture open to the use of models
- new theory: online, stochastic, robust programming, etc. . .

Decision models: terminology

System is the portion of the world affected by the decision

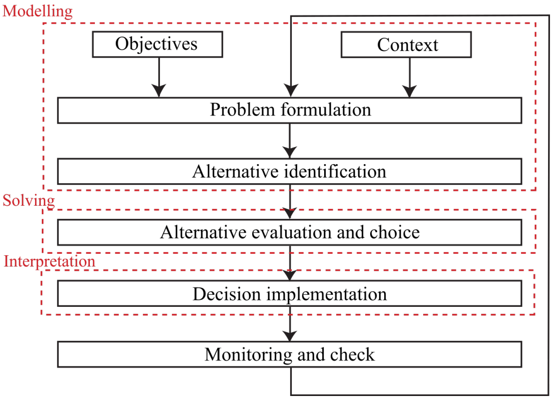
- a system should allow different configurations that combine
  - an alternative or solution, describing all its controllable aspects
  - a scenario or outcome, describing all its uncontrollable aspects
- each configuration is associated to an impact, that describes all aspects which are relevant for the decision
- decision-maker or stakeholder is everybody who contributes to the choice of an alternative
- the preference describes the relative satisfaction between impacts

A decision problem requires to choose an alternative

- so as to move the system into a configuration
- such that the decision-makers prefer its associated impact to those of the other configurations
- keeping into account that the actual configuration depends on the alternative, but also on the scenario

Feedback loops in the decision process

The decision process often occurs by an iterative correction approach



H. A. Simon, "Models of bounded rationality" (1982)

Phases of the decision process

1. Problem formulation: delineate the system, the decision-makers, the scenarios (Context), the impacts and preferences (Objectives)
2. Alternative identification: define the set of feasible alternatives
3. Alternative evaluation and choice:
    - evaluate the impact associated to each alternative and scenario
    - select an alternative based on the preferences of the decision-makers
4. Decision implementation:
   apply in practice, or simulate, the alternative selected
5. Monitoring and check:
    - observe (or simulate) the outcomes of the choice
    - if unsatisfactory, correct and repeat the process with new or modified alternatives, scenarios, decision-makers, objectives, models, methods

Why a formal approach?

A formal approach allows to

1. predict in a more certain and precise way the impact of a decision using descriptive models instead of intuition and experience
2. accelerate the decision process using informatic tools
3. consider a much larger number of possible alternatives
4. clarifying and certifying the decision process
   - explicitating the assumptions made on alternatives, scenarios, preferences and decision-makers and their relative weight
     Sometimes it is required to prove that a decision was based on data
   - guarantee the repeatability of the process
   - allow focused changes to the process without restarting from scratch

The formal approach is based on

- models, to take decisions and to predict their outcomes
- methods, to build models, to solve them (algorithms) and to interpret their results

Prescriptive and descriptive models

Decision models usually combine

1. prescriptive models that
   - receive impacts and preferences in input
   - return a suggested alternative in output

   If this is the case, you should do that

2. descriptive/predictive models
   - receive the system, an alternative and a scenario in input
   - return an impact in output

   If you do this and if this happens, you will obtain that

The descriptive ones range from trivial (e.g.: $c_{\text{tot}} = c_1 x_1 + c_2 x_2$)
to very refined (e.g.: simulation models, differential equations, etc...)

The two families of models can have subtle and complex interactions
For instance:

- a model prescribes a decision (*close or open streets*)
- based on models that describe a system (*the amount of traffic*)
- including decisions prescribed by models (*satellite navigators*)

Complicated decision problems: examples (1)

1. the search for a parking before a meeting:
   - system is the local street network,
     with the set of all potential parking places
   - alternative is every possible trajectory of the car (path and schedule)
   - scenario is every possible distribution of the free parking places
     over space and time
   - impact are the driving time and the walking time after parking
   - decision-maker is the driver (or also the passengers?)
2. the thermostat regulation:
   - system is the classroom
   - alternative is the position of the thermostat knob
   - scenario is the external temperature
     and the exposition of the classroom to the sun
   - impact is the internal temperature of the classroom
     (but its humidity is also relevant)
   - decision-makers are the people dwelling in the classroom
     (all of them, or just the teacher?)

Complicated decision problems: examples (2)

3. buying a car:
   - system is the local market of cars, petrol, repairs, etc...
   - alternative is the car bought (possibly, more than one or none!)
   - scenario are the stock and the prices of the car dealers, the occurrence of accidents, the prices of petrol and car repairs, etc...
   - impact are the characteristics of the car throughout its life-cycle
   - decision-maker is the buyer (and possibly other family members)
4. playing a Risiko round:
   - system is the map with the distribution of territories, armies, cards
   - alternative are the attacking and defending territories and the number of attacking and defending armies in each round
   - scenario is the outcome of the dice at each attack
   - impact is the number of armies lost by each player
   - decision-makers are the players

What makes a decision problem complicated?

A decision problem can be complicated due to

1. an insufficient model of the system:
2. complicating features of the model
   - complex preference structure, insufficient to define an optimum
   - uncertain environment: the impact depends also on an unknown scenario
   - multiple decision-makers, with potentially conflicting preferences
3. a computationally complex model
   - everything is clearly defined, but no efficient algorithm is known to solve the problem

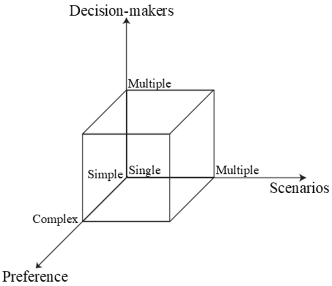
This course focuses on the second source of complexity

Classification of decision problems

The three main complexity sources for decision problems are

1. preference structure: simple or complex
2. uncertainty: a single scenario or many
3. decision-makers: a single decision-maker or many

give rise to $2^3 = 8$ families of decision problems



Syllabus

We consider four basic families of prescriptive models

1. simple preference, a single scenario and a single decision-maker
    - mathematical programming
    - multiple-attribute utility theory
2. complex preference, a single scenario and a single decision-maker:
    - Paretian preferences
    - weak rationality models (*AHP* and *ELECTRE* methods)
3. simple preference, multiple scenarios and a single decision-maker:
    - decisions in conditions of ignorance (robust programming)
    - decisions in conditions of risk (stochastic programming)
4. simple preference, a single scenario and multiple decision-makers:
    - independent decision-makers (game theory)
    - cooperating decision-makers (group decisions)

The lecture notes also consider some important families of descriptive models (*not required for the exam*)