Solved exercises for the course of

Foundations of Operations Research

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Project planning

The preparation of the apple pie has long been a tradition for the Smiths family. First, the weight of the ingredients has to be determined: flour, sugar, butter, eggs, apples, cream. Then, we must melt down the butter and add it to a mixture of flour, sugar, and eggs. Apples must be added to this new mixture, once they have been peeled and cut into thin slices. The mixture can then be cooked, in the already heatened oven. It is adviced to whip the cream only after the apple slices have been added to the mixture. Once the cake is cooked, the cream is used to garnish it.

The following table reports the time needed for each activity.

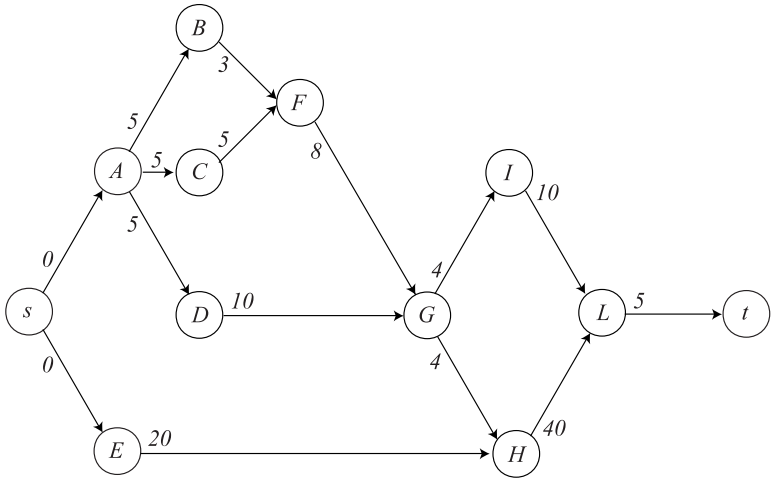
	Activity	Time (min)
A	Weight the ingredients	5
B	Meld the butter down	3
C	Mix flour, eggs, and sugar	5
D	Peel and cut the apples into slices	10
E	Heaten the oven	20
F	Add butter to the mixture	8
G	Add apples to the mixture	4
H	Cook the mixture in the oven	40
I	Whip the cream	10
L	Garnish	5

Draw the graph of the priorities, determine the earliest and latest start times, $\tau_i^{\min}$ and $\tau_i^{\max}$, by which each activity must begin and end so as to minimize the overall project duration, and identify the critical paths. Draw the "at the earliest" Gantt diagram.

Solution

From the textual description of the problem, it is possible to derive the following precedence relations: $A \preceq B$, $A \preceq C$, $A \preceq D$, $B \preceq F$, $C \preceq F$, $D \preceq G$, $F \preceq G$, $E \preceq H$, $G \preceq H$, $G \preceq I$, $H \preceq L$, $I \preceq L$.

The following figure represents the priority graph, in the "activity-on-node" format: the starting time of each activity corresponds to a node (plus additional nodes s and t for the start and end of the whole project), each precedence relation corresponds to an arc (plus the precedences between s and all activity nodes, and between all activity nodes and t, from which we remove the redundant ones). The cost of each arc is the duration of the activity associated to its tail node.



The alphabetic order is one of the possible topological orders for the nodes of this graph. We adopt it while applying the Critical Path Method. The earliest start times and latest start times are reported in the following figure. The critical activities are A, C, F, G, H and L. They identify a single critical path, reported in red in the figure.